Shocks and Trends in Global Equity Markets

by

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Abstract

My research investigates the shocks and linkages that drive the stock indexes of eighteen developed economies and attempts to use this information to assess the forecastability of real stock returns. I first explore whether there is spatial cointegration and therefore, forecastability across country stock indexes. Using both a parametric and nonparametric approach, I find prevalent presence of idiosyncratic stochastic trends that cause stock indexes to diverge and reject spatial cointegration. In light of the lack of spatial cointegration, I investigate whether there is cointegration between country stock indexes and their respective output levels. I present a model, following Balvers et al. (1990), to motivate cointegration between stock indexes and outputs. I then provide the empirical evidences to support cointegration. My results lead to further research that decomposes the system of stock indexes and outputs into structural shocks with forecasting power for stock indexes.
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1 Introduction

Equity has historically offered substantial premiums over many other asset classes (Mehra and Prescott, 1985). Unsurprisingly, in those countries with developed equity markets, there has been a drastic increase in the proportion of equity in the household wealth portfolio. According to the U.S. Census Bureau, in 1984, only 7 percent of U.S. total wealth was held in stocks and mutual funds. In 2011, stocks and mutual funds accounted for 15 percent of all wealth. Hence, it is important from both household and government perspectives to understand the potential shocks and global linkages that drive stock indexes. Empirical evidence shows that macroeconomic variables have explanatory power for stock returns. Fama (1981, 1990), Chen et al. (1986), Barro (1990), Schwert (1990), and Ferson and Harvey (1991) have found that U.S. stock returns and its aggregate real activity are correlated. Asprem (1989), Ferson and Harvey (1993), have reached a similar conclusion using other international market data. These studies focus on estimating the short-run stationary risk factors that correlate with the stochastic discount factor. My thesis takes a different approach to explore the long-run linkages of stock indexes with one another as well as with a macroeconomic variable; industrial production. This follows the tradition of Kasa (1992), Richards (1995), Nasseh and Strauss (2000), and Lettau and Ludvigson (2001, 2004, 2011). Most previous studies of long-run stock market linkages are conducted either for a single country with its macroeconomic variables or for a small number of stock indexes with one another. My thesis emphasizes the use of non-stationary panel techniques to conduct hypothesis testing and time-series analysis on a large panel of stock indexes and their associated macroeconomic data. From an econometric perspective, this offers more reliable estimates. From a pure economic perspective, to the best of my knowledge, I am yet aware of other research that applies these new techniques to understand the long-run patterns in stock indexes for as many member countries and over such long timespan.
The first question to ask is whether spatial cointegration and therefore, forecastibility exist across global stock indexes as a result of globalization and increasing integration of economies. After the early 1980s recession, coordinated actions by governments in developed countries sent global stock markets rising as the economy recovered. This motivated many research in this period to test the cointegration relationship between global stock indexes. Among them, Kasa (1992) reports the strongest evidence, that from 1974 to 1990, the quarterly and monthly price (and dividend) indexes for the equity markets of five major industrial countries, U.S., Japan, England, Germany, and Canada, are all cointegrated around a single common stochastic trend. This finding would imply that returns in these markets may follow different patterns in the short term, but that in the long run the levels of total return indexes (i.e., prices plus reinvested dividends) are very closely linked. Using simulation, Richards (1995) suggests that the finding of such a strong cointegrating relationship is due to a failure to adjust asymptotic critical values to take account of the small number of degrees of freedom that remain in the Johansen (1988) multivariate estimation procedure. In order to study long-run international equity linkages, it is imperative to have a clear perspective on these contrary results. In addition, a limitation of these approaches is that they cannot test for spatial cointegration in panels with large cross-sectional dimensions due to high computational requirement. I consider two popular panel unit root tests, namely IPS (Im, Pesaran, and Shin, 1997) and cross sectionally augmented ADF (CADF - Im, Pesaran, and Shin, 2003). IPS assumes a pairwise cointegrating vector of $[1, -1]$ between stock indexes, which is deficient under a modeling perspective. CADF is a more desirable test that allows cointegrating vectors to be unrestricted. A recent development in nonstationary panel unit root test, the nonparametric MMIB rank test (Pedroni et al., 2013), also offers a robust testing procedure that avoids usual assumptions made by IPS and CADF about the cross-sectional dependence between data series. In addition, it has power to test for the alternative hypothesis of a specific number of common stochastic trends in the system. This feature is lacking in IPS and CADF which only has power over the alternative hypothesis of one single common
stochastic trend. Both CADF and MMIB support Richards (1995) that there is hardly any evidence of spatial cointegration in international stock indexes.

The evidence that spatial cointegration does not generally exist between return series is hardly surprising, given that basic models of asset pricing would preclude spatial cointegration. Even if markets become more integrated and share some common stochastic trends, for a non-stationary panel to cointegrate with one single common stochastic trend, it is necessary that all components of the return series cointegrate. This would imply that all country-specific returns shocks are always followed by unexpected but exactly offsetting shocks. However, according to Solnik (1991, p. 46), ‘Some investigators have attempted to find leads or lags between markets. However, no evidence of a systematic delayed reaction of one national market to another has ever been found. The existence of such simple market inefficiencies is, indeed, unlikely, since it would be so easy to exploit them to make an abnormal profit.’

My finding of no spatial cointegration between national stock indexes supports a more segmented view of global equity even in the long run. In particular, while it is reasonable to assume some common global shock that drives national stock indexes, it is important to recognize the presence of idiosyncratic shocks that cause stock indexes to diverge in the long run. A natural next step is then to investigate whether there is any cointegration between stock indexes and some national measures of real economic activities. The simplest variable is output, proxied by industrial production. My results support cointegration between stock indexes and outputs. This points to further research that identifies potential structural shocks which can potentially forecast real stock returns.
2 Cointegration and Its Relation to Stock Price

2.1 Background on Cointegration

Let $y_t$ and $x_t$ be two time series which are both integrated of order $d$, denoted as $I(d)$. In general, any linear combinations of these two series will be $I(d)$. However, if there exists a linear combination of the variables with a reduced order of integration, they are said to be cointegrated (Engle and Granger, 1987). In practice, most financial and economic time series, such as stock index and industrial production, are considered $I(1)$ processes. Any order of integration larger than 1 implies an explosive process that is not typically realistic. If two $I(1)$ processes are cointegrated, the resulting linear combination is $I(0)$ which is stationary. The presence of such cointegration implies a long-run equilibrium between the two variables. If two variables are cointegrated, there exist an infinite number of cointegrating vectors of coefficients. We usually normalize the cointegrating vector so that one of the coefficients is equal to 1. For instance, let $y_t$ and $x_t$ be two unit root processes,

$$y_t \sim I(1), \quad x_t \sim I(1).$$

If $y_t$ and $x_t$ are cointegrated,

$$\exists \beta \text{ s.t. } y_t - \beta x_t \sim I(0).$$

The cointegrating vector is normalized to be $[1, -\beta]$. To test for cointegration, Engle & Granger suggest the following two-step procedure. First, estimate the cointegrating coefficients by regression with ordinary least squares (OLS),

