LV

$$\frac{dR}{dt} = k_o R - k_d R W$$

$$\frac{dW}{dt} = \alpha k_d R W - k_f W$$

1920

$$\frac{dR}{dt} = R\left(k_o - k_d W\right) \equiv F(R,W)$$

$$\frac{dW}{dt} = W\left(\alpha k_d R - k_f\right) \equiv G(R,W)$$

SS

$\bar{W} = \frac{k_o}{k_d}$

$\bar{R} = \frac{k_f}{\alpha k_d}$

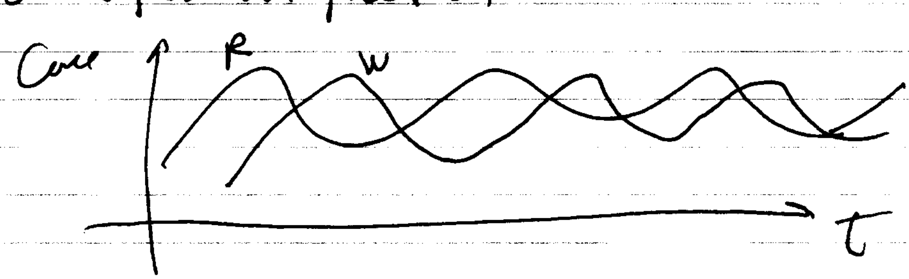
$$\Rightarrow \quad \frac{dR}{dt} = 0 = \bar{R}\left(k_o - k_d \bar{W}\right) \cancel{= F_\ell}$$

$$\frac{dW}{dt} = 0 = \bar{W}\left(\alpha k_d \bar{R} - k_f\right)$$

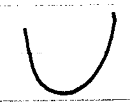
(0, 0) Total extinction

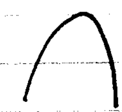
$(\bar{R}, \bar{W})$ coexistence

But after integration



SS are not stable

Stability stable 

unstable 

$$\text{Jacobian} = \begin{pmatrix} \left.\frac{dF}{dR}\right|_{\bar R \bar w} ; & \left.\frac{dF}{dw}\right|_{\bar R \bar w} \\ \left.\frac{dG}{dR}\right|_{\bar R \bar w} ; & \left.\frac{dG}{dw}\right|_{\bar R \bar w} \end{pmatrix} \equiv J$$

Look for eigenvalues of J!

Real $\lambda > 0$ UNS

Real $\lambda < 0$ STA

$$\begin{pmatrix} k_0 - k_d \bar w ; & -k_d \bar R \\ \alpha k_d \bar w ; & \alpha k_d \bar R - k_f \end{pmatrix}$$

$$J_{00} = \begin{pmatrix} k_0 & 0 \\ 0 & -k_f \end{pmatrix}$$

$\lambda_1 = k_0 > 0$ UNSTABLE

$\lambda_2 = -k_f < 0$ STABLE

LV (1920)

$$J_{11} = k_0 - k_d W \qquad J_{12} = -k_d R$$

$$J_{21} = \alpha k_d W \qquad J_{22} = \alpha k_d R - k_f$$

(0, 0)

$$\det \begin{vmatrix} k_0 - \lambda & 0 \\ 0 & -k_f - \lambda \end{vmatrix} = 0 = (-1)(k_f + \lambda)(k_0 - \lambda)$$

$$\Rightarrow \quad \lambda_1 = -k_f \qquad \text{STABLE} \quad Re(\lambda) < 0$$
$$\lambda_2 = k_0 \qquad \text{UNSTABLE} \quad Re(\lambda) > 0$$

$(\bar{R}, \bar{\omega})$

$$\det \begin{vmatrix} 0 - \lambda & -k_d \bar{R} \\ \alpha k_d \bar{\omega} & 0 - \lambda \end{vmatrix} = 0 = \lambda^2 + \alpha k_d^2 \bar{R} \bar{\omega}$$

$$\lambda^2 = -\alpha k_d^2 \bar{R} \bar{\omega} = -\alpha k_d k_0 \bar{R}$$

$$\lambda^2 = -\cancel{\alpha k_d} k_0 \frac{k_f}{\cancel{\alpha k_d}} = -k_0 k_f$$

$$\lambda_{\pm} = \pm i \sqrt{k_0 k_f} \qquad Re(\lambda) = 0$$

MARGINAL STABILITY

$$J_{\bar{R}\bar{\omega}} = \begin{pmatrix} 0 & -k_d \bar{R} \\ \alpha k_d \bar{\omega} & 0 \end{pmatrix}$$

$$\det \begin{vmatrix} -\lambda & -k_d \bar{R} \\ \alpha k_d \bar{\omega} & -\lambda \end{vmatrix} = 0$$

$$\lambda^2 + \alpha k_d^2 \bar{R} \bar{\omega} = 0$$

$$\lambda^2 = - \alpha k_d^2 \bar{R} \bar{\omega} = -\alpha k_d k_o \bar{R}$$

$$\lambda^2 = - k_o k_f$$

$$\boxed{\lambda = \pm i \sqrt{k_o k_f}}$$ No Real part!

Marginal stability ⇒ OSC FOR ANY initial condition.

Glycolisis ~ 1960's

$$\text{Glucose} \xrightarrow[\text{hexokinase}]{} \text{G-6-P} \rightleftharpoons \overset{(A)}{\text{F-6-P}} \xrightarrow[\text{hexofructose kinase}]{\oplus} \overset{(B)}{\text{F-6-bP}} \Rightarrow$$

$$\boxed{\begin{aligned} \frac{dA}{dt} &= r - kAB \\ \frac{dB}{dt} &= kAB - \frac{VB}{K+B} \end{aligned}}$$