Solutions to Assignment # 2, Econ 253

2.16

$E(X) = (-20)(0.10) + (-10)(0.15) + (10)(0.45) + (25)(0.25) + (30)(0.05) = 8.75\%$.

$E(X^2) = (-20)^2(0.1) + (-10)^2(0.15) + (10)^2(0.45) + (25)^2(0.25) + (30)^2(0.05) = 301.25$ percent squared.

$Var(X) = E(X^2) - \{E(X)\}^2 = 224.6875$ percent squared.

S.D (X) = Positive square root of variance = 14.989%.

2.19

(a) $Var(3X + 2) = 9\,Var(X) = 36$; $E(Y) = E(3X + 2) = 2 + 3E(X) = 2 + 24 = 26$.
(b) $Var(0.6X - 4) = (0.36)(Var(X)) = 1.44$; $E(Y) = E(0.6X - 4) = -4 + 0.6E(X) = 0.8$
(c) $Var(Y/4) = (1/16)Var(Y) = 0.25$; $E(Y) = E(X/4) = 1/4E(X) = 2$.
(d) $Var(aX+b) = Var(aX) = a^2Var(X)=a^2*4$; $E(aX+b)=a*8+b$

2.22 $Var(X + Y) = Var(X) + Var(Y) + 2\rho_{x,y}\sigma_x\sigma_y$

Therefore, $Var(X + Y) = 16 + 9 + 2(-0.8)(4)(3) = 5.8$.

If the money is divided equally between the two securities, we need

$Var(0.5X + 0.5Y)$

$= (1/4)Var(X) + (1/4)Var(Y) + 2\rho_{x,y}(0.5)\sigma_x(0.5)\sigma_y$

$= (1/4)(16) + (1/4)(9) + 2(-0.8)(0.5)(4)(0.5)(3) = 1.45$.

Clearly, diversification has reduced risk. Whether this is the right investment depends on the expected values of the returns to the two stocks as well as the investor's attitude to risk.

Additional Problems

1.
a. $Corr(X,Y) = Cov(X,Y)/\sigma_x\sigma_y$, and

$Cov(X,Y) = E(XY) - E(X)E(Y)$.

First obtain the marginal distributions of X and Y. Thus, P (X = 0) = 0.20 + 0.10 + 0.05 = 0.35, and so on. Once this is done, compute the expected values and standard deviations of X and Y using the same approach as in the problems above.

E (X) = 0.95, Var (X) = 0.6475, S.D (X) = 0.8046

Similarly, E (Y) = 1.05, V (Y) = 0.6475, S.D (X) = 0.8046

To compute E (XY) we can ignore all the instances where X= 0 or Y = 0. We get

E (XY) = (1)(0.20) + 2(0.10) + 2(0.05) + (4) (0.20) = 1.3

Cov (X,Y) = 1.3 - (0.95)(1.05) = 0.3025

Corr (X, Y) = 0.3025/(0.8046)(0.8046) = 0.467.

b. Start by contructing conditional pdf

x_i	$P(X=x_i \mid Y=2)$	$XiP(X=x_i \mid Y=2)$
0	0.05/0.35	0
1	0.1/0.35	10/35
2	0.2/0.35	40/35

E(X|Y=2)=50/35=1.42