$$y_t = \alpha + \beta x_t + \epsilon_t.$$

The residuals from the regression are then tested for stationarity using a variant of the Augmented Dicker-Fuller (ADF) unit-root test. The most general form involves fitting
the following model by OLS,

$$\epsilon_t = \alpha + \beta \epsilon_{t-1} + \delta t + v_t$$

or equivalently,

$$\Delta \epsilon_t = \alpha + \rho \epsilon_{t-1} + \delta t + v_t,$$

$$v_t = \sum_{j=1}^{K} \theta_j \Delta \epsilon_{t-j} + u_t$$

where $\Delta \epsilon_t = \epsilon_t - \epsilon_{t-1}$ and $\rho = \beta - 1$. If there is no cointegration, $\epsilon_t \sim I(1)$ and there exists a unit root; $\beta = 1$ and $\rho = 0$. If the two series are cointegrated, $\epsilon_t \sim I(0)$, $\beta < 1$ and $\rho < 0$. The intercept $\alpha$ allows for a time drift and $t$, deterministic trend, in the time series of $\epsilon_t$. Depending on the data, either $\alpha$ or $\delta$ can be set to zero.\footnote{Throughout my thesis, there is no real reason to believe that the time series data are trending. Hence, I set $\delta$ to zero in all ADF tests.} If $v_t$ is serially correlated, lags of $\Delta \epsilon_t$ are added so that the error term $u_t$ is Gaussian white noise. Hamilton (1994, chap. 17) shows that except when only $\alpha$ is included, the t-statistic used to test $H_0 : \rho = 0$, also known as the Dicker-Fuller statistic, does not have a standard distribution. MacKinnon (1994) shows how to approximate the p-values on the basis of a regression surface. Cheung and Lai (1995) points out that the lag order $K$, in addition to sample size, can affect the finite-sample behavior of the test. The authors suggest the use of reponse surface analysis to correct for the effect of lag order in finite sample. MacKinnon (2010) provides updated tables of robust critical values for standard ADF tests. As robust critical values vary for different samples, I consider the robust critical values based on MacKinnon (2010) and report the approximate p-value based on MacKinnon (1994) for standard ADF tests.

### 2.2 Spatial Cointegration of Stock Indexes

The two most popular debates regarding cointegration and stock prices include: (1) whether cointegration of stock prices implies market inefficiency, and (2) whether inte-
Granger financial markets mean cointegration of stock indexes. In this section, I provide some perspectives on these two issues.

Granger (1986) argues that two price series that are determined in efficient markets cannot be cointegrated because cointegration and the correspondent error correction implies predictability. However, as pointed out by Richards (1995), predictability does not necessarily imply inefficiency. For instance, stock prices will be highly predictable on days when stocks go ex-dividend though total returns including dividends should not be predictable. Second, predictability of stock prices bears no implication about efficiency unless returns are risk-adjusted.

To shed light on the second issue, I again follow Richards (1995) to argue that cointegration between stock indexes is unlikely. Let $s_{i,t}$ be the log of stock index $i$ at time $t$ and $r_{i,t}$ be continuously compounded return in excess of risk-free rate. The price process of the stock index is

$$s_{i,t} = s_{i,t-1} + E_{t-1}r_{i,t} + \epsilon_{i,t}$$

or

$$s_{i,t} = s_{i,0} + \sum_{j=1}^{t} E_{j-1}r_{i,j} + \sum_{j=1}^{t} \epsilon_{i,j}$$

where $\epsilon_{i,t}$ is the unexpected returns in period $t$. If expectations are rational, $\epsilon_{i,t}$ is white noise. For simplicity, the analysis assumes: risk-free rate is constant and can be ignored when cointegration is considered; the indexes are denominated in a common currency. A model with time-varying risk-free rate and local currency will make cointegration even more difficult. Let’s assume that excess returns are generated by CAPM:

$$E_{t-1}r_{i,t} = \beta_{i,t}E_{t-1}r_{gm,t}$$
where \( r_{gm,t} \) denotes the continuously compounded global market risk premium and the market \( \beta_{i,t} \) is not necessarily constant. A constant \( \beta_{i,t} = \beta_i \) assumed by basic CAPM is just a more restrictive case and does not alter the argument. Cointegration of the stock indexes of two countries; for instance \((i = \text{US, JP})\), with a cointegrating vector of \((1, -\alpha)\) would require that

\[
(\beta_{US,t} - \alpha \beta_{JP,t}) \sum_{j=1}^{t} E_{j-1} r_{gm,j} + \left( \sum_{j=1}^{t} \epsilon_{US,j} - \alpha \sum_{j=1}^{t} \epsilon_{JP,j} \right) \sim I(0).
\]

As the cumulated risk premium term is nonstationary and \( \epsilon_{i,t} \) is independent, cointegration has two implications. The first implication is plausible that \( \beta_{US,t} - \alpha \beta_{JP,t} \sim I(0) \) with zero mean, implying an exact relationship between the long-run expected excess returns of two cointegrated stock indexes. However, the second implication requires

\[
\sum_{j=1}^{t} \epsilon_{US,j} - \alpha \sum_{j=1}^{t} \epsilon_{JP,j} \sim I(0),
\]

implying that one of the \( \epsilon_{i,t} \) terms will predict the other which is directly at odds with rational expectations. In addition, it requires that any unexpected returns in different countries are driven by a single common permanent shock. This would rule out the possibility of any permanent country-specific innovations. Last but not least, the comment by Solnik (1991) as mentioned before adds an empirical argument against cointegration. The next section discusses potential tests for spatial cointegration.

3 Tests for Spatial Cointegration

3.1 Johansen (1988)

Using the Johansen (1988) multivariate estimation procedure and data from 1974 to 1990, Kasa (1992) finds evidence of a single common stochastic trend among five major national stock indexes: U.S., Japan, England, Germany, and Canada. This approach starts with a \( p \)th-order VAR model for \( X_t \), a vector of \( N I(1) \) variables, with independent Gaussian
errors,

\[ X_t = \mu + \sum_{j=1}^{p} \theta_j X_{t-j} + v_t, \]

which can be expressed in vector error-correction (VECM) form,

\[ \Delta X_t = \mu + \Pi X_{t-1} + \sum_{j=1}^{p-1} F_j \Delta X_{t-j} + v_t \]

where

\[ F_j = -\sum_{i=j+1}^{p} \theta_i, \quad \Pi = -(I - \sum_{j=1}^{p} \theta_j) \]

The Johansen procedure is based on rank analysis of the matrix \( \Pi \). If \( \Pi = 0 \), there is no cointegration and nonstationarity of \( X_t \) vanishes by taking differences. If \( \Pi \) has full rank, multiplying both sides of the VECM by \( \Pi^{-1} \) indicates that \( X_t \) is stationary. Cointegration implies rank reduction in matrix \( \Pi \); if \( rank(\Pi) = r < N \), there are \( r \) cointegrating vectors. Johansen (1988) proposes two different likelihood-ratio tests of the significance of \( \Pi \)'s reduced rank, namely, the trace test and maximum eigenvalue test. Richards (1995) argues that Kasa (1992)'s finding of such a strong cointegrating relationship is due to a failure to adjust asymptotic critical values to take account of the small number of degrees of freedom. This problem is exacerbated when \( N \) is large and VECM order is high. Breitung and Cubadda (2011) point out that the Johansen test is only suitable for data sets including a fairly large number of time periods (usually 100-300) and a small number of variables (less than 6). Hence, in order to test for spatial cointegration in a panel with moderate number of countries (typically between 10 and 50), other panel unit root tests are considered.

