24th Green Chicken Contest - Solutions

1. \[10 + 11 + \ldots + 99 = \frac{90(10 + 99)}{2} = 4950\]
   So palindromes must begin with 4. Hence it is 4884. So the missing number = 4950 - 4884 = 66.

2. \[p(\text{next two rolls are 3, 4}) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36}\]
   To find \(g\) (roll 3, 4, 5 (given sum 12)), count all ways sum could be 12. There is 1 way to get 3, 4, 5; 6 ways to get 2, 3, 4, 5; 3 ways to get 2, 3, 6; 6 ways to get 1, 5, 6. So \(g(3, 4, 5) \text{ sum 12} = \frac{1}{25}\).
   So more likely that his three rolls were all 3.

3. \(f_1(n) = 2, f_2(n) = 4, f_3(n) = 16, f_4(n) = 37, f_5(n) = 55, f_6(n) = 89, f_7(n) = 144, f_8(n) = 233, f_9(n) = 377\).
   \(f_{10}(n) = f_9(n) + f_8(n)\). Then \(f_{2001}(10) = 2000\).

4. \[x^2 + y^2 = x^3 + 3y^3 = (x^7)(x^2 - 7x + 49)\] for \((x^7)/(x^3 + y^3)\). For all \(x, y \in \mathbb{Z}\), divide \(x^2 + y^2\), then \((x^7)/(x^3 + y^3)\) for all \(x, y \in \mathbb{Z}\).
   The largest such \(x\) is \(217\).

   \(I + 2001A = (I + 2001B + C + 2001B)\).
   \(I + 2001A = (I + 2001B) + C = I + 2001B\).
   \(I + 2001C = I + 2001B\).
   \(A = C\).

   \(I + 2001C = I + 2001B\).
   \(C = B\).

The discriminant of \(f(x) = y = x^2 + (2b - 2a)x + (1 - 2ab)\) is \(D = b^2 - 4ac = (2b - 2a)^2 - 4(1 - 2ab) = 4(a^2 - 2ab + b^2) - 4 + 8ab = 4(a^2 + b^2 - 1)\).
   \(S\) is the region where \(D < 0\).
    \(a^2 + b^2 < 1\).
   So \(S\) is the interior of a circle of radius 1.
   Area of \(S\) is \(\pi\).