

## Problem Set 1

P2

a)

$$P_{H_2} = \frac{n_{H_2}RT}{V} = \frac{1.00/2.016 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 6.24 \times 10^5 \text{ Pa}$$

$$P_{O_2} = \frac{n_{O_2}RT}{V} = \frac{1.00/32.00 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 3.90 \times 10^4 \text{ Pa}$$

$$P_{\text{total}} = 6.57 \times 10^5 \text{ Pa}$$

$$\text{mol \% } H_2 = 100 \times \frac{\text{mol } H_2}{\text{mol } H_2 + \text{mol } O_2} = 100 \times \frac{1.00/2.016}{1.00/2.016 + 1.00/32.00} = 94.1\%$$

$$\text{mol \% } O_2 = 100 \times \frac{\text{mol } O_2}{\text{mol } H_2 + \text{mol } O_2} = 100 \times \frac{1.00/32.00}{1.00/2.016 + 1.00/32.00} = 5.9\%$$

b)

$$P_{N_2} = \frac{n_{N_2}RT}{V} = \frac{1.00/28.02 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 4.45 \times 10^4 \text{ Pa}$$

$$P_{O_2} = \frac{n_{O_2}RT}{V} = \frac{1.00/32.00 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 3.90 \times 10^4 \text{ Pa}$$

$$P_{\text{total}} = 8.35 \times 10^4 \text{ Pa}$$

$$\text{mol \% } N_2 = 100 \times \frac{\text{mol } N_2}{\text{mol } N_2 + \text{mol } O_2} = 100 \times \frac{1.00/28.02}{1.00/28.02 + 1.00/32.00} = 53.3\%$$

$$\text{mol \% } O_2 = 100 \times \frac{\text{mol } O_2}{\text{mol } N_2 + \text{mol } O_2} = 100 \times \frac{1.00/32.00}{1.00/28.02 + 1.00/32.00} = 46.7\%$$

c)

$$P_{NH_3} = \frac{n_{NH_3}RT}{V} = \frac{1.00/17.03 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 7.32 \times 10^4 \text{ Pa}$$

$$P_{CH_4} = \frac{n_{CH_4}RT}{V} = \frac{1.00/16.04 \text{ mol} \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{2.00 \times 10^{-3} \text{ m}^3} = 7.77 \times 10^4 \text{ Pa}$$

$$P_{\text{total}} = 1.51 \times 10^5 \text{ Pa}$$

$$\text{mol \% } NH_3 = 100 \times \frac{\text{mol } NH_3}{\text{mol } NH_3 + \text{mol } CH_4} = 100 \times \frac{1.00/17.03}{1.00/17.03 + 1.00/16.04} = 48.5\%$$

$$\text{mol \% } O_2 = 100 \times \frac{\text{mol } CH_4}{\text{mol } NH_3 + \text{mol } CH_4} = 100 \times \frac{1.00/16.04}{1.00/17.03 + 1.00/16.04} = 51.5\%$$

**P3**

Expressing the ideal gas law in the form  $PV = R(t + a) = m(t + a)$ ,

$$m = \frac{\Delta PV}{\Delta t} = \frac{(31.18 - 22.98) \text{ l atm mol}^{-1}}{(100 - 0)^\circ \text{ C}} = 0.08200 \text{ atm mol}^{-1} \text{ }^\circ \text{ C}^{-1} = R$$

$$a = \frac{PV}{R} - t = \frac{31.18 \text{ l atm mol}^{-1}}{0.08200 \text{ atm mol}^{-1} \text{ }^\circ \text{ C}^{-1}} - 100^\circ \text{ C} = 280.2^\circ \text{ C}$$

