

Difference in Means Test

Econ 253

This handout describes the procedure that can be used to test hypotheses about the difference between means of two populations: $X_1 \sim N(\mu_1, \sigma_1^2), X_2 \sim N(\mu_2, \sigma_2^2)$. Suppose we have a random sample of n_1 observations from distribution X_1 and n_2 observations from distribution X_2 . We calculate sample means and sample standard deviations: $\bar{x}_1, \bar{x}_2, s_1$ and s_2 . For large samples ($n_1, n_2 > 30$) the following test statistic is distributed standard normal:

$$Z = \frac{(\bar{x}_1 - \bar{x}_2) - (\mu_1 - \mu_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}}$$

The test can be summarized in the following table:

null hypothesis H_0	alternative hypothesis H_1	critical region reject H_0 if
$\mu_1 = \mu_2$	$\mu_1 > \mu_2$	$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} > Z_\alpha$
$\mu_1 = \mu_2$	$\mu_1 < \mu_2$	$Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} < -Z_\alpha$
$\mu_1 = \mu_2$	$\mu_1 \neq \mu_2$	$ Z = \frac{(\bar{x}_1 - \bar{x}_2)}{\sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}} > Z_{\alpha/2}$

where Z_α is such the area under the $N(0, 1)$ curve to the right of Z_α is α . Conventional values for α , the significance level, are 0.05 and 0.01.

Example: Suppose that a researcher collects a sample of male and female students and finds that on average female students had higher test scores than male students. Specifically, the researcher collected the following data:

males	females
$n_1 = 50$	$n_2 = 40$
$\bar{x}_1 = 80$	$\bar{x}_2 = 85$
$s_1 = 5$	$s_2 = 8$

Is the difference in average scores statistically significant?

$$H_0 : \mu_1 - \mu_2 = 0$$

$$H_1 : \mu_1 - \mu_2 \neq 0$$

The test statistic $Z = \frac{(80-85)}{\sqrt{5^2/50+8^2/40}} = -3.45$. We test H_0 at 5% level of significance. The 5% critical regions for normal distribution and a 2-tailed test are $(-\infty, -1.96)$ and $(1.96, \infty)$. Our statistic $Z < -1.96$, and falls in the critical region, hence we reject the null hypothesis that population means of scores of male and females students are equal.