### 3.2 IPS (Im, Pesaran, and Shin - 1997)

IPS is a general panel unit root testing procedure that tests the null hypothesis of all unit roots in panel members against the alternative hypothesis of all or some stationary processes, depending on the question to be investigated. The test pools the Dicker-Fuller
statistic across panel members to compute group-mean test statistic. Inference is based on the group-mean test statistic adjusted by its asymptotic mean and variance. Spatial cointegration with a single common stochastic trend as documented by Kasa (1992) is equivalent to pairwise cointegration between stock indexes. The most common way to test for spatial pairwise cointegration is to use the IPS panel unit root test. Let \( p_{i,t} \) be country \( i \)'s stock index at time \( t \) and \( s_{i,t} = \ln p_{i,t} \). As shown below, I can use IPS to test the following hypothesis,

\[
H_0 : \quad s_{i,t} - s_{j,t} - \alpha_{i,j} \sim I(1) \forall i, j
\]

\[
H_1 : \quad s_{i,t} - s_{j,t} - \alpha_{i,j} \sim I(0) \forall i, j.
\]

Under the alternative hypothesis,

\[
s_{i,t} - s_{j,t} - \alpha_{i,j} \sim I(0) \forall i, j \iff s_{i,t} - \bar{s}_t - \bar{\alpha}_i \sim I(0) \forall i
\]

where with \( N \) countries, the common time effect, \( \bar{s}_t \), and the country fixed effect, \( \bar{\alpha}_i \), are defined as:

\[
\bar{s}_t = \frac{1}{N} \sum_{j=1}^{N} s_{j,t}, \quad \bar{\alpha}_i = \frac{1}{N} \sum_{j=1}^{N} \alpha_{i,j}.
\]

\[\Rightarrow\]:

\[
s_{i,t} - s_{j,t} - \alpha_{i,j} \sim I(0) \forall i, j \Rightarrow \frac{1}{N} \sum_{j=1}^{N} (s_{i,t} - s_{j,t} - \alpha_{i,j}) \sim I(0) \forall i, j
\]

\[
\Rightarrow s_{i,t} - \bar{s}_t - \bar{\alpha}_i \sim I(0) \forall i.
\]

\[\Leftarrow\]:

\[
s_{i,t} - \bar{s}_t - \bar{\alpha}_i \sim I(0) \forall i \Rightarrow s_{i,t} - \bar{s}_t - \bar{\alpha}_i - (s_{j,t} - \bar{s}_t - \bar{\alpha}_j) \sim I(0) \forall i, j
\]

\[
\Rightarrow s_{i,t} - s_{j,t} - \alpha_{i,j} \sim I(0) \forall i, j.
\]
Equation (1) indicates that if a single common stochastic trend exists, it can be captured by the common time effect. It also allows us to conveniently reduce the number of panel members when using IPS to test for pairwise cointegration; from $(N-1)! (s_{i,t} - s_{j,t}, i \neq j)$ to just $N (s_{i,t} - \bar{s}_t)$ . This results from the implicit assumption that the cointegrating vector between indexes is $[1, -1]$. Later, I argue that this is a strong restriction in our context and propose ways to relax this restriction. We may use the following testing procedure as proposed by Im, Pesaran, and Shin (1997). First extract the common time effect for cross sectional dependence.

$$\tilde{s}_{i,t} = s_{i,t} - \bar{s}_t.$$ 

We then estimate ADF regression for each indexes $i$:

$$\Delta \tilde{s}_{i,t} = \tilde{\alpha}_i + \rho_i \tilde{s}_{i,t-1} + \sum_{k=1}^{K_i} \theta_{i,k} \Delta \tilde{s}_{i,t-k} + \epsilon_{i,t}.$$ 

$\tilde{\alpha}_i$ absorbs any country-specific deterministics. We estimate by OLS individually for each country. $K_i$ is chosen by lag truncation process in which we start with an arbitrarily large number of lags and reduce it until the largest lag is significant\(^2\). We pull the t-statistic, $t_{\rho_i}$, for $H_0 : \rho_i = 0$ for each country $i$ and use to compute:

$$\bar{t}_\rho = \frac{1}{N} \sum_{i=1}^{N} t_{\rho_i}.$$ 

The group-mean panel unit root test statistic is computed by adjusting for asymptotic mean, $\mu$, and variance, $\nu$, under the null $H_0 : \rho_i = 0$.

$$Z_{NT}^{IPS} = \sqrt{\frac{N}{\nu}} (\bar{t}_\rho - \mu)$$

\(^2\)For later analysis, I set the largest lag to be two years: 24 months for monthly data and 8 quarters for quarterly data. The critical value for last lag used is 1.64.
where

\[
\mu = E[t_{\rho}], \quad \nu = Var(t_{\rho}).
\]

The pairwise cointegration test is equivalent to:

\[
H_0 : \quad \rho_i = 0 \quad \forall \ i, \quad Z_{NT}^{IPS} \sim N(0, 1)
\]
\[
H_1 : \quad \rho_i < 0 \quad \forall \ i, \quad Z_{NT}^{IPS} \to -\infty.
\]

Hence, large negative values imply rejection of unit root. In our context, IPS is not desirable because it assumes a cointegrating vector of \([1, -1]\) between each pair of indexes as well as between each index with the single common stochastic trend. One implication is that in the stead state, index returns are exactly the same. To see this, for simplicity and without loss of generality (WLGO), assume that \(\tilde{s}_{ij,t} = s_{i,t} - s_{j,t}\) follows an autoregressive process with order \(p\), denoted as AR\((p)\). From the alternative hypothesis,

\[
\tilde{s}_{ij,t} = \alpha_{i,j} + \sum_{l=1}^{p} \theta_l \tilde{s}_{ij,t-l} + \epsilon_{ij,t}
\]

where \(\sum_{l=1}^{p} |\theta_l| < 1\). We know that in the steady state, \(\tilde{s}_{ij,t+1} = \tilde{s}_{ij,t} = E\tilde{s}_{ij,t}\) and \(\epsilon_{ij,t} = 0\). That is, the stationary process \(\tilde{s}_{ij,t}\) reverts to its mean and there is no more shock to \(\tilde{s}_{ij,t}\):

\[
E\tilde{s}_{ij,t} = \alpha_{i,j} + \sum_{l=1}^{p} \theta_l E\tilde{s}_{ij,t-l} = \frac{\alpha_{i,j}}{1 - \sum_{l=1}^{p} \theta_l}.
\]
In the steady state, \( \tilde{s}_{ij,t} = E\tilde{s}_{ij,t} \), then

\[
\tilde{s}_{ij,t+1} = \alpha_{i,j} + \sum_{l=1}^{p} \theta_l E\tilde{s}_{ij,t-l}
\]

\[
= \alpha_{i,j} + \alpha_{i,j} \sum_{l=1}^{p} \theta_l \frac{1}{1 - \sum_{l=1}^{p} \theta_l}
\]

\[
= \frac{\alpha_{i,j}}{1 - \sum_{l=1}^{p} \theta_l}
\]

\[
= \tilde{s}_{ij,t}.
\]

Also, \( \tilde{s}_{ij,t} = \ln \frac{p_{i,t}}{p_{j,t}} \). This means that in the steady state, the spread between the two indexes is constant; they must grow at the same rate to maintain this constant spread and therefore, exactly same returns. This is difficult to justify; country stock indexes have empirically displayed significant risk premia relative to each other because of either their idiosyncratic characteristics or their different contributions to the variance of the global market portfolio in the long run. Hence, I will only use IPS to test for unit roots in panel members (without subtracting the common time effect). Then, the alternative hypothesis is that there exist some stationary processes against the null hypothesis of all unit root processes. The next two tests are proposed to test for spatial cointegration.

