

## Midterm I

Econ 253

Spring 2001

*You have 60 minutes to answer the following 6 questions. Each question is worth 20 points, so you should plan to spend an equal amount of time on each one. You may use a calculator and a single-sided sheet of paper with notes and formulas. Any collaboration is considered a violation of the honor code, as is discussing the exam with students in other sections who have not yet taken the exam. Good luck!*

1. The following table describes the joint probability distribution of stock price/earnings ratios ( $PE$ ) and their annual returns ( $R$ ).

		$R$		
		0%	1%	5%
$PE$	1	0	0.1	0.1
	10	0.1	0	0.2
	100	0.1	0.4	0

Answer the following questions:

- (a) What is the expected value of stock returns?
  - (b) What is the expected value of stock returns given that the  $PE$  ratio is equal to 100?
2. You need 18 computer memory chips to install in the motherboard of a PC. You order 20 memory chips because you know that 10% of all chips are defective.
    - (a) What is the probability that all chips you ordered will work?
    - (b) What is the probability that exactly 18 chips will work?
    - (c) What is the probability that you will be able to make your computer work?

3. A random sample of 500 owners of single-family homes is drawn from the population of a city. Let the random variable  $X$  denote annual household income in thousands of dollars, and the random variable  $Y$  denote the value of a house, also in thousands of dollars. The following information is available:

$$\begin{aligned}
 n &= 500 & \sum_{i=1}^n x_i &= 24,838 & \sum_{i=1}^n y_i &= 107,226 \\
 \sum (x_i - \bar{x})^2 &= 66,398 & \sum (y_i - \bar{y})^2 &= 1,398,308 \\
 \sum (x_i - \bar{x})(y_i - \bar{y}) &= 194,293
 \end{aligned}$$

Answer the following questions:

- Compute the sample mean and standard deviation of the value of the houses in this sample. Do the same for household income.
  - Compute the correlation between income and house value.
  - Construct a 95% confidence interval for the mean value of the houses. What assumptions do you have to make to do this?
4. Daily sales in a convenience store are normally independently distributed with an unknown mean  $\mu$  and a variance  $\sigma^2$ . In January the store was open 28 days and average daily sales were  $\bar{x}_J = \frac{1}{28} \sum_{i=1}^{28} x_i$ . In February the store was open only 20 days and average daily sales were  $\bar{x}_F = \frac{1}{20} \sum_{i=29}^{48} x_i$ . Consider two estimators of the population mean  $\mu$ :

$$\begin{aligned}
 \bar{x} &= \frac{1}{48} \sum_{i=1}^{48} x_i \\
 \tilde{x} &= \frac{1}{2}(\bar{x}_J + \bar{x}_F)
 \end{aligned}$$

Thus,  $\bar{x}$  is the sample mean over all observations, while  $\tilde{x}$  is the average of the two averages. Answer the following questions:

- Are both estimators unbiased?
- Which estimator is more efficient?

5. A credit card company surveyed 225 of its customers and found that their average income is \$35,000 with a sample standard deviation of \$5,102.
- (a) Construct a 95% confidence interval for population mean of income.
  - (b) How many customers should the company survey so that the 95% interval is within  $\pm\$100$  of the population mean?
6. Suppose that  $X$  denotes annual incomes, in thousands of dollars, and that  $X$  is normally distributed with mean 26 and variance 36. A random sample of 25 persons is drawn from the group. What is the probability that the average income is between \$25,000 and \$29,000?