**P6**

a)

$$\text{mol \% N}_2 = 100 \times \frac{P_{\text{N}_2}}{P_{\text{total}}} = 100 \times \frac{0.78 \times 0.88 \text{ atm}}{1 \text{ atm}} = 68.6\%$$

$$\text{mol \% O}_2 = 100 \times \frac{P_{\text{O}_2}}{P_{\text{total}}} = 100 \times \frac{0.21 \times 0.88 \text{ atm}}{1 \text{ atm}} = 18.5\%$$

$$\text{mol \% Ar} = 100 \times \frac{P_{\text{Ar}}}{P_{\text{total}}} = 100 \times \frac{0.01 \times 0.88 \text{ atm}}{1 \text{ atm}} = 0.9\%$$

$$\text{mol \% H}_2\text{O} = 100 \times \frac{P_{\text{H}_2\text{O}}}{P_{\text{total}}} = 100 \times \frac{0.12 \text{ atm}}{1 \text{ atm}} = 12.0\%$$

b)

$$P_{\text{H}_2\text{O}}V = \frac{n_{\text{H}_2\text{O}}RT}{V}$$

$P'_{\text{H}_2\text{O}}V' = P_{\text{H}_2\text{O}}V$  where the primed quantities refer to 100% RH

$$V' = \frac{P_{\text{H}_2\text{O}}V}{P'_{\text{H}_2\text{O}}} = \frac{0.12 \text{ atm} \times 20.0 \text{ L}}{0.197 \text{ atm}} = 12.2 \text{ L}$$

c)

If all the water remained in the gas phase, the partial pressure of water at a total pressure of 200 atm would be

$$P_{\text{H}_2\text{O}} = P_{\text{total}} \times \text{mol fraction H}_2\text{O} = 200 \text{ atm} \times 0.12 = 24.0 \text{ atm}$$

However, the partial pressure of water cannot be greater than 0.197 atm, and the excess will condense. The fraction that condenses is given by

$$\text{fraction condensed} = \frac{24.0 \text{ atm} - 0.197 \text{ atm}}{24.0 \text{ atm}} = 0.992$$

**P8**

$$n_{N_2} = \frac{\rho_{N_2} V_{wg}}{M_{N_2}}$$

$$V_{N_2} = \frac{n_{N_2} RT}{P} = \frac{\rho_{N_2} V_{wg} RT}{M_{N_2} P}$$

$$= \frac{875.4 \text{ g L}^{-1} \times 2.00 \times 10^{-3} \text{ L} \times 8.2057 \times 10^{-2} \text{ L atm K}^{-1} \text{ mol}^{-1} \times (273.15 + 18.5) \text{ K}}{28.01 \text{ g mol}^{-1} \times 1 \text{ atm}}$$

$$= 1.50 \text{ L}$$

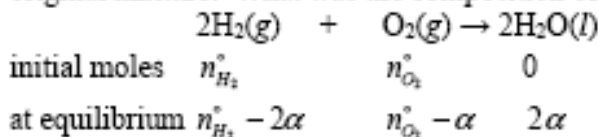
**P10**

The reaction is  $\text{C}_3\text{H}_6(\text{g}) + 5\text{O}_2(\text{g}) \rightarrow 3\text{CO}_2(\text{g}) + 4\text{H}_2\text{O}(\text{g})$ . If  $m$  mol of propene are present initially, there must be  $5m$  mol of  $\text{O}_2$ . After the reaction is complete, there are  $3m$  mol of  $\text{CO}_2$ ,  $4m$  mol of  $\text{H}_2\text{O}$ , and  $m$  mol of  $\text{O}_2$ .

$$x_{\text{CO}_2} = \frac{3m}{17m} = 0.176; \quad x_{\text{H}_2\text{O}} = \frac{4m}{17m} = 0.235; \quad x_{\text{O}_2} = \frac{m}{17m} = 0.0588$$

**P12**

ORIGINAL Mixture. What was the composition of the original mixture in mole percent?



If the  $\text{O}_2$  is completely consumed,  $n_{\text{O}_2}^0 - \alpha = 0$  or  $\alpha = n_{\text{O}_2}^0$ . The number of moles of  $\text{H}_2$  remaining is  $n_{\text{H}_2}^0 - 2\alpha = n_{\text{H}_2}^0 - 2n_{\text{O}_2}^0$ .

