

## Parameters of a Strömgren Sphere

Let's assume that we have a 40,000 K B-main sequence star surrounded by an interstellar medium composed entirely of hydrogen (density  $N(H) = 1$  particles  $\text{cm}^{-3}$ ).

*How long will a neutral hydrogen atom stay neutral?* The photon luminosity of a B main sequence star is

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = 5 \times 10^{48} \text{ photons s}^{-1} \quad (19.01)$$

where  $\nu_0$  is the ionization frequency for hydrogen. Let's now assume all the ionizing radiation comes from the star. The photoionization rate at a distance  $r$  from the star is

$$P = N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu \quad (19.02)$$

Note the terms in the equation. The photoionization rate is proportional to the number of neutral atoms that are around to be photoionized, the number of photons available for photoionization, and the cross section for ionization. For a precise calculation, we should integrate this equation, but for now, we can just adopt an “average” value of  $a_\nu$ . For a 40,000 K star, only the high-end tail of the blackbody function extends shortward of 912 Å, hence almost all ionizations occur with  $a_\nu \approx 5 \times 10^{-18} \text{ cm}^2$ . So

$$P = N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu \approx \frac{N(H^0) \bar{a}_\nu}{4\pi r^2} \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu \quad (19.03)$$

If we now pick a position, say, 5 pc away from the central star, then we can just plug in the numbers to get  $P \sim 10^{-8}$  ionizations  $\text{cm}^{-3} \text{ s}^{-1}$ . In other words, in a cubic centimeter of space, the time between ionizations is  $10^8$  seconds: a neutral hydrogen atom can stay neutral for over a year!

*What is the distribution of states for hydrogen?* The time it takes an atom to decay from state  $nL$  to state  $n'L'$  is just one over the Einstein  $A$  value. The time it takes an atom to decay from state  $nL$  to *any* state is therefore

$$\tau_{nL} = \frac{1}{\sum_{n' < n} \sum_{L' = L \pm 1} A_{nL, n'L'}} \quad (19.04)$$

(Note that by specifying that  $L' = L \pm 1$ , we are neglecting the forbidden transitions.). Typical  $A$  values for permitted transitions are  $10^4 \lesssim A_{nL, n'L'} \lesssim 10^8 \text{ sec}^{-1}$ , so  $10^{-4} \gtrsim \tau_{nL} \gtrsim 10^{-8}$  seconds.

There is one exception to this. A hydrogen atom in the  $2 \ ^2S$  state, has no permitted way to decay down to  $1 \ ^2S$ . Atoms in this state must either be a) collided out of the state, or b) decay via a low probability ( $A = 8.23 \text{ sec}^{-1}$ ) two-photon emission, where the atom makes a temporary state for itself between  $1 \ ^2S$  and  $2 \ ^2S$ . If this occurs, the atom will be trapped in  $2 \ ^2S$  for  $\tau = 0.12$  seconds.

In any event, the time for a hydrogen atom to decay is less than a second, whereas it must wait a year before a neutral atom gets photoionized. Consequently, all photoionizations occur from the ground state.

*What fraction of hydrogen atoms will be neutral?* Let  $\xi$  be the fraction of neutral hydrogen, *i.e.*,  $\xi = N(H^0)/N(H)$ , and again, let's consider a spot 5 pc from a 40,000 K main sequence star. As we have seen, at that location

$$P = N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu \approx N(H^0) 10^{-8} \text{ sec}^{-1} \quad (19.05)$$

For an emission region to be in equilibrium, the number of ionizations (in each cubic centimeter) must equal the number recombinations. The latter number is given by

$$R = N_e N_p \alpha_A \quad (19.06)$$

In other words, the recombination rate is proportional to the density of free electrons, the density of hydrogen atoms needing an electron, and the recombination coefficient,  $\alpha_A \sim 4 \times 10^{-13} \text{ cm}^3 \text{ s}^{-1}$ . Ionization balance implies

$$N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{4\pi r^2 h\nu} a_\nu d\nu = N_e N_p \alpha_A \quad (19.07)$$

In a pure hydrogen nebula,  $N_e = N_p$ , and  $N_p = (1 - \xi)N(H)$ . Thus

$$\xi N(H)(10^{-8}) = (1 - \xi)^2 N(H)^2 (4 \times 10^{-13}) \quad (19.08)$$

or

$$2.5 \times 10^4 \xi = (1 - \xi)^2 N(H) \quad (19.09)$$

For a typical ISM value of  $N(H) \sim 1$ ,  $\xi = 4 \times 10^{-5}$ . Virtually all the hydrogen is ionized!