### 3.3 CADF (Pesaran - 2007)

Unlike IPS, CADF directly tests the null hypothesis of all unit roots against the alternative hypothesis of a single common stochastic trend. CADF offers a more general testing approach which only employs the fact that pairwise cointegration is equivalent to spatial cointegration with a single common stochastic trend. Pesaran (2007) shows that the common stochastic trend can be proxied by the cross-section mean, \( \bar{s}_t \), and its lagged value(s), \( \bar{s}_{t-1}, \bar{s}_{t-2}, \ldots \) (or equivalently \( \bar{s}_{t-1} \) and \( \Delta\bar{s}_t, \Delta\bar{s}_{t-1}, \ldots \)), for \( N \) sufficiently large. The cointegrating vector between a stock index \( s_{i,t} \) and the common stochastic trend is allowed to be unrestricted. Rather than using ADF with \( \tilde{s}_{i,t} \) as in the case of IPS, CADF
directly includes $\bar{s}_t$ into the regression as follows,

\[
\Delta s_{i,t} = \alpha_i + \rho_i s_{i,t-1} + \sum_{j=1}^{K} \theta_j \Delta s_{i,t-j} + f_t + e_{i,t}
\]

\[
f_t = \beta_i \bar{s}_{t-1} + \sum_{j=0}^{K} \gamma_j \Delta \bar{s}_{i,t-j}
\]

where $e_{i,t}$ is the idiosyncratic error and $f_t$ captures the common stochastic trend. Under the null hypothesis of cointegration with a single common stochastic trend, conditioned on $f_t$, individual index $s_{i,t}$ is stationary. Pesaran (2007) considers the pooled statistic $\overline{CADF}$, the average OLS t-ratio of $\rho_i$. Large negative values imply rejection of unit root in favor of spatial pairwise cointegration. $\overline{CADF}$ does not follow $\mathcal{N}(0,1)$ but its critical values can be derived by Monte Carlo simulation.

### 3.4 MMIB (Pedroni, Vogelsang, Wagner, and Westerlund - 2013)

The essence of CADF is to capture the cross-sectional contemporaneous dependence between the indexes by conditioning on the common stochastic trend, the term $f_t$, also known as the common dynamic factor. Breitung and Cubadda (2011) argues that if the indexes are cointegrated due to a more complex dynamic relationship; for example due to (Granger) causality among the indexes, then the introduction of common dynamic factor is not sufficient to obtain a pooled test statistic that is free of nuisance parameters. Pedroni et al. (2013) introduces MMIB, a nonparametric rank test that allows for unrestricted cross-sectional dependence and dynamic heterogeneity among the indexes. In addition, instead of assuming a single common stochastic trend, MMIB employs an iterative process to explicitly test for a specific number of common stochastic trends in the system. This allows us to gauge the degree of cointegration in the panel. I refer the readers to Pedroni et al. (2013) for the full technical details of the MMIB test. Intuitively, panel cointegration implies rank reduction in the long-run covariance matrix of the panel. Then, MMIB is essentially a rank test for the number of common stochastic
trends. Consider a panel of $N$ members. The full rank is $N$. The rank $r$ is equivalent to the number of shared stochastic trends and $N - r$, the number of cointegrating vectors.

3.5 Empirical Results for Spatial Cointegration

I collect data on national stock indexes (including dividends) from Morgan Stanly Capital Indexes (MSCI). The indexes used in this section are those of 18 developed economies: Austria (AT), Australia (AU), Belgium (BE), Canada (CA), Switzerland (CH), Germany (DE), Denmark (DK), Spain (ES), France (FR), United Kingdom (GB), Hong Kong (HK), Italy (IT), Japan (JP), Netherland (NL), Norway (NO), Sweden (SE), Singapore (SG), and United States (US). While it is possible to include more stock indexes (up to 45) into the panel, these 18 economies have the longest history (from December 1969) and most integrated financial markets; they are most likely to exhibit evidence of spatial cointegration. To avoid the initial arbitrary value when indexes are constructed, I allow the indexes to grow and only conduct analysis from January 1975 till August 2013, a total of 464 months.

I first conduct panel unit root tests on the (log) nominal indexes denominated in U.S. dollars. Because inflation is a potential trivial unit root process that can affect results, I also report panel unit root tests for real indexes. There are two approaches to convert a nominal stock index to its real value: deflate the nominal U.S. dollar-denominated stock index by U.S. price index, or deflate the nominal stock index denominated in local currency by local price index. The two approaches are exactly the same if purchasing power parity (PPP) holds. In practice, because PPP is usually violated, the latter approach will include a component of PPP deviation. I report panel unit root tests for both approaches. MSCI provides country stock indexes denominated in both nominal U.S. dollars and local currencies. I use local consumer price indexes (CPI) reported in the ALFRED dataset as price deflators.\[^{3}\] CPI is not seasonally adjusted. Because CPI data of Singapore and

\[^{3}\] The dating convention used by ALFRED is beginning of period for actual end of period value.
Hong Kong are only available at annual frequency and Australia, quarterly, the panel of local inflation adjusted indexes only includes the other 15 countries. Figure 1 shows Japan’s MSCI stock index in U.S. dollar nominal, U.S. inflation adjusted, and local inflation adjusted terms from 1969 to 2013 as an example.

Japan stock index grows drastically from 1969 to peak in 1988. Since then, it experiences decades of stagnation. None of the time series seem to display mean reversion or stationarity. Despite starting at the same initial value, the U.S. inflation adjusted index deviates persistently from the local inflation adjusted index.

I first report CADF test results. Kasa (1992) finds evidence of spatial cointegration with a single common stochastic trend for data from 1974 to 1990. I thereby report my results for three subsamples: 1975-1990 (192 months), 1991-2013 (272 months), and 1975-2013 (full sample, 464 months). To test for spatial cointegration, any procedures first require all panel members to be unit root processes. Stock indexes are generally considered unit root processes because firms that comprise the indexes grow over time. A simple panel unit root test for stock indexes is IPS without substracting the common time effect and
with the group mean statistic adjusted for asymptotic values as discussed in section 3.2. Table 1 reports IPS test results for presence of unit roots in all stock indexes and CADF test results for spatial pairwise cointegration. IPS generally fails to reject the null that all panel members contain unit root processes. While $Z_{NT}^{IPS}$ for local inflation adjusted indexes is significant at 10% level post-1990 periods, the test statistics in full sample are highly positive. This satisfies the first requirement for analysis that stock indexes are unit root processes. CADF test statistics hardly show any evidence of pairwise cointegration with a single common stochastic trend. Only the test statistic for U.S. dollar nominal indexes is barely significant at 10% level in full sample. After removing inflation, the result is no longer significant.