Let  $P_1$  be the initial total pressure and  $P_2$  be the total pressure after all the  $\text{O}_2$  is consumed.

$$P_1 = (n_{\text{H}_2}^0 + n_{\text{O}_2}^0) \frac{RT}{V} \quad \text{and} \quad P_2 = (n_{\text{H}_2}^0 - 2n_{\text{O}_2}^0) \frac{RT}{V}$$

Dividing the second equation by the first

$$\frac{P_2}{P_1} = \frac{n_{\text{H}_2}^0}{n_{\text{H}_2}^0 + n_{\text{O}_2}^0} - 2 \frac{n_{\text{O}_2}^0}{n_{\text{H}_2}^0 + n_{\text{O}_2}^0} = x_{\text{H}_2} - 2x_{\text{O}_2} = 1 - x_{\text{O}_2} - 2x_{\text{O}_2} = 1 - 3x_{\text{O}_2}$$

$$x_{\text{O}_2} = \frac{1}{3} \left( 1 - \frac{P_2}{P_1} \right) = \frac{1}{3} \left( 1 - \frac{0.400 \text{ atm}}{1.00 \text{ atm}_1} \right) = 0.20; \quad x_{\text{H}_2} = 0.80$$

**P14**

The total number of Ar atoms in the atmosphere is  $N_{Ar} = \int_0^{\infty} \tilde{N}_{Ar} A dz$  where  $\tilde{N}_{Ar}$  is the number of Ar atoms per  $\text{m}^3$  at the surface of the earth.  $\tilde{N}_{Ar}$  is given by

$$\tilde{N}_{Ar} = \frac{N_A P_{Ar}}{RT} = \frac{6.023 \times 10^{23} \times 0.0100 \times 1 \times 10^5 \text{ Pa}}{8.314 \text{ J K}^{-1} \text{ mol}^{-1} \times 300 \text{ K}} = 2.41 \times 10^{23} \text{ m}^{-3}$$

The total number of Ar atoms in the atmosphere is

$$\begin{aligned} N_{Ar} &= \int_0^{\infty} \tilde{N}_{Ar} A dz = \tilde{N}_{Ar} \int_0^{\infty} e^{-\frac{M_{Ar}gz}{RT}} A dz = \tilde{N}_{Ar} A \frac{RT}{M_{Ar}g} \\ &= \frac{2.41 \times 10^{23} \text{ m}^{-3} \times 4\pi \times (6.37 \times 10^6 \text{ m})^2 \times 8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{39.9 \times 10^{-3} \text{ kg} \times 9.81 \text{ m s}^{-2}} \\ &= 7.85 \times 10^{41} \end{aligned}$$

The fraction of these atoms that came from Caesar's last breath,  $f$ , is given by

$$f = \frac{\tilde{N}_{Ar} V}{N_{Ar}} = \frac{2.41 \times 10^{23} \text{ m}^{-3} \times 0.500 \times 10^{-3} \text{ m}^3}{7.85 \times 10^{41}} = 1.53 \times 10^{-22}$$

The number of Ar atoms that we inhale with each breath is

$$N = N_A \frac{PV}{RT} = 6.023 \times 10^{23} \times \frac{10^{-2} \times 1 \times 10^5 \text{ Pa} \times 0.500 \times 10^{-3} \text{ m}^3}{8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}} = 1.21 \times 10^{20}$$

The number of these that came from Caesar's last breath is  $fN$

$$fN = 1.53 \times 10^{-22} \times 1.21 \times 10^{20} = 1.85 \times 10^{-2}$$

The reciprocal of this result, or 54, is the number of breaths needed to inhale one Ar atom that Caesar exhaled in his last breath.

