Solution Set, Assignment # 3, Econ 253

Problems from textbook

3.13 (a) If X is the amount of toothpaste in a tube, we need $P(X < 6)$. Subtracting mean and dividing by standard dev., we need $P(Z < (6 - 6.5)/0.8) = P(Z < -0.625)$.

Rounding off to –0.63 we get $(0.5 - 0.2357) = 0.2643$.

Out of 1000 tubes we would expect 264 to have less than 6 ounces.

3.16. Of each dollar invested, ½ each is going into the two stocks. The rate of return on the portfolio is therefore $G = (1/2)X + (1/2)Y$. Since X and Y are normal, G, which is a linear function of X and Y, is also normal.

$E(G) = E(X/2 + Y/2) = (1/2)E(X) + (1/2)E(Y) = 11.5$

$Var(G) = Var(X/2 + Y/2) = (1/4)Var(X + Y)$.

Recall that $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X,Y)$.

Note that $Cov(X,Y) = \sigma_x \sigma_y \rho_{x,y}$

Substituting the appropriate values we get

$Var(X + Y) = 25 + 4 + (2)(-0.4)(5)(2) = 21$. Hence $Var(G) = 21/4 = 5.25$.

X has high expected return and high risk; Y has low expected return and low risk. G is an intermediate case, with moderate risk and return, compared to the other two. The appropriate choice obviously depends on the tastes of the investor: "conservative" investors would pick Y, "aggressive" investors would pick X, and "moderates" might pick G. Of course various other types of diversification are possible, besides G.

3.14 (d) Let $G = 4X + 5Y$

Since G is a linear combination of two normal variables, it is also normal.

$E(4X + 5Y) = E(4X) + E(5Y) = 4E(X) + 5E(Y) = 40 + 75 = 115$

$Var(4X + 5Y) = Var(4X) + Var(5Y)$ (no covariance term because X and Y are independent).

This implies

$Var(4X + 5Y) = 16\ Var(X) + 25\ Var(Y) = 48 + 200 = 248.$

Thus $4X + 5Y$ is distributed normally, with mean 115 and variance 248.

Additional Problems

1. This is case 1 in the handout, confidence interval for population mean, population standard deviation known.

Note that the sample mean is (11,600)/50 = 232. Using the method from the handout, we have: $232 + 1.96 (20)/\sqrt{50}, 232 - 1.96 (20)/\sqrt{50}$.

2. Since X_1 and X_2 are a random sample they are independently and identically distributed. Let $G = X_1 - X_2$.

Since G is a linear combination of two random variables, it is also normal.

$E(G) = E(X_1) - E(X_2) = 0$ (because X_1 and X_2 have the same mean).

$Var(G) = Var(X_1) + Var(X_2) = 5000$ (we ignore the covariance term because X_1 and X_2 are independent)

We now need $P(-50 \leq G \leq 50)$. Convert G into a standard normal variable by subtracting the mean and dividing by the standard deviation. We need

$P(-50/70.71 \leq Z \leq 50/70.71) = 0.52.$

3. Case 1 from the handout (confidence interval for population mean, variance known). Use normal distribution.

$110.72 + (1.96)(46)/\sqrt{100}, 110.72 - (1.96)(46)/\sqrt{100}$

Computer exercises:

1.

```
b. summarize stud

Variable |     Obs        Mean    Std. Dev.       Min        Max
---------+-----------------------------------------------------
    stud |      23    18.91304      10.054          6         50
```

```
c. . ci stud, level(95)

Variable |      Obs        Mean    Std. Err.       [95% Conf. Interval]
---------+-------------------------------------------------------------
    stud |       23    18.91304    2.096405        14.56537    23.26072
```

d. H0: mu=20, H1: mu<20

t=(18.91-20)/(10.054/sqrt(23))=-1.09/2.096=0.52
the critical region is (-infty, -1.717). The test statistic is outside of the critical region hence we accept H0.

2. Monthly return on S&P500 is equal to percentage growth in the price index.

rsp=(SP500-SP500[_n-1])/SP500[_n-1]*100

```
summarize rsp

Variable |      Obs        Mean   Std. Dev.       Min        Max
---------+-----------------------------------------------------
     rsp |      557    .7638307   4.171158  -21.76304   16.30469
```

The probability that mean of returns is less negative is:

$$P(\mu < 0) = P(-\mu > 0) = P(\frac{\bar{x} - \mu}{s/\sqrt{n}} > \frac{\bar{x}}{s/\sqrt{n}})$$

$$= P(Z > 4.32) = 0$$

The confidence interval for S&P500 returns is (0.41,1.11). This means that with probability 95% the monthly return on S&P500 will be somewhere between 0.41% and 1.11%.