<table>
<thead>
<tr>
<th>Stock Index Type</th>
<th>1975-1990</th>
<th>1991-2013</th>
<th>1975-2013</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>$Z_{NT}^{IPS}$ - Member Unit Roots</strong></td>
<td></td>
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<tr>
<td>USD Nominal</td>
<td>4.26</td>
<td>0.554</td>
<td>2.07</td>
</tr>
<tr>
<td>US Inflation Adjusted</td>
<td>3.36</td>
<td>-0.640</td>
<td>2.48</td>
</tr>
<tr>
<td>Local Inflation Adjusted</td>
<td>2.70</td>
<td>-1.57*</td>
<td>2.05</td>
</tr>
<tr>
<td><strong>CADF - Pairwise Cointegration</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>USD Nominal</td>
<td>-1.36</td>
<td>-1.08</td>
<td>-1.52*</td>
</tr>
<tr>
<td>US Inflation Adjusted</td>
<td>-1.28</td>
<td>-1.00</td>
<td>-1.40</td>
</tr>
<tr>
<td>Local Inflation Adjusted</td>
<td>-1.48</td>
<td>-0.585</td>
<td>-1.22</td>
</tr>
<tr>
<td>T (months)</td>
<td>192</td>
<td>272</td>
<td>464</td>
</tr>
<tr>
<td>N (countries)</td>
<td>15-18</td>
<td>15-18</td>
<td>15-18</td>
</tr>
</tbody>
</table>

Table 1: IPS and CADF Panel Unit Root Test for 18 Developed Stock Indexes

U.S. dollar nominal and U.S. inflation adjusted panels contain 18 indexes: AT, AU, BE, CA, CH, DE, DK, ES, FR, HK, UK, IT, JP, NL, NO, SE, SG, and US. Local inflation adjusted panel contains 15 indexes excluding AU, HK, and SG. Max lag is set to be 24 for monthly data. Critical value for last lag is 1.64. Critical values of CADF are obtained from Pesaran (2007); for the panels with 18 members, -1.79 (1%), -1.61 (5%), -1.50 (10%); for the panel with 15 members, -1.85 (1%), -1.65 (5%), -1.53 (10%). Critical values of $Z_{NT}^{IPS}$ are those of one-sided $\mathcal{N}(0,1)$; -1.28 (10%), -1.65 (5%), -2.33 (1%). Levels of significance are coded as: *** (1%), ** (5%), * (10%).
While CADF only has the power to test for the alternative hypothesis of a single common stochastic trend, MMIB has the power to test for a specific number of shared stochastic trends among panel members. This enables us to gauge the degree of cointegration in the panel. Because MMIB requires simulation to derive the critical values for different combinations of the null rank, \( c \), and the time dimension, \( T \), I fix \( T \) and report test results for the number of significant common stochastic trends on rolling 10-year windows (120 months) for the same three sets of indexes as reported with CADF. Table 3 of appendix A shows the critical values from \( 10^4 \) simulations for different null ranks \( c \) with \( T \) fixed to 120.

Figure 2: MMIB test for the number of common stochastic trends present in the panel of stock indexes on rolling 10-year windows. U.S. dollar nominal and U.S. inflation adjusted panels contain 18 indexes: AT, AU, BE, CA, CH, DE, DK, ES, FR, HK, UK, IT, JP, NL, NO, SE, SG, and US. Local inflation adjusted panel contains 15 indexes excluding AU, HK, and SG. Critical values from \( 10^4 \) simulations are reported in Table 3 of appendix A.
Similar to the data used in CADF, both panels of U.S. dollar nominal and U.S. inflation adjusted stock indexes contain 18 country members. Hence, their full ranks are 18; each panel is driven by at most 18 stochastic trends. The panel of local inflation adjusted indexes contain 15 country members, excluding AU, HK, and SG due to unavailable monthly data. The panel is driven by at most 15 stochastic trends. The blue dots in Figure 2 show the rank, corresponding with the number of shared stochastic trends MMIB fails to reject, in each subsample of 120 months. At any point in time, MMIB shows at most 2 cointegrating vectors in all three tested panels. This indicates the presence of numerous stochastic trends and a very low degree of spatial cointegration in the panels.

The fact that both CADF and MMIB show evidence against spatial cointegration with a single common stochastic trend supports the arguments in section 2.2. While it is reasonable to assume some common global shock that drives country stock indexes, it is important to recognize the presence of idiosyncratic stochastic trends that do not cointegrate and cause stock indexes to diverge in the long run.

4 Cointegration between Stock Index and Output

4.1 Cointegration Model for Output and Stock Index

I present the model, a modification of that by Balvers et al. (1990), to motivate cointegration between stock index and industrial production as a proxy for output. The intuition underlying the model arises from consumption smoothing by households. For simplicity, assume the only investment opportunity for households is equity. Consider households aggregate wealth to comprise of human wealth and asset wealth (stock) whose dividends equal labor income and stock dividend respectively. Output determines labor income and hence, consumption. To maximize utility, households attempt to smooth

---

4All variables discussed in this section are in real terms.
consumption by adjusting their required rate of returns on asset wealth. For example, households, anticipating lower output in the next period, will attempt to transfer wealth to this anticipated period of scarcity by investing in more assets. This increases asset prices contemporaneously and lower asset returns. This motivates the linkage between output and stock index. Assume for the moment a closed economy that can be divided into a representative firm and a representative household.

4.1.1 The Representative Firm

Firm produces output, \( y_t \), by a stochastic, decreasing returns to scale, Cobb-Douglas technology. The decreasing returns to scale allow profit-making and hence, the incentive for firms to issue shares. The only input relevant to decision-making is capital, \( k_t \). For simplicity, capital is assumed to depreciate fully each period. Firms allocate output to re-investment, \( i_t \), and dividends, \( d_t \). There is a gestation lag of one period before investment becomes productive as capital in the production function. There exists multiplicative serially uncorrelated uncertainty in production, \( \theta_t \), such that \( E\theta_t = 1 \). Firm observes this shock before determining investment, \( i_t \).

The timing of events are shown in Figure 3. Firm produces output, \( y_t \), at time \( t \) and divides output into dividends, \( d_t \), and investment, \( i_t \). Ex-dividend share price, \( p_t \), reflects this information. Firm pays out dividends to investors and converts the investment into capital one period later, \( k_{t+1} \). Output, \( y_{t+1} \), is realized after the random productivity shock, \( \theta_{t+1} \), is revealed. Firm then allocates output, \( y_{t+1} \), between investment, \( i_{t+1} \), and dividends, \( d_{t+1} \). Share price changes to \( p_{t+1} \) to reflect new information. The gross one-period return (including dividends) is denoted as \( R_{t+1} \).

