Constructing Confidence Intervals[*]

Econ 253

Case 1: 95% Confidence Interval for Population Mean, Variance Known

Since $\frac{\overline{x}-\mu}{\sigma/\sqrt{n}}$ is distributed standard normal, the $P(\overline{x}-1.96\sigma/\sqrt{n} < \mu < \overline{x}+1.96\sigma/\sqrt{n}) = 0.95$ so the confidence interval for population mean is:$(\overline{x} - 1.96\sigma/\sqrt{n}, \quad \overline{x} + 1.96\sigma/\sqrt{n})$.

Q: In a random sample of 50 undergraduate men at Williams, the sample mean height was 6 feet. If the population standard deviation is known to be 0.1 feet, construct a 95% confidence interval for the population mean.

A: Using the formula above, the 95% confidence interval is $(6 - (1.96)(0.1)/\sqrt{50}, \quad 6 + (1.96)(0.1)/\sqrt{50})$.

Case 2: 95% Confidence Interval for Population Mean, Variance Unknown

In this case we have to replace the population standard deviation σ with sample standard deviation s. Then $\frac{\overline{x}-\mu}{s/\sqrt{n}}$ is distributed t_{n-1}. We can find number A such that $P(-A < \frac{\overline{x}-\mu}{s/\sqrt{n}} < A) = 0.95$. [†] The inequality inside the probability can be written as $P(\overline{x} - As/\sqrt{n} < \mu < \overline{x} + As/\sqrt{n}) = 0.95$. Hence, the confidence interval for population mean is: $(\overline{x} - As/\sqrt{n}, \quad \overline{x} + As/\sqrt{n})$

Q: In a random sample of 41 undergraduate men at Williams, the sample mean height was 6 feet. If the sample standard deviation s is 0.5 feet, construct a 95% confidence interval for the population mean.

A: Since the sample size is 41, we need t_{40}. In the t-table (p. 501) first find the degrees of freedom (40), in the extreme left column. Then, in the second row of numbers in bold at the top of the table, find the number 0.05. The number in the table corresponding to this row and column is 2.021. This means a variable distributed t_{40} lies between -2.021 and 2.021 with probability 95%. The confidence interval for population mean is $(\overline{x}-2.021s/\sqrt{n}, \quad \overline{x}+$

[*]Notation: $\overline{x}$ is the sample mean, μ is the population mean, n is the sample size, s^2 is the sample variance, σ^2 is the population variance, s is the sample standard deviation, and σ is the population standard deviation.

[†]A is the 97.5 percentile of the t_{n-1} distribution, i.e. 97.5% of the distribution lies to the left of $t_{n-1,(0.975)}$.

$2.021s/\sqrt{n}$). Plugging in the appropriate values, the 95% confidence interval is (5.842, 6.158).

Case 3: 95% Confidence Interval for Variance

We know that $(n-1)\frac{s^2}{\sigma^2}$ is distributed χ^2_{n-1}. Using tables for χ^2 we can find numbers A and B such that $P(A < (n-1)\frac{s^2}{\sigma^2} < B) = 0.95$. [‡] We can manipulate the inequalities inside the probability to get the 95% confidence interval for population variance:$((n-1)\frac{s^2}{B}, (n-1)\frac{s^2}{A})$.

Q: A random sample of heights of 51 undergraduate men at Williams yields a sample standard deviation of 3 inches. Assume heights are normally distributed, and compute a 95% confidence interval for the population variance.

A: We know $s^2 = 3$, and $n - 1 = 50$. We need to find A and B. In the Chi-Squared table, (p. 508), find the degrees of freedom in the column on the extreme left (50). In this row, A and B are in the columns with the headings 0.975 and 0.025. Thus A and B are 32.357, and 71.420. Our confidence interval is: (50)(9)/32.357, (50)(9)/71.420.

[‡] A and B are the 2.5 and 97.5 percentiles of the χ^2_{n-1} distribution.