Problem Set 3 – Answers
Econ 253

1. Let X be the amount of coffee in the jar. X is normally distributed with mean 8 and variance 0.04. We need P ($X \leq 7.8$). Subtracting the mean and dividing by the standard deviation we need P ($Z \leq (7.8 - 8)/0.2$) = P ($Z \leq -1$) = 0.5 - 0.3413 = 0.1587.

2. Let X be the life of the first bulb. Mr. Edison will be satisfied if it lasts more than 1200 hours. So the probability he will be satisfied with the first bulb is P ($X \geq 1200$). Again, subtract mean, divide by S.D. and use the standard normal table. We find P ($X \geq 1200$) = 0.1587. Similarly, for each of the other three bulbs the probability that they last 1200 hours or more is 0.1587. Since they are independent events, the probability that all four bulbs will last 1200 hours or more is $(0.1587)^4 = 0.00063$.

3. Let X be the amount of time taken by Ike and Y be the amount of time taken by Mike. We need P (X - Y) < 0. Let G = X - Y. First note that, since X and Y are normal, G is also normal. Using the properties of expectations, E (G) = -2. Since X and Y are independent, the variance of G is the sum of their variances. Thus Var (G) = $(3)^2 + (4)^2 = 25$. It follows that the standard deviation of G is 5. We now need P (G < 0) = P (Z < {0 - (-2)}/5) = P (Z < 0.4) = 0.6554.

4. If the total weight of 36 crates exceeds 8000 pounds, the mean weight must exceed 222.22. Using the Central Limit Theorem, the sample mean is distributed normally with mean 240 and variance $(66)^2/36$. If X* is the sample mean, we need P($X^* \geq 222.22$) = P ($Z \geq (222.22 - 240)/(66/6)$) = P ($Z \geq -1.616$) = 0.9474.