As defined,

\[
R_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t}.
\]
Figure 3: Time line of output allocation. \( y_t \) = output at time \( t \), \( d_t \) = dividends, \( i_t \) = investment, \( p_t \) = share price, \( k_t \) = physical capital, \( \theta_t \) = random productivity shock, \( R_{t+1} \) = realized gross return (including dividends) on shares from period \( t \) to \( t + 1 \). (Source: Balvers et al., 1990).

Denote the present as \( t = 0 \). Ex-post pricing is as follows:

\[
R_1 = \frac{p_1 + d_1}{p_0} \quad \text{and} \quad p_0 = \frac{p_1 + d_1}{R_1}.
\]

Solve this equation forward and recognize that \( \lim_{t \to \infty} \left[ \prod_{h=1}^{t} R_h^{-1} \right] p_t = 0 \),

\[
p_0 = \sum_{t=1}^{\infty} \left[ \prod_{h=1}^{t} R_h^{-1} \right] d_t
\]

or

\[
d_0 + p_0 = \sum_{t=0}^{\infty} \left[ \prod_{h=0}^{t} R_h^{-1} \right] d_t.
\]

Conditional on ex-ante information,

\[
V_0 = d_0 + p_0 = E_0 \sum_{t=0}^{\infty} \left[ \prod_{h=0}^{t} R_h^{-1} \right] d_t
\]
By second separation principle (Fama and Miller, 1972), at any point in time, Firm’s optimal investment decision involves maximizing the market value of outstanding shares and is independent of shareholder tastes. Firm’s optimization problem is as follows:

\[
\max_{i_t = k_{t+1}} V_0 = E_0 \sum_{t=0}^{\infty} \left[ \prod_{h=0}^{t} R_{h}^{-1} \right] d_t
\]

subject to

\[
d_t = y_t - i_t = y_t - k_{t+1}
\]

\[
y_t = AL^t \theta_t k_t^\alpha,
\]

where \(A\) and \(L\) are positive constants, \(R_0 = 1\), and \(\alpha \in (0, 1)\). Substitute the constraints into the objective function and solve the optimization problem at a particular time \(t\).

\[
\frac{\partial V_t}{\partial i_t} = \frac{\partial V_t}{\partial k_{t+1}}
\]

\[
= \frac{\partial E_t \left[ d_t + R_{t+1}^{-1} d_{t+1} \right]}{\partial k_{t+1}}
\]

\[
= \frac{\partial E_t \left[ -k_{t+1} + R_{t+1}^{-1} y_{t+1} \right]}{\partial k_{t+1}}
\]

\[
= E_t \left[ \frac{\alpha y_{t+1}}{R_{t+1} k_{t+1}} - 1 \right].
\]

Setting the derivative to zero yields the stochastic Euler condition:

\[
E_t \left[ \frac{\alpha y_{t+1}}{R_{t+1} k_{t+1}} \right] = 1.
\]

That is, the expected marginal product of investment, properly discounted, must equal the one unit of the consumption good sacrificed in favor of investment.
4.1.2 The Representative Household

Given that Household owns $\xi_t$ shares of Firm at the beginning of each period, the former maximizes the Von Neumann-Morgenstern utility function by allocating between consumption, $c_t$, and the number of shares invested next period, $\xi_{t+1}$.

$$\max_{\xi_{t+1}} \quad U_0 = E_0 \sum_{t=0}^{\infty} \beta^t u(c_t)$$

subject to

$$c_t = (p_t + d_t)\xi_t - p_t \xi_{t+1},$$

where $\beta \in (0,1)$ represents the subjective discount factor for utility. Substitute the constraints into the objective function and solve the optimization problem at a particular time $t$.

$$\frac{\partial U_t}{\partial \xi_{t+1}} = \frac{\partial E_t [u(c_t) + \beta u(c_{t+1})]}{\partial \xi_{t+1}} = E_t [-u'(c_t)p_t + \beta u'(c_{t+1})(p_{t+1} + d_{t+1})].$$

Setting the derivative to zero yields the stochastic Euler condition:

$$p_t u'(c_t) = E_t [\beta u'(c_{t+1})(p_{t+1} + d_{t+1})].$$

$p_t u'(c_t)$ is the loss in utility if Household buys another unit of asset. $E_t [\beta u'(c_{t+1})(p_{t+1} + d_{t+1})]$ is the increase in (expected, discounted) utility Household obtains from the extra payoff at $t+1$. Household continues to buy/sell the asset until marginal loss is equal to marginal gain in utility. Rewrite the condition as follows

$$p_t = E_t \left[ \beta \frac{u'(c_{t+1})}{u'(c_t)} (p_{t+1} + d_{t+1}) \right].$$
Solve the equation forward and recognize that \( \lim_{h \to \infty} \beta^h p_{t+h} = 0 \) to yield the general expression for ex-dividend share price:

\[
p_t = E_t \sum_{h=1}^{\infty} \beta^h \frac{u'(c_{t+h})}{u'(c_t)} d_{t+h}.
\]  

(3)

**4.1.3 The General Equilibrium Model**

In order to solve the general equilibrium model, assume a power utility function,

\[ u(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}, \quad \gamma \in (0,1) \]

that satisfies two common conditions,

- **Non-satiation**: \( u'(c_t) = c_t^{-\gamma} > 0 \),
- **Risk-aversion**: \( u''(c_t) = -\gamma c_t^{-2-\gamma} < 0 \)

In addition, assume that consumption is growing at a constant rate \( G \):

\[ c_{t+1} = G c_t, \quad G > 1 \]

Because shares are just claims to Firm’s dividends, we can normalize the number of shares such that \( s_t = 1 \ \forall t \) and let dividends adjust to clear market. Then, the Household’s budget constraint implies that \( c_t = d_t \) at market equilibrium. Applying the utility function
to equation (3) yields

\[ p_t = E_t \sum_{h=1}^{\infty} \beta^h \frac{c_t^{\gamma}}{c_{t+h}^{\gamma}} d_{t+h} \]

\[ = E_t \sum_{h=1}^{\infty} \beta^h d_t^{1-\gamma} \]

\[ = E_t \sum_{h=1}^{\infty} \beta^h (G^h d_t)^{1-\gamma} \]

\[ = E_t \sum_{h=1}^{\infty} (\beta G^{1-\gamma})^h d_t \]

Let \( \beta^* = \beta G^{1-\gamma} < 1 \) so that stock price \( p_t \) is finite. Solving the geometric sum for stock price yields

\[ p_t = \frac{\beta^*}{1 - \beta^*} d_t. \]

Furthermore,

\[ R_{t+1} = \frac{p_{t+1} + d_{t+1}}{p_t} \]

\[ = \frac{\left(\frac{\beta^*}{1 - \beta^*} + 1\right) d_{t+1}}{\beta^* d_t} \]

\[ = \frac{1}{\beta^*} \frac{d_{t+1}}{d_t}. \]

The solution to (2) and (4) is

\[ k_{t+1} = \alpha \beta^* y_t. \]

