$$\Delta S = c_p \ln\left\{1 + \left(\frac{\delta}{2T_c}\right)^2\right\} > 0$$

b) $T_f = T_h + \delta > T_h > T_c > T_c - \delta$

$$\left(\frac{T_h+\delta}{T_h}\right)\left(\frac{T_c-\delta}{T_c}\right) = \left(1+\frac{\delta}{T_c}\right)\left(1-\frac{\delta}{T_c}\right)$$

$$= 1 - \left(\frac{\delta}{T_c} - \frac{\delta}{T_c}\right) - \frac{\delta^2}{T_c^2}$$

$$= 1 - \delta\left(\frac{1}{T_c} - \frac{1}{T_h}\right) - \frac{\delta^2}{T_c^2} < 1$$

$$\Delta S_B = c_p \ln\left(\frac{T_f}{T_h}\frac{T_f'}{T_c}\right) < 0$$

Clausius

$$dU = \text{đ}q - P_{ext}\, dV$$

$$dU = \text{đ}q_{rev} - P\, dV$$

$$dU = T\, dS - P\, dV$$

$U(S, V)$

$$\left(\frac{\partial U}{\partial S}\right)_T = T$$

$$\left(\frac{\partial U}{\partial V}\right)_S = -P$$

$$\boxed{\left(\frac{\partial T}{\partial V}\right)_S = -\left(\frac{\partial P}{\partial S}\right)_T}$$

$$\text{đ}q_{rev} - \text{đ}q = (P - P_{ext})\, dV > 0$$

$$T\, dS > \text{đ}q$$

$$\boxed{dS > \frac{\text{đ}q}{T}}$$

CLAUSIUS

$$dS \geq \frac{dq}{T}$$

$$\Delta S \geq \int \frac{dq}{T}$$

FOR AN ISOLATED SYSTEM

$$dq = 0$$

$$\Delta S \geq 0$$

EQUALITY FOR REVERSIBLE PROCESS

$$\Delta S_U \geq 0$$

$$\Delta S_{SYS} + \Delta S_{SUR} \geq 0$$

LARGE SURR SUCH THAT T_{SUR} = CONST

$$\Delta S_{SUR} = \frac{q_{SUR}}{T_{SUR}} = \frac{-q_{SYS}}{T_{SUR}}$$

$$dS = \frac{1}{T} dU + \frac{p}{T} dV$$

$S(T, V)$

$$dS = \left(\frac{\partial S}{\partial T}\right)_V dT + \left(\frac{\partial S}{\partial V}\right)_T dV$$

$$dU = C_v \, dT + \left(\frac{\partial U}{\partial V}\right)_T dV$$

$$dS = \frac{C_v}{T} dT + \frac{1}{T}\left[p + \left(\frac{\partial U}{\partial V}\right)_T\right] dV$$

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_V = \frac{C_v}{T}}$$

$$\boxed{\left(\frac{\partial S}{\partial V}\right)_T = \frac{1}{T}\left[p + \left(\frac{\partial U}{\partial V}\right)_T\right]}$$

$$\left(\frac{\partial}{\partial V}\left(\frac{\partial S}{\partial T}\right)_V\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V$$

$$\boxed{\left(\frac{\partial}{\partial V}\frac{C_V}{T}\right)_T} = \frac{1}{T}\left(\frac{\partial}{\partial V}C_V\right)_T$$

$$= \boxed{\frac{1}{T}\left(\frac{\partial}{\partial V}\left(\frac{\partial U}{\partial T}\right)_V\right)_T}$$

$$\boxed{\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial V}\right)_T\right)_V} = \left(\frac{\partial}{\partial T}\frac{1}{T}\left(p+\left(\frac{\partial U}{\partial V}\right)_T\right)\right)_V$$

$$= -\frac{1}{T^2}\left[p+\left(\frac{\partial U}{\partial V}\right)_T\right]$$

$$+\frac{1}{T}\left[\frac{\partial}{\partial T}\left(p+\left(\frac{\partial U}{\partial V}\right)_T\right)\right]_V$$

$$= \boxed{-\frac{1}{T^2}\left[p+\left(\frac{\partial U}{\partial V}\right)_T\right]+\frac{1}{T}\left(\frac{\partial p}{\partial T}\right)_V + \frac{1}{T}\left(\frac{\partial}{\partial T}\left(\frac{\partial U}{\partial V}\right)_T\right)_V}$$

$$\cancel{\frac{1}{T}}\left(\frac{\partial p}{\partial T}\right)_V - \frac{1}{T^{\cancel{2}}}\left[p + \left(\frac{\partial U}{\partial V}\right)_T\right] = 0$$

$$\boxed{T\left(\frac{\partial p}{\partial T}\right)_V - p = \left(\frac{\partial U}{\partial V}\right)_T} \quad !$$

