Econ 233, Problem Set 2
Externalities in a Market Economy - Answers

a) The expected payoff from production is $yP = 5 * \frac{1}{2} = 2.5$. Since the cost of production is distributed uniformly on interval (0,10); only 0.25 agents will find that their cost of production is less than the expected payoff. Hence, the value of the aggregate output is $5 * 0.25 = \frac{5}{4}$.

b) In this case there are positive externalities because each agent's action has a positive effect on other agents' expected payoff. The number of people who will climb the tree is $\frac{yP-0}{10-0}$. This is because the costs of climbing the tree are distributed uniformly on the interval (0,10). Anyone who is to the left of yP will climb the tree. The aggregate output is the solution to the following equation:

$$\begin{aligned} Y &= y(\#\text{ of agents with } c_i < yP(Y)) \\ Y &= y\frac{yP(Y)}{10} \\ Y &= 5\frac{5\frac{Y}{Y+1}}{10} \\ Y &= \frac{3}{2} \end{aligned}$$

The equilibrium is unique because the equation has only one solution.

c) Once again there are positive externalities, because the probability of making a trade *increases* with aggregate output. The equation for aggregate output is:

$$\begin{aligned} Y &= 5\frac{5\frac{Y-\frac{1}{5}}{Y}}{10} \\ 0 &= Y^2 - \tfrac{5}{2}Y + \tfrac{1}{2} \end{aligned}$$

which has two solutions: a high level of output $Y_H = 2.28$; and a low level of output $Y_L = 0.22$. There are multiple equilibria and there is the potential for a useful government action to try to move the economy from a low to a high activity equilibrium.

d) In this case each agent's action affects other agents' payoffs negatively. The aggregate output lowers the probability of trade, hence, there are *negative* externalities. This situation is likely to arise when there are congestion effects. It may be harder to produce if road or phones are congested with other people trying to produce or trade. In summary, both positive and negative externalities present a market failure.