To see this, substituting the solution into Firm’s budget constraint yields

\[ d_t = (1 - \alpha \beta^*) y_t. \]

This says that at equilibrium, Household requires less dividends for present consumption if there is high cost of capital (\( \alpha \)), high preference for future consumption (\( \beta \)), high consumption growth rate (\( G \)), and low risk aversion (\( \gamma \)). Substitute this into the returns
equation

$$R_{t+1} = \frac{1}{\beta^*} \frac{d_{t+1}}{d_t} = \frac{1}{\beta^*} \frac{y_{t+1}}{y_t}. \quad (4)$$

Substitute (4) into (2) to verify the solution. Overall, stock price is related to output as follows

$$p_t = \frac{\beta^* (1 - \alpha \beta^*)}{1 - \beta^*} y_t. \quad (5)$$

Taking the log of (5) yields the cointegrating relationship,

$$\ln p_t = \ln \frac{\beta^* (1 - \alpha \beta^*)}{1 - \beta^*} + \ln y_t. \quad (6)$$

The model predicts that in a closed economy, stock index and output will cointegrate with cointegrating vector $[1, -1]$ and an intercept.

### 4.2 Panel Cointegration Test

In order to test the model’s hypothesis, I employ the Pedroni heterogeneous panel cointegration test (1999) that computes the group mean parametric t-statistic. Equation (6) motivates the following regression model for a specific country $i$:

$$\ln p_{i,t} = \ln \frac{\beta^*_i (1 - \alpha_i \beta^*_i)}{1 - \beta^*_i} + \ln y_{i,t} + \epsilon_{i,t}$$

If cointegration exists, $\epsilon_{i,t} \sim I(0) \forall i$. Because the data used for country stock prices and industrial production are indexes (rather than raw values) with different compositions, it will be difficult to test the exact numerical values of the coefficients. However, it is still reasonable to expect cointegration to exist. Let $ip_{i,t}$ be the log industrial production.
index of country $i$. The testing model is specified as

$$s_{i,t} = \alpha_i + \beta_i i p_{i,t} + e_{i,t}$$

where $e_{i,t} \sim I(0) \forall i$. I estimate $e_{i,t}$ by OLS for each member country individually. Then, testing for panel cointegration is equivalent to applying IPS to $\hat{e}_{i,t}$. Pedroni (1999) reports the adjusted values for the asymptotic mean, $\mu$, and variance, $\nu$. I refer to this as panel cointegration at composite level. Because I derive equation (6) under the context of a closed economy, in order for it to apply at a global level, we need to account for cross-dependence between countries. Following Pedroni (2013) and the discussion in section 2.2, I argue that the composite stock index $s_{i,t}$ and industrial production $ip_{i,t}$ can be decomposed into their common and idiosyncratic components as follows,

$$s_{i,t} = \tilde{s}_{i,t} + \bar{s}_{i,t}, \quad ip_{i,t} = \tilde{ip}_{i,t} + \bar{ip}_{i,t}$$

where $\bar{s}_{i,t} = \frac{1}{N} \sum_{i=1}^{N} s_{i,t}$ and $\bar{ip}_{i,t} = \frac{1}{N} \sum_{i=1}^{N} ip_{i,t}$ (common components) are the common time effects to account for any cross-dependence between countries. $\tilde{s}_{i,t}$ and $\tilde{ip}_{i,t}$ are called the country-specific or idiosyncratic components. Then, panel cointegration test can be applied directly to the idiosyncratic components. For the common components, I apply the usual Engle & Granger cointegration test. For stock indexes and industrial production to cointegrate at composite level, their idiosyncratic (common) components should cointegrate and the cointegrating vector should be the same for both components. This process will help us understand how cointegration exists in an international context and hence, aids in future modelling of long-run dynamics.

### 4.3 Empirical Results

I collect industrial production data from International Financial Statistics (IFS). Because I want to analyze the trends in industrial production, the data are seasonally adjusted.
IFS reports industrial production data in real terms with 2005 as the base year. Data are collected by governments in local currencies and deflated by local price indexes. To make the data compatible, I use local inflation adjusted stock indexes in this analysis. There are 15 developed economies with industrial production data available at quarterly frequency: Austria, Australia, Belgium, Canada, Switzerland, Germany, Denmark, Spain, France, United Kingdom, Italy, Japan, Netherlands, Norway, and United States. They exclude Singapore, Hong Kong, and Sweden. Sweden only has industrial production data since 1995. Because the analysis will be at quarterly frequency, we can add Australia’s local inflation adjusted stock index. Overall, the panel consists of the 15 countries mentioned above. Both stock indexes and industrial production data are in local inflation adjusted terms and at quarterly frequency. I conduct analysis for the same three subsamples used in section 3.5: 1975-1990 (64 quarters), 1991-2013 (90 quarters), and 1975-2013 (full sample, 154 quarters).

I first confirm that the composite, common, and idiosyncratic components of both stock indexes and industrial production are driven by unit root processes. For composite and idiosyncratic components, I report IPS panel unit root test. For the common components, I report the usual ADF test. Table 5 and 6 in Appendix B report the results. In all subsamples, IPS fails to reject the null of all unit root processes in both composite and idiosyncratic components of industrial production. IPS results for quarterly stock indexes at composite level are similar to those reported in section 3.5 for monthly local inflation adjusted stock indexes. Interestingly, IPS is able to reject the null for the idiosyncratic components of stock indexes at 5% and 1% over the 1975-1990 subsample and the full sample respectively. First, the way idiosyncratic components of stock indexes are constructed makes its IPS test similar to IPS for spatial pairwise cointegration. However, because there is no reason to impose a cointegrating vector of [1, −1] between stock indexes as argued in section 3.2, this is not a pairwise cointegration test. Rather, it is a general panel unit test with the alternative hypothesis of some stationary processes.
Then, it is more useful to consider the more powerful MMIB rank test for the number of shared stochastic trends in the panel. Second, since IPS test result for idiosyncratic components of stock indexes in the 1991-2013 subsample is very insignificant, the full-sample result is likely to be driven by the 1975-1990 subsample. Figure 4 in Appendix B reports the MMIB rank test results on rolling 10-year windows (40 quarters). Contrast to IPS, MMIB shows absolutely zero cointegrating vector for the idiosyncratic components of stock indexes. MMIB indicates up to only 2 cointegrating vectors in industrial production at composite level, the highest among all panels considered. In general, MMIB, as a more powerful and reliable test than IPS, supports that the composite and idiosyncratic components of stock indexes and industrial production are unit root processes. As shown in table 6 of Appendix B, ADF fails to reject the null of unit roots in the common components of both industrial production and stock indexes. The overall results continue to support my argument in section 2.2 because the idiosyncratic components of stock indexes do not cointegrate, one should not expect spatial cointegration of stock indexes at composite level.

