(1) Root and Integer Test

(2) Method of Least Squares

Newtons Raphson

Change of C

(3) Combined Sphere/Coil (Coils)

(4) General ladder/Other method

Miscellaneous

Math 150
Vector has length (often called magnitude) and direction

\[
\{ (x \in \mathbb{R} : (z \cdot x) = z) \} = \mathbb{R}
\]

\[
\{ (x \in \mathbb{R} : (\vec{b} \cdot x) = v) \} = \mathbb{R}
\]

\[
\{ (x \in \mathbb{R} : (\vec{X} \cdot x) = \vec{y}) \} = \mathbb{R} \implies n \text{ copies of } \mathbb{R}
\]

Let \( A \) mean \( f \) takes elements of \( A \) as input and output elements of \( B \). 

\[
\{ b \in \mathbb{R} : x + x = z \} = \mathbb{C}
\]

\{ complete numbers \}

\{ real numbers \}

\{ objects \}

\{ integers \}

\{ natural numbers \}

\{ definition \}

Test: Febrary 20th cumulative last of Calc I and II

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Calculus 150 Notes
\[ \langle 4, 3 \rangle \]
\[ \{ \mathbf{a} \cdot \mathbf{a} = 1, \mathbf{a} \cdot \mathbf{a} = 1 \} = \mathbb{R} \]
\[ \langle 4, 3 \rangle = \mathbb{R} \]

\[ \langle 1, 0, 0 \rangle \] unit vector in z axis direction
\[ \langle 0, 1, 0 \rangle \] unit vector in y axis direction
\[ \langle 0, 0, 1 \rangle \] unit vector in x axis direction

\[ \forall \text{ vector from } A \text{ to } B \]

\[ \langle 8, 6 \rangle = \langle 4 \cdot 2, 3 \cdot 2 \rangle = \langle 8, 6 \rangle \]
\[ \langle 8, 6 \rangle = \langle 2 + 4 \cdot 8 + 3 \rangle = \langle 2, 8 \rangle + \langle 4, 3 \rangle \]
\[ \langle 4, 3 \rangle = \langle 4 - 3, 3 - 3 \rangle = \langle 1, 3 \rangle = \frac{1}{2} \]
\[ \frac{1}{2} \mathbf{a} + \frac{1}{2} \mathbf{a} = \langle 1, 0 \rangle \mathbf{a} + \langle 0, 1 \rangle \mathbf{a} = \langle 4, 3 \rangle = \mathbb{R} \]

Other Notations

\[ \begin{aligned}
\text{directions} \\
\text{in the coordinate units vectors} \\
\end{aligned} \]

\[ \begin{array}{c}
\{ \mathbf{a} \} \ni \langle 0, 0, 0, 1 \rangle = \frac{1}{2} \\
\{ \mathbf{a} \} \ni \langle 1, 0, 0, 0 \rangle = \frac{1}{2} \\
\{ \mathbf{a} \} \ni \langle 0, 1, 0, 0 \rangle = \frac{1}{2} \\
\end{array} \]
\((\vec{O}, \ldots, \vec{O}) = \vec{O}\)
\((\vec{d}, \ldots, \vec{d}) = \vec{d}\)

Dot product generalizes multiplication:

\(\vec{O} = (\vec{d} - \vec{O}) + \vec{d}\)
\((\vec{d}, \vec{O}, \vec{d}) = \vec{O}\)
\((\vec{d} - \vec{O}, \vec{d} - \vec{O}, \vec{d} - \vec{O}) = \vec{d} - \vec{O}\)
\((\vec{d} \cdot \vec{d}) = \vec{d}\)

\[\langle \vec{v}, \vec{v} \rangle = \frac{n}{n} = n \text{ in direction of } \vec{u}\]
\[z = \epsilon \vec{z}^e = \epsilon \vec{v} + \epsilon \vec{z}^e = || \vec{u} ||\]

Unit Vectors:
\[\epsilon \vec{u} + \cdots + \epsilon \vec{z}^e + \epsilon \vec{z}^e = || \vec{u} ||\]

For this vector, the magnitude will equal 2. We can solve for the magnitude by:

The magnitude of the \(\vec{u}\) is denoted by its length \(\vec{u}\) as:

\[\vec{a} \quad 3 \quad 2 \quad 1 \quad 1 - 1\]

\[4 \quad 3 \quad 2 \quad 1 \quad 1 - 1\]

\[4 \quad 3 \quad 2 \quad 1 \quad 1 - 1\]
Vectors are parallel when the dot product is 0. Vectors are perpendicular when the dot product is 1.