**P15**

$$P_{N_2} = P_{N_2}^0 e^{\frac{M_{N_2} g z}{RT}} = 0.600 \times 1.0125 \times 10^5 \text{ Pa} \exp\left(-\frac{28.04 \times 10^{-3} \text{ kg} \times 9.81 \text{ m s}^{-2} \times 50 \times 10^3 \text{ m}}{8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}\right)$$

$$= 242 \text{ Pa}$$

$$P_{CO_2} = P_{CO_2}^0 e^{\frac{M_{CO_2} g z}{RT}} = 0.400 \times 1.0125 \times 10^5 \text{ Pa} \exp\left(-\frac{44.04 \times 10^{-3} \text{ kg} \times 9.81 \text{ m s}^{-2} \times 50 \times 10^3 \text{ m}}{8.314 \text{ J mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}\right)$$

$$= 6.93 \text{ Pa}$$

$$x_{CO_2} = \frac{P_{CO_2}}{P_{CO_2} + P_{N_2}} = \frac{6.93}{6.93 + 242} = 0.028 \quad x_{N_2} = 1 - x_{CO_2} = 0.972$$

**P17**

$$P = \frac{RT}{V_m - b} - \frac{a}{V_m^2}$$

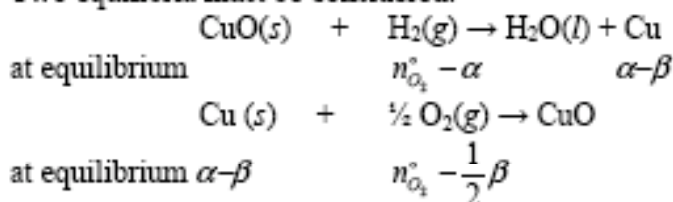
$$= \frac{8.314 \times 10^{-2} \text{ bar dm}^3 \text{ mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{1.42 \text{ dm}^3 \text{ mol}^{-1} - 0.0321 \text{ dm}^3 \text{ mol}^{-1}} - \frac{1.355 \text{ bar dm}^6 \text{ mol}^{-2}}{(1.42 \text{ dm}^3 \text{ mol}^{-1})^2} = 17.3 \text{ bar}$$

$$P_{ideal} = \frac{RT}{V} = \frac{8.3145 \times 10^{-2} \times \text{L bar mol}^{-1} \text{ K}^{-1} \times 300 \text{ K}}{1.42 \text{ L}} = 17.6 \text{ bar}$$

Because  $P < P_{ideal}$ , the attractive part of the potential dominates.

**P20**

Two equilibria must be considered:



In the final state, only  $\text{O}_2$  is present. Therefore  $\alpha = n_{\text{O}_2}^\circ$ . In an excess of  $\text{O}_2$ , all the copper is oxidized. Therefore  $\alpha - \beta = 0$  or  $\beta = n_{\text{O}_2}^\circ$ . We conclude that  $n_{\text{O}_2} = n_{\text{O}_2}^\circ - \frac{1}{2}n_{\text{H}_2}^\circ$ .

Let  $V_1$  and  $V_2$  be the initial and final volumes.

$$V_1 = (n_{\text{H}_2}^\circ + n_{\text{O}_2}^\circ) \frac{RT}{P} \quad V_2 = \left( n_{\text{O}_2}^\circ - \frac{1}{2}n_{\text{H}_2}^\circ \right) \frac{RT}{P}$$

Dividing the second equation by the first yields

$$\frac{V_2}{V_1} = \frac{n_{\text{O}_2}^\circ}{n_{\text{H}_2}^\circ + n_{\text{O}_2}^\circ} - \frac{1}{2} \frac{n_{\text{H}_2}^\circ}{n_{\text{H}_2}^\circ + n_{\text{O}_2}^\circ} = x_{\text{O}_2}^\circ - \frac{1}{2}x_{\text{H}_2}^\circ = 1 - x_{\text{H}_2}^\circ - \frac{1}{2}x_{\text{H}_2}^\circ = 1 - \frac{3}{2}x_{\text{H}_2}^\circ$$

$$x_{\text{H}_2}^\circ = \frac{2}{3} \left( 1 - \frac{V_2}{V_1} \right) = \frac{2}{3} \left( 1 - \frac{84.5 \text{ cm}^3}{100.0 \text{ cm}^3} \right) = 0.103; \quad x_{\text{O}_2}^\circ = 1 - x_{\text{H}_2}^\circ = 0.897$$