*How thick is the transition region between the ionized and neutral material?* The mean free path of an ionizing photon is given by

$$\ell = \frac{1}{N(H^0)a_\nu} \quad (19.10)$$

(the larger the cross section, or higher the density of neutral atoms, the shorter the path). By definition, in the transition region, half of the hydrogen is neutral. Thus the mean free path of an ionizing photon in this region is

$$\ell_{trans} \sim \frac{1}{0.5N(H)a_\nu} \quad (19.11)$$

For  $a_\nu \sim 5 \times 10^{-18} \text{ cm}^2$  and  $N(H) \sim 1$ ,  $\ell_{trans} \approx 4 \times 10^{17} \text{ cm}$ , or 0.1 pc. Thus, when the material starts to become neutral, the ionizing photons are rapidly eaten up. Considering that the entire H II region might be  $\sim 10 \text{ pc}$  across, the transition region is very thin.

(Note: if the source of ionizing radiation is very hard (for instance, as in the power law continuum of an AGN), then the effective  $a_\nu$  can be significantly smaller. This can translate into a large transition region.)

*What is the size of an ionized region?* To calculate this, let's start with the full equation for ionization balance. The ionization rate at every location in the nebula must be equal to the recombination rate at that location.

$$N(H^0) \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu d\nu = N_e N_p \alpha_A \quad (19.12)$$

Here,  $J_\nu$  is the mean intensity of radiation, in units of energy per unit area, per unit time, per unit frequency, per unit solid angle. (The  $4\pi$  term is the implicit integration over all directions.) Note that  $J_\nu$  has two components. Ionizing radiation can come from the central star, but it can also come from a diffuse component. If, somewhere in the nebula, an electron recombines from the continuum directly into the ground state, the photon emitted will have enough energy to ionize another atom.

Keeping track of the diffuse ionizing field is fairly complicated. However, for some nebulae, there is an excellent approximation. Most recombinations will produce a photon with  $\nu \approx \nu_0$ ; such a photon has a high cross section for re-absorption. Therefore, let us make the **on-the spot assumption** that **all photons produced by ground state recombinations are absorbed locally**. Any recombination to the ground state produces a photon that is immediately re-absorbed by a neighboring atom: it's as if no ground state recombination ever occurred.

(In low density nebulae with  $N(H) \sim 1$ , this approximation is not very good, since, as we have seen, the mean free path for an ionizing photon is

$$\ell = \frac{1}{N(H^0)a_\nu} \quad (19.10)$$

If the neutral fraction is small,  $\xi \sim 10^{-4}$ , then  $\ell$  can be large.

However, as the density of particles increases, the mean free path decreases, and the approximation becomes better and better.)

If we neglect the diffuse component of the ionizing radiation field, then the only ionizing photons are those that come from the central star. If each square centimeter of the star produces a monochromatic flux of  $\pi F_\nu$  (ergs cm<sup>-2</sup> s<sup>-1</sup> Hz<sup>-1</sup>), then any point in the nebula sees this field, diluted by a geometrical term, and by the attenuation that has already occurred. In other words,

$$N(H^0) \int_{\nu_0}^{\infty} \frac{4\pi J_\nu}{h\nu} a_\nu d\nu = N(H^0) \int_{\nu_0}^{\infty} \frac{\pi F_\nu}{h\nu} \left( \frac{R^2}{r^2} \right) a_\nu e^{-\tau_\nu} d\nu \quad (19.13)$$

where  $R$  is the radius of the star,  $r$  is the distance to the star, and  $\tau$ , the optical depth, reflects the fact that some photons from the star have already been absorbed before they reach  $r$ . Specifically, the equation for optical depth is

$$\tau_\nu(r) = \int_0^r N(H^0)(r') a_\nu dr' \quad (19.14)$$

Since the luminosity of the star is

$$L_\nu = 4\pi R^2(\pi F_\nu) \quad (19.15)$$

the equation of ionization balance simplifies to

$$\frac{N(H^0)}{4\pi r^2} \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} d\nu = N_e N_p \alpha_B \quad (19.16)$$

where now we are using  $\alpha_B$ , which is the recombination rate to all levels except the ground state of hydrogen.