\[
0 = \hat{\mathbf{v}} \cdot \hat{\mathbf{d}}
\]

\[
(\hat{\mathbf{v}}, 0, 1) = \hat{\mathbf{d}}
\]

\[
\varepsilon \hat{x} \hat{d} + \cdots + \varepsilon \hat{z} \hat{d} + \varepsilon \hat{d} \wedge = \| \hat{d} \| \|
\]

\[
\frac{\| \hat{d} \| \hat{d}}{\hat{d} \cdot \hat{d}} = \hat{d} \hat{d} \cos
\]

\[
\hat{d} \hat{d} + \cdots + \varepsilon \hat{z} \hat{d} + \varepsilon \hat{d} = \hat{d} \cdot \hat{d}
\]

Please note that it doesn't matter WHICH way we measure as theta and -theta have the same cosine, and theta is all we can compute.
\[
\int_0^\phi \int_0^R \rho \, \rho \, \sin \phi \, \cos \phi \, \phi \, d\phi \, d\theta = \frac{2\pi R^2}{3} \rho
\]

\[
\rho = \frac{Z^2}{r^2 + p^2 - r^2}
\]

\[
\cos \phi = \frac{r^2 + p^2 - r^2}{2rp}
\]

\[
\rho \sin \phi \cos \phi \phi \, d\phi \, d\theta = \frac{6M_r M_m}{G \sqrt{R^2 - z^2}} \cos \alpha
\]

\[
\text{Newton's Law of Gravitation:}
\]

\[
\text{Force of Magnitude:}
\]

\[
\text{Connecting Line:}
\]

\[
\text{Law of G:}
\]

\[
M_3 = \frac{4}{3} \pi R^3
\]

\[
\text{Mass:} \quad \text{Vol} \text{(sphere) \times \text{radius)} \times \text{density}
\]

\[
\text{Note: All mass may be regarded to lie in center}
\]

\[
\text{Newton's Result}
\]
\( x = \theta \cos \phi \)
\( \frac{dx}{d\theta} = -\phi \sin \phi \cos \theta \)
\( \frac{dx}{d\phi} = \cos \phi \sin \theta \)
\( y = \theta \sin \phi \)
\( \frac{dy}{d\theta} = \phi \cos \phi \sin \theta \)
\( \frac{dy}{d\phi} = \cos \phi \cos \theta \)
\( z = \theta \)
\( \frac{dz}{d\theta} = 1 \)

Cylindrical coordinates

\( f(x,y,z) = \rho \) (distance from origin)
\( \phi \) (angle from positive x-axis)
\( \theta \) (angle from positive z-axis)

Change of variables in cylindrical coordinates.
\[ \mathbf{r} = \mathbf{r}_0 \pm \mathbf{P} \]

\[ z = \frac{2}{W_m^2} \]

\[ \mathbf{r} = \mathbf{r}_0 + \frac{1}{2} \mathbf{P} \]

\[ \mathbf{F} = \mathbf{F}_0 + \frac{1}{2} \mathbf{P} \]

\[ \mathbf{F} = \mathbf{F}_0 + \frac{1}{2} \mathbf{P} \]

\[ \mathbf{F} = \mathbf{F}_0 + \frac{1}{2} \mathbf{P} \]

\[ \phi \]
The method of least squares involves finding the line of best fit of the form:

\[ y = a_1 f_1(x) + a_2 f_2(x) + \cdots + a_n f_n(x) \]

such that for \( 1 \leq j \leq N \), among the pairs \( (x_j, y_j) \) observed, the least sum of the squares of the residuals is obtained. This is a way of determining the line of best fit for a set of data.

The next step is to minimize the error, which we do by setting our derivative function to zero and solving:

\[
0 = \frac{\partial}{\partial a_k} \sum_{j=1}^{N} (y_j - a_1 f_1(x_j) - a_2 f_2(x_j) - \cdots - a_n f_n(x_j))^2
\]

for both \( a_k \) and solving:

\[
\begin{align*}
\frac{\partial}{\partial a_k} & \sum_{j=1}^{N} (y_j - a_1 f_1(x_j) - a_2 f_2(x_j) - \cdots - a_n f_n(x_j))^2 \\
& = 0
\end{align*}
\]
Step 5: solve systems of equations

\[
\begin{bmatrix}
\frac{\partial^2 f}{\partial x^2} \\
\frac{\partial^2 f}{\partial y^2}
\end{bmatrix}
= \begin{bmatrix} a \\
0 \end{bmatrix}
\begin{bmatrix}
\frac{\partial^2 f}{\partial x \partial y} \\
\frac{\partial^2 f}{\partial y \partial x}
\end{bmatrix}
\]

