1. Jeremy has ten rods, having lengths $1, 2, ..., 10$. How many different ways are there to make a triangle by choosing three appropriate rods?

2. Show that there are infinitely many solutions in positive integers $x < y < z$ for which

$$x!y! = z!$$

3. Evaluate $\int_{0}^{\frac{\pi}{2}} f(x) \, dx$ where

$$f(x) = \frac{\sin^3 x}{\sin^3 x + \cos^3 x}$$

Hint: Consider $f\left(\frac{\pi}{2} - x\right)$.

4. Let $1, 3, 5, ..., (2n-1)$ be the first $n$ odd numbers. Show that their product is less than $n^n$.

5. No matter which 1001 distinct positive integers are chosen from $\{1, 2, ..., 1991\}$, prove that two must have difference 9.

6. What is the largest even integer not expressible as the sum of two odd composite numbers? Prove it.