THEREFORE

$$\left(\frac{\partial S}{\partial V}\right)_T = \left(\frac{\partial p}{\partial T}\right)_V = -\left(\frac{\partial T}{\partial V}\right)_p\left(\frac{\partial V}{\partial p}\right)_T$$

$$= -\frac{\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T}{\frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p} = \frac{\alpha}{\kappa}$$

$$\Rightarrow \boxed{dS = \frac{C_V}{T}dT + \frac{\alpha}{\kappa}dV} \quad S(T,V)$$

$$\Delta S = \int_{T_i}^{T_f}\frac{C_V}{T}dT + \int_{V_i}^{V_f}\frac{\alpha}{\kappa}dV$$

$S(T, P)$

$$dS = \left(\frac{\partial S}{\partial T}\right)_P dT + \left(\frac{\partial S}{\partial P}\right)_T dP$$

a) $H = U + PV$

b) $dH = dU + PdV + VdP$

c) $dU = TdS - PdV$

$$dH = TdS + VdP$$

c) $dS = \frac{1}{T} dH - \frac{V}{T} dP$

$$dH = C_p dT + \left(\frac{\partial H}{\partial P}\right)_T dP$$

$$dS = \frac{C_p}{T} dT + \frac{1}{T}\left[\left(\frac{\partial H}{\partial P}\right)_T - V\right] dP$$

$$\boxed{\left(\frac{\partial S}{\partial T}\right)_P = \frac{C_p}{T}} \; ; \; \left(\frac{\partial S}{\partial P}\right)_T = \frac{1}{T}\left[\left(\frac{\partial H}{\partial P}\right)_T - V\right]$$

$$\left(\frac{\partial}{\partial p}\left(\frac{\partial S}{\partial T}\right)_p\right)_T = \left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial p}\right)_T\right)_p$$

$$\left(\frac{\partial}{\partial p}\left(\frac{\partial S}{\partial T}\right)_p\right)_T = \left(\frac{\partial}{\partial p}\ \frac{C_p}{T}\right)_T = \frac{1}{T}\left(\frac{\partial C_p}{\partial p}\right)_T = \frac{1}{T}\left(\frac{\partial}{\partial p}\left(\frac{\partial H}{\partial T}\right)_p\right)_T$$

$$\left(\frac{\partial}{\partial T}\left(\frac{\partial S}{\partial p}\right)_T\right)_p = \left(\frac{\partial}{\partial T}\left(\frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right]\right)\right)_p$$

$$= -\frac{1}{T^2}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right] + \frac{1}{T}\left(\frac{\partial}{\partial T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right]\right)_p$$

$$= -\frac{1}{T^2}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right] - \frac{1}{T}\left(\frac{\partial V}{\partial T}\right)_p + \frac{1}{T}\left(\frac{\partial}{\partial T}\left(\frac{\partial H}{\partial p}\right)_T\right)_p$$

$$\Rightarrow \left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{T}\left[\left(\frac{\partial H}{\partial p}\right)_T - V\right]$$

$$\boxed{\left(\frac{\partial H}{\partial p}\right)_T = V - T\left(\frac{\partial V}{\partial T}\right)_p}$$

$$\boxed{\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p = -V\beta}$$

$$\boxed{dS = \frac{C_p}{T}dT - V\beta\, dP} \quad S(T,P)$$

$$\Delta S = \int_{T_i}^{T_f} \frac{C_p}{T}dT - \int_{P_i}^{P_f} V\beta\, dP$$

$$\Delta S_{SYS} + \Delta S_{SURR} \geq 0$$

$$\Delta S_{SYS} - \frac{q_{SYS}}{T_{SURR}} \geq 0$$

CONST P $\Longrightarrow$ $q_{SYS} = \Delta H_{SYS}$

$$\Delta S_{SYS} - \frac{\Delta H_{SYS}}{T_{SURR}} \geq 0$$

CONST T $\Longrightarrow$ $T_{SYS} = T_{SURR}$

$$\boxed{\Delta H_{SYS} - T_{SYS}\,\Delta S_{SYS} \leq 0}$$ (T, P) CONST

$$\Delta G_{SYS} \leq 0$$