Solutions to Assignment # 4, Econ 253

4.20. (a) Suppose we call Standard values "X" and Petrocoal values "Y."

H