II Rotational

IN THE CASE OF ROTATIONS WE CONSIDER THE ELECTRIC FIELD IN THE Z-DIRECTION

$$\underset{\sim}{E}(t) = |E_0|\hat{k}\cos(\omega t)$$

THEREFORE

$$\underset{\sim}{\mu}\cdot\underset{\sim}{E} \simeq |E_0|\mu_z(z)\cos(\omega t)$$

AND

$$\mu_z(z) = \mu_z^0 + \left.\frac{d\mu_z}{dz}\right|_0 z + \frac{1}{2}\left.\frac{d^2\mu_z}{dz^2}\right|_0 z^2 + \ldots$$

NOW

$$z = r\cos\theta \sim r\,Y_1^0(\theta,\varphi)$$

$$\hat{h}_{10} \longrightarrow \langle l', m'|\hat{h}(z)|l, m\rangle$$

$$\langle l', m'|\,|E_0|\,g\,\hat{\mu}_z|\,l, m\rangle = g|E_0|\,\langle l', m'|\hat{\mu}_z|\,l, m\rangle$$

IF $l' \neq l$ AND $m' \neq m$

WE GET TO LOWEST APPROXIMATION

$$\langle l', m' | \hat{z} | l, m \rangle = \langle l', m' | r\cos\theta | l, m \rangle$$

$$= r \langle l', m' | \cos\theta | l, m \rangle$$

$$\sim r \langle l', m' | Y_1^0 | l, m \rangle$$

$$= r \int_0^{\pi} \int_0^{2\pi} Y_{l'}^{m'*}(\theta,\varphi) \left(Y_1^0(\theta,\varphi) \, Y_l^m(\theta,\varphi) \right) \sin\theta \, d\theta \, d\varphi$$

BUT

$$Y_1^0 Y_l^m \sim a \, Y_{l+1}^m + b \, Y_{l-1}^m$$

$$\langle l', m' | \hat{z} | l, m \rangle \sim a \langle l', m' | l+1, m \rangle +$$

$$b \langle l', m' | l-1, m \rangle$$

Integration over $\varphi \Rightarrow \quad m' = m$

AND $\quad \Delta l = \pm 1$ FOR ALLOWED TRANSITIONS

$$\frac{\partial^2 \Psi}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 \Psi}{\partial t^2} \qquad \text{WAVE EQ.}$$

$$\Psi(x,t) = \phi(x)\, e^{i2\pi\nu t}$$

$$e^{i2\pi\nu t}\frac{d^2\phi}{dx^2} = \frac{(i2\pi\nu)^2}{v^2} e^{i2\pi\nu t}\phi$$

(I) $$v \longrightarrow \frac{E}{\sqrt{2m(E-V)}}$$

OPTICS $\longleftrightarrow$ CLASSICAL MECHANICS

$$e^{i2\pi\nu t}\frac{d^2\phi}{dx^2} = -\frac{(2\pi)^2\nu^2[2m(E-V)]}{E^2}\phi\, e^{i2\pi\nu t}\quad \frac{h^2}{h^2}$$

(II) $$E = h\nu \Rightarrow \nu = \frac{E}{h}$$

$$e^{i\frac{E}{\hbar}t}\frac{d^2\phi}{dx^2} = -\frac{2m}{\hbar^2}(E-V)\phi\, e^{i\frac{E}{\hbar}t}$$

$$-\frac{\hbar^2}{2m}\left[e^{i\frac{E}{\hbar}t}\frac{d^2\phi}{dx^2}\right] + V e^{i\frac{E}{\hbar}t}\phi = E e^{i\frac{E}{\hbar}t}\phi$$

$$= \frac{\hbar}{i}\frac{\partial}{\partial t}\left[e^{i\frac{E}{\hbar}t}\phi\right]$$

$$\boxed{-\frac{\hbar^2}{2m}\frac{\partial^2 \Psi}{\partial x^2} + V\Psi = \frac{\hbar}{i}\frac{\partial \Psi}{\partial t}}$$

TISE

$$-\frac{\hbar^2}{2m}\frac{d^2\phi}{dx^2} + V\phi = E\phi$$

$$(\hat{K}+\hat{V})\phi = \hat{H}\phi = E\phi$$

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + \hat{V}$$

FOR HYDROGEN ATOM

$$\boxed{\hat{V}(\underset{\sim}{r}) = +\frac{Z e^* q}{4\pi\epsilon_0 |\underset{\sim}{r}|} = -\frac{Z|e|^2}{4\pi\epsilon_0 r}}$$

$$\hat{K} = -\frac{\hbar^2}{2\mu r^2}\frac{\partial}{\partial r} r^2 \frac{\partial}{\partial r} + \frac{1}{2I}\hat{L}^2$$

$$\phi(r,\theta,\varphi) = R(r)\, Y_l^m(\theta,\varphi)$$

$$-\frac{\hbar^2}{2\mu r^2}\frac{d}{dr}\left[r^2\frac{dR}{dr}\right] + \left[\frac{\hbar^2 l(l+1)}{2I} - \frac{|e|^2 Z}{4\pi\epsilon_0 r}\right]R = ER$$

$$E_n = -\frac{\mu|e|^4}{8\epsilon_0^2 h^2}\frac{1}{n^2}$$

Hydrogenlike Atoms

$$\Psi_{n\ell m}(r,\theta,\varphi) = \cancel{\phi_{n\ell}}\, R_{n\ell}(r)\, Y_{\ell}^{m}(\theta,\varphi)$$

$$Y_{\ell}^{m}(\theta,\varphi) = (-1)^{m}\sqrt{\frac{(2\ell+1)(\ell-|m|)!}{4\pi(\ell+|m|)!}}\, P_{\ell}^{|m|}(\cos\theta)\, e^{im\varphi}$$

$$\int_0^{2\pi}\int_0^{\pi} Y_{\ell'}^{m'*}(\theta,\varphi)\, Y_{\ell}^{m}(\theta,\varphi)\sin\theta\, d\theta\, d\varphi = \delta_{\ell,\ell'}\,\delta_{m,m'}$$

FOR A GIVEN $n \rightarrow \ell = 0, 1, 2, \dots, n-1$ AND $m = 0, \pm 1, \dots, \pm \ell$

(Arfken)

$$R_{n\ell}(r) = \sqrt{\left(\frac{2Z}{na_0}\right)^3 \frac{(n-\ell-1)!}{2n[(n+\ell)!]^3}}\,(\alpha r)^{\ell}\, e^{-\alpha r/2}\, L_{n-\ell-1}^{2\ell+1}(\alpha r)$$

$$\alpha = \frac{2Z}{na_0}$$

$$\int_0^{\infty} e^{-x} x^{k} L_n^{k}(x)\, L_m^{k}(x)\, dx = \frac{(n+k)!}{n!}\,\delta_{m,n}$$

$$\int_0^{\infty} e^{-x} x^{k+1} L_n^{k}(x)\, L_n^{k}(x) = \frac{(n+k)!}{n!}(2n+k+1)$$