Distances in Astronomy

One of the most important (and difficult!) problems in astronomy is the accurate determination of distances to planets, stars, galaxies, etc. Different methods are used at different distances, but most are based on either trigonometry or . The purpose of this exercise is to show that:

- the relationship between size, angular size, and distance can be used to determine distances
- parallax can be used to determine distances
- the relationship between apparent brightness and distance can be used to determine distances

Background

A. Physical Size, Angular Size, and Distance

When you look at an object, you don't see its physical size (in, e.g., meters). You see the angle that it occupies, from your vantage point. A full Moon (~ 1/2 degree) looks much smaller than your hand (~ 10 degrees), but only because it is far away. This is called *angular size*, (or, in the case of the spacing between two objects, *angular separation*) and it is the only kind of size that any optical imaging system, from your eye to the largest telescope, can measure. The angular size θ of an object depends on its physical size a and its distance d, as shown in Figure 1,

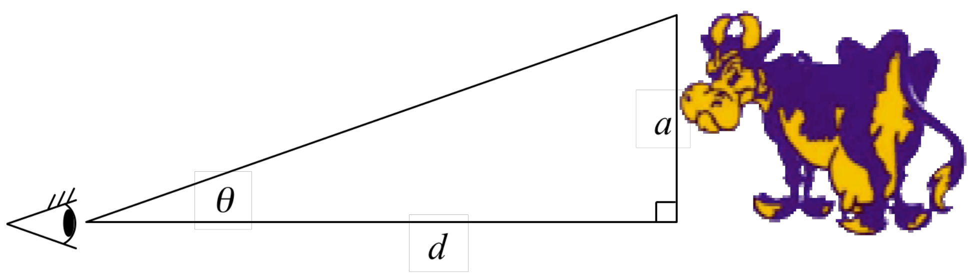

Figure 1: Apparent Size of a Purple Cow

and their mathematical relationship is

$$\tan\theta = a / d .$$

If you know any two of these numbers, you can compute the third. In astronomy the angle θ is often rather small, so <u>if θ is expressed in radians</u>

$$\theta \approx a / d .$$

Recall that a complete circle is 2π radians = 360 degrees. Angular sizes or separations may also be measured in *arcminutes* (60 arcmin = 1 degree) or *arcseconds* (60 arcsec = 1 arcmin).

B. Parallax

Look at the tip of your nose, first with your left eye, and then with your right. Notice that the background directly behind your nose appears to change. This effect is called *parallax*. Now imagine observing a field of stars two times during a year, when the Earth is at different points in its orbit. Most stars are so far away that they do not appear to change position between the two observations. Some, though, appear to move against the background of more-distant stars. The angle that a star appears to move depends on how far away it is; a closer star will appear to move more than a more distant star. This is just like the situation in Figure 1, with the cow replaced by half of the Earth's orbit, as in Figure 2.

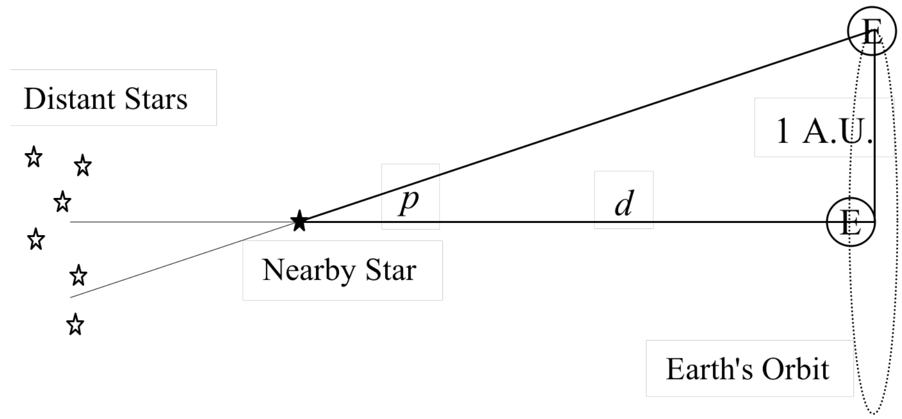

Figure 2: Stellar Parallax

For a simulation of this effect, go to

http://instruct1.cit.cornell.edu/courses/astro101/java/parallax/parallax.html

and play with the simulation until you clearly see what's going on.

Starting with the second equation above, replacing θ with p (the parallax angle, now in arcsec) and setting a to 1 A.U., the distance d, in *parsecs*, of an object is the inverse of its parallax angle

$$d = 1 / p \quad .$$

Therefore, a star that appears to move 1 arcsec while the Earth moves laterally by its orbital radius is at a distance of 1 parsec. One parsec is about 3.26 light-years (the distance that light travels in one year), or 30,856,780,000,000 km.

C. How Apparent Brightness Relates to Distance

Imagine that a lamp illuminates a painting on a wall, and provides exactly the right amount of light. The lamp is moved twice as far from the painting, which now seems dimly lit. This is because the apparent brightness B of a point source of light is inversely proportional to the square of its distance d:

$$B \propto 1/d^2$$

At twice the distance the brightness has fallen off by a factor of $2^2 = 4$.

This applies directly to finding distances to astronomical objects. By comparing an object's known brightness (from its absolute magnitude M) to its apparent brightness (from its apparent magnitude m), and knowing the relation above, its distance can be determined. The actual relationship between apparent magnitude, absolute magnitude, and distance is

$$d = 10^{\left(\frac{m-M+5}{5}\right)} \quad ,$$

where d is the distance in parsecs.