Step 4: write in matrix form

\[
\frac{\partial^2 f}{\partial x^2} = q \left( \frac{\partial^2 f}{\partial x \partial y} \right) + a \left( \frac{\partial^2 f}{\partial y \partial x} \right) = \frac{aq}{\sqrt{3} \pi}
\]

\[
\frac{\partial^2 f}{\partial y^2} = q \left( \frac{\partial^2 f}{\partial y \partial x} \right) + 0 \left( \frac{\partial^2 f}{\partial x \partial y} \right) = \frac{aq}{\sqrt{3} \pi}
\]

Step 3: rearrange

\[0 = (q - a) \frac{\partial^2 f}{\partial x^2} = \frac{aq}{\sqrt{3} \pi}
\]

\[0 = (q - a) \frac{\partial^2 f}{\partial y^2} = \frac{aq}{\sqrt{3} \pi}
\]

Step 2: differentiate (and 4y - 2)

\[0 = (q - a)(x - b)(-1 - 1) \frac{\partial^2 f}{\partial x \partial y} = \frac{aq}{\sqrt{3} \pi}
\]

\[0 = (q - a)(-x - b) \frac{\partial^2 f}{\partial y \partial x} = \frac{aq}{\sqrt{3} \pi}
\]

Step 1: differentiate
If the curve under the curve will be a little more than area.

The area of the shaded region will be:

Try the visualizing game.

Can be written, \( f'(x) = \frac{1}{x^2} \)

Ex: \( a = \frac{1}{4}, \quad f(x) = \frac{x}{4} \quad x \neq 0 \)

Strategy: Take the sequence of \( f(x) \) and replace every \( n \) with \( x \)

Strategy: Part of interval test: finding the function \( f(x) \)

\[ \int_{a}^{\infty} f(x) \, dx = \lim_{b \to \infty} \int_{a}^{b} f(x) \, dx = \lim_{b \to \infty} \left[ \frac{x^2}{2} \right]_{a}^{b} = \lim_{b \to \infty} \left( \frac{b^2}{2} - \frac{a^2}{2} \right) = \frac{1}{2} \]

Assume \( a \) is a non-increasing \( f(x) \). Then \( f(x) \) is non-increasing. (A non-decreasing sequence will certainly diverge.)

Assume we have a non-increasing or non-decreasing sequence.

\[ \int_{a}^{\infty} f(x) \, dx \]

Notes for May 7th (Cont'd)
\[
\frac{u_0}{u} > \frac{S}{B} \quad \text{for} \quad x = z_n
\]

Proof by example:

\[
\frac{u_0}{u} = \frac{n}{1}
\]
\[ x = \frac{u}{v} \]

\[ \log x = \log \frac{u}{v} = \log u - \log v \]

\[ \log(x^n) = n \log x \]

Take \( u^x = e^x \)

Show this by studying \( \lim u \rightarrow e \) of \( \frac{\log u}{\log x} \)

we can see the value is rapidly approaching one.

\[ \frac{\log u}{\log x} \rightarrow 1 \]

\[ \lim u^x \]

Try plugging in numbers -- for instance take \( u = 10 \)

\[ \lim \frac{u^n}{x^n} = \lim \frac{10^n}{x^n} \]

\[ \lim \frac{10}{x} \]

Example: the harmonic series, \( \frac{1}{n} \)

\[ \sum_{n=1}^{\infty} \frac{1}{n} = \frac{\log(2)}{1} \]

\[ p = \frac{1}{n} \]

Then we gain no information.

If \( p < 1 \)

Then \( \sum a_n \) converges

If \( p > 1 \)

Then \( \sum a_n \) diverges

Let \( \lim a_n = 0 \) (if such a limit exists)

\[ \sum a_n \]

If \( \sum a_n \)

Be a sequence of positive numbers

Root Test:

\[ a_n^{1/n} \]

Notes for May 7th: Root Test and Integral Test

Load Testing
Reca: think: need to be able to do the calculation

Good thing: automatic, just like the title

- have other strategies to see if the terms tend to 0
- only use other thing other similar technique

Final note on Red Test:

\[ \lim_{n \to \infty} \frac{a_n}{b_n} = \frac{1}{1} = 1 \]

For \( n \to \infty \):

Now go back to the start of the problem and plug in:

\[ \lim_{n \to \infty} a_n = 1 \]

We just showed above that this became:

Now plug this limit back into \( \frac{a_n}{b_n} \) again:

\[ 0 = \frac{u}{n - 1} = \frac{1}{\frac{1}{n} - 1} = \frac{1}{\frac{1}{n} - 1} \]