In table 2 I report Pedroni panel cointegration test for composite and idiosyncratic components of stock indexes and industrial production. I report the Engle & Granger cointegration test for the common components. In the 1975-1990 subsample, all components of stock indexes and industrial production show significant evidence of cointegration. The statistic on the composite level is slightly smaller in absolute value. In the 1991-2013 subsample and the full sample, there is hardly any evidence of cointegration between the composite stock indexes and industrial production. However, both the idiosyncratic and common components of the two variables exhibit significant evidence of cointegration. As discussed in section 4.2 this implies different cointegrating vectors between stock index and industrial production across common and idiosyncratic components.
Table 2: Cointegration Tests for Quarterly Stock Indexes and Industrial Production

<table>
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<tr>
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<tbody>
<tr>
<td>Pedroni Panel Cointegration</td>
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<td></td>
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<tr>
<td>Composite</td>
<td>-1.85**</td>
<td>-0.01</td>
<td>-1.20</td>
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<tr>
<td>Idiosyncratic</td>
<td>-2.53***</td>
<td>-2.11**</td>
<td>-3.21***</td>
</tr>
<tr>
<td>Engle &amp; Granger</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Common</td>
<td>-2.61***</td>
<td>-3.07***</td>
<td>-4.14***</td>
</tr>
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<td></td>
<td>(0.01)</td>
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<td>(0.00)</td>
</tr>
<tr>
<td>T (quarters)</td>
<td>64</td>
<td>90</td>
<td>154</td>
</tr>
<tr>
<td>N (countries)</td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Both stock indexes and industrial production data are in local inflation adjusted terms and at quarterly frequency. The panel consists of 15 countries: AT, AU, BE, CA, CH, DE, DK, ES, FR, UK, IT, JP, NL, NO, and US. Max lag is set to be 8 for quarterly data. Critical value for last lag is 1.64. Critical values of Pedroni panel cointegration test are those of one-sided $N(0, 1)$: -1.28 (10%), -1.65 (5%), -2.33 (1%). I consider the robust critical values based on MacKinnon (2010) and report the approximate p-value based on MacKinnon (1994) in brackets underneath the Dicker-Fuller statistics for Engle and Granger cointegration test. Levels of significance are coded as: ***(1%)**, **(5%)**, *(10%)*. 

5 Concluding Remarks and Further Research

In conclusion, I confirm that there is no spatial cointegration between global stock indexes. Stock indexes and outputs show significant evidence of cointegration when carefully considering their common and idiosyncratic components. The cointegration results point to further research to identify structural shocks that can potentially forecast stock returns. The permanent-transitory decomposition suggested by Gonzalo and Ng (2001) allows us to identify permanent and transitory shocks in a cointegrated system. Panel structural VAR by Pedroni (2013) allows us to identify the common and idiosyncratic shocks. Combining these two techniques allows us to identify four types of shocks that potentially drive stock indexes: permanent-common, permanent-idiosyncratic, transitory-common, and transitory-idiosyncratic. Analysis of impulse responses, variance decomposition and
empirical backtesting can point us to the important shocks with forecasting power.

References


32


A Critical Values for MMIB by Simulation

Table 3: Critical Values for MMIB by Simulation, $T = 120$

<table>
<thead>
<tr>
<th>Null Rank ($c$)</th>
<th>18</th>
<th>17</th>
<th>16</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
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<td>Critical Values</td>
<td>53660</td>
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<td>39602</td>
<td>33555</td>
<td>28202</td>
<td>23216</td>
<td>18842</td>
</tr>
</tbody>
</table>

Critical values are derived from $10^4$ simulations following Pedroni et al. (2013) without including a deterministic trend. $T$ is fixed at 120.

Table 4: Critical Values for MMIB by Simulation, $T = 40$

<table>
<thead>
<tr>
<th>Null Rank ($c$)</th>
<th>15</th>
<th>14</th>
<th>13</th>
<th>12</th>
<th>11</th>
<th>10</th>
<th>9</th>
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<tr>
<td>Critical Values</td>
<td>21446</td>
<td>18420</td>
<td>15799</td>
<td>13280</td>
<td>11049</td>
<td>8961</td>
<td>7085</td>
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</tbody>
</table>

Critical values are derived from $10^4$ simulations following Pedroni et al. (2013) without including a deterministic trend. $T$ is fixed at 40.
### B Unit Root Tests for Quarterly IP & Stock

Table 5: IPS Panel Unit Root Tests for Quarterly Stock Indexes and Industrial Production

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<thead>
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<td><strong>Industrial Production</strong></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>2.37</td>
<td>0.04</td>
<td>0.24</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>-1.05</td>
<td>3.33</td>
<td>4.69</td>
</tr>
<tr>
<td><strong>Stock Index</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Composite</td>
<td>2.54</td>
<td>-1.39*</td>
<td>1.69</td>
</tr>
<tr>
<td>Idiosyncratic</td>
<td>-2.21**</td>
<td>-0.21</td>
<td>-2.62**</td>
</tr>
<tr>
<td><strong>T (quarters)</strong></td>
<td>64</td>
<td>90</td>
<td>154</td>
</tr>
<tr>
<td><strong>N (countries)</strong></td>
<td>15</td>
<td>15</td>
<td>15</td>
</tr>
</tbody>
</table>

Both stock indexes and industrial production data are in local inflation adjusted terms and at quarterly frequency. The panel consists of 15 countries: AT, AU, BE, CA, CH, DE, DK, ES, FR, UK, IT, JP, NL, NO, and US. Max lag is set to be 8 for quarterly data. Critical value for last lag is 1.64. Critical values of IPS panel unit root test are those of one-sided $N(0, 1)$; -1.28 (10%), -1.65 (5%), -2.33 (1%). Levels of significance are coded as: ***(1%)**, **(5%)**, *(10%)*. 

Table 6: ADF Test for Stock Indexes and Industrial Production Common Components

<table>
<thead>
<tr>
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<tr>
<td><strong>Industrial Production</strong></td>
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<tr>
<td></td>
<td>-0.22</td>
<td>-1.56</td>
<td>-1.85</td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td>(0.51)</td>
<td>(0.35)</td>
</tr>
<tr>
<td><strong>Stock Index</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-0.20</td>
<td>-1.71</td>
<td>-0.99</td>
</tr>
<tr>
<td></td>
<td>(0.94)</td>
<td>(0.43)</td>
<td>(0.76)</td>
</tr>
<tr>
<td><strong>T (quarters)</strong></td>
<td>64</td>
<td>90</td>
<td>154</td>
</tr>
</tbody>
</table>

Both stock indexes and industrial production data are in local inflation adjusted terms and at quarterly frequency. The panel consists of 15 countries: AT, AU, BE, CA, CH, DE, DK, ES, FR, UK, IT, JP, NL, NO, and US. Max lag is set to be 8 for quarterly data. Critical value for last lag is 1.64. I consider the robust critical values based on MacKinnon (2010) and report the approximate p-value based on MacKinnon (1994) in brackets underneath the Dicker-Fuller statistics. Levels of significance are coded as: ***(1%)**, **(5%)**, *(10%)*. 

35
Figure 4: MMIB test for the number of common stochastic trends present in the panel of local inflation adjusted stock indexes and industrial production on rolling 10-year windows. The panels include quarterly data of 15 countries: AT, AU, BE, CA, CH, DE, DK, ES, FR, UK, IT, JP, NL, NO, and US. Critical values from $10^4$ simulations are reported in Table 4 of appendix A.