Now, let's assume that the density of the emission region is roughly constant. We can re-write (19.16) as

$$N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} d\nu = 4\pi r^2 N_e N_p \alpha_B \quad (19.17)$$

Now let's integrate both sides over  $r$ , and let  $\mathcal{R}$  represent the extent of the nebula.

$$\int_0^{\mathcal{R}} N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} d\nu dr = \int_0^{\mathcal{R}} 4\pi r^2 N_e N_p \alpha_B dr \quad (19.18)$$

We can replace  $dr$  with  $d\tau/N(H^0)a_\nu$ , since

$$\tau_\nu(r) = \int_0^r N(H^0)(r') a_\nu dr' \implies \frac{d\tau}{dr} = N(H^0) a_\nu \quad (19.19)$$

And, at the edge of the nebula  $\tau = \infty$  (by definition), so

$$\int_0^{\mathcal{R}} N(H^0) \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} a_\nu e^{-\tau_\nu} \frac{d\tau}{N(H^0)a_\nu} = \int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu \int_0^{\infty} e^{-\tau_\nu} d\tau \quad (19.20)$$

or just

$$\int_{\nu_0}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(H^0) \quad (19.21)$$

In other words, the left hand side of (19.17) is just the number of ionizing photons coming from the star. (This should not be too surprising, since, one way or the other, all the photons must be eaten up.) In the case of a constant density nebula, the right hand side of (19.17) is straightforward to integrate, and, since the ionization fraction inside the nebula is extremely high,  $N_p \approx N_e \approx N(H)$ . So

$$Q(H^0) = \frac{4}{3} \pi \mathcal{R}^3 N(H)^2 \alpha_B \quad (19.22)$$

Thus, we have a very simple expression for the size of the nebula. Our 40,000 B-main sequence star will generate an H II region  $\sim 50$  pc in radius.

## Modifications Due to Helium

Photons with energies greater than 24.6 eV (504 Å) can ionize helium and, due to the  $\nu^{-3}$  dependence on cross section, such photons are  $\sim 8$  times more likely to ionize helium than hydrogen. At first glance, this would seem to mean that helium can significantly reduce the size of a Strömngren sphere. However, the energy difference between the  $n = 1$  state of helium, and the  $n = 2$  state is about 19.9 eV. (The exact difference depends on whether we are talking about singlet states, or triplet states, and whether the  $n = 2$  term is a  $P$  or an  $S$ .) Thus, almost every recombination of a helium atom will produce another photon capable of ionizing hydrogen. Consequently, the effect of helium on the size of the Strömngren sphere is minor.

(Note: some interesting physics is associated with the He I atom. Since there are two electrons, He I has singlet levels (opposite spin electrons) and triplet levels (same spin electrons). Like hydrogen, singlet decays from  $2^1P$  to  $1^1S$  are no problem, but  $2^1S$  to  $1^1S$  is forbidden; these atoms can only decay by creating a temporary state somewhere between  $2^1S$  and  $1^1S$  and emitting 2 photons ( $A = 51 \text{ sec}^{-1}$ ). When this occurs, 56% of the time, one of the photons will have an energy greater than 13.6 eV. Atoms in the triplet state, however, have a harder problem: once they get to  $2^3S$ , the only way for them to decay is to have one electron change its spin. The  $A$  value for this is  $A = 1.27 \times 10^{-4} \text{ sec}^{-1}$ . If the spin-change does occur, then the resulting photon can ionize hydrogen, but since the timescale for this is long ( $\tau \approx 2.2$  hours), it's possible that the atom will get collided out of  $2^3S$  before it can decay. In fact, some of these collisions will dump the electron directly in  $1^1S$ . If this occurs, an ionizing photon will be lost. Depending on the density, He I will eat up between from 4% to 34% of hydrogen's ionizing photons.)



The accurate computation of ionization balance for a hydrogen-helium nebula can be moderately complicated, but the problem can be approximated by noting that  $a_\nu(He^0)$  is always significantly larger than  $a_\nu(H)$ . Thus, if you make the assumption that all photons capable of ionizing helium do ionize helium, then

$$\int_{\nu_1}^{\infty} \frac{L_\nu}{h\nu} d\nu = Q(He^0) = \frac{4}{3}\pi N(He^+) N_e \alpha_B(He^0) \mathcal{R}_{He^+}^3 \quad (19.23)$$

where  $\nu_1$  is the ionization frequency of helium, and  $\mathcal{R}_{He^+}$  is the radius of the He I Strömngren sphere.

If the central star is hot enough to produce photons with energies greater than 54.4 eV (228 Å), then helium can become doubly ionized. Because of the extremely large differences between the energy levels of He II, a recombination to He II will produce, on average, 1.25 photons capable of ionizing hydrogen.