Take the limit and case
\[
\lim_{n \to \infty} \frac{\log(n)}{n^2} = 0
\]

\[
0 = \lim_{n \to \infty} \frac{\log(n)}{n^2}
\]

\[
G = \lim_{n \to \infty} \left( \frac{1}{n^2} \right) = \lim_{n \to \infty} \left( \frac{\log(n)}{n^2} \right)
\]

\[
x = \lim_{n \to \infty} \left( \frac{\log(n)}{n^2} \right) = \left( \frac{x}{\infty} \right) = \left( \frac{\log(n)}{\infty \cdot x} \right)
\]

\[
\text{Take limits of both sides:}
\]

\[
\text{Note: } \lim_{n \to \infty} \left( \frac{\log(n)}{n^2} \right) = 0
\]

\[
\text{Study limits:}
\]

\[
\lim_{n \to \infty} \left( \frac{\log(n)}{n^2} \right) = 0
\]

\[
\lim_{n \to \infty} \left( \frac{1}{n^2} \right) = \lim_{n \to \infty} \left( \frac{\log(n)}{n^2} \right)
\]

\[
\text{Example: }
\]

\[
\text{If } p \leq 1 \text{ then converges, diverges otherwise}
\]

\[
\text{If } p > 1 \text{ then converges}
\]

\[
\text{If } p < 1 \text{ then diverges}
\]

\[
\text{If } p = 1 \text{ then diverges (if } p \text{ exists)}
\]

\[
\text{Can't converge of positive numbers, let}
\]

\[
\text{Root Test:}
\]

\[
\text{If } a_n \text{ and } a_n^p \text{ exists, then}
\]

\[
\text{If } a_n \to 0 \text{ then converges}
\]

\[
\text{If } a_n \to L \text{ then diverges}
\]

\[
\text{If } a_n \to \infty \text{ then diverges (if } p \text{ exists)}
\]

\[
\text{Can't converge of positive numbers, let}
\]

\[
\text{Serena}
\]

\[
\text{Rodriguez}
\]

\[
\text{May 2, 2014}
\]
Changes

If \( n = 1 \), \( \frac{1}{n} \geq \frac{1}{2} \to \infty \)

Replace sum with integral (Cauchy)

\[
\sum_{n=1}^{\infty} \frac{1}{n^2} \geq \int_{1}^{\infty} \frac{1}{x^2} \, dx = \frac{1}{x} \bigg|_{1}^{\infty} = 1
\]

Proof by example: \( a_n = 1/n \)

\[
\frac{1}{n} \geq \frac{1}{2} \to \frac{1}{2} \geq \int_{1}^{\infty} \frac{1}{x^2} \, dx = \frac{1}{x} \bigg|_{1}^{\infty} = 1
\]

What's hard: finding \( f(x) \) usually replace with \( x \)

\[
\int_{1}^{\infty} \frac{1}{x^2} \, dx = \frac{1}{x} \bigg|_{1}^{\infty} = 1
\]

(2)

Assume \( a_n = \frac{1}{n} \) and \( f(x) \) are functions (f) that non-decreasing

\[
\int_{1}^{\infty} \frac{1}{x^2} \, dx = \frac{1}{x} \bigg|_{1}^{\infty} = 1
\]

(3)

(4) Assume \( a_n \) is non-increasing

Inequality Test
\[ N^n \geq (N_{\text{min}})^n + \mathcal{O}(1) \quad \text{for all } N \geq N_{\text{min}} \]

\[ \log N \leq 1 \quad \text{for all } N \geq 1 \]

\[ \int x \log x \, dx = x \log x - x + c \]

\[ (\log(x))^2 + 1 \geq \int x \log(x) \, dx + 1 \]

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \frac{1}{n} = \gamma \]

\[ \text{for large } n \]

\[ \frac{1}{N} \sum_{n=1}^{N} \frac{1}{n} = \log N + \gamma + \frac{1}{2N} + \mathcal{O}(N^{-2}) \]

\[ \prod_{p \leq N} p = N^{\frac{1}{2} \log \ln N + \mathcal{O}(1)} \quad \text{for large } N \]

\[ \prod_{n=1}^{N} n = \exp \left( N \log N - N + \frac{1}{2} \log(2\pi N) + \mathcal{O}(1) \right) \]

\[ \frac{1}{N} \sum_{n=1}^{N} \frac{1}{n} = \log N + \gamma + \frac{1}{2N} + \mathcal{O}(N^{-2}) \]

\[ \text{for large } N \]

\[ \lim_{N \to \infty} \frac{1}{N} \sum_{n=1}^{N} \frac{1}{n} = \gamma \]

\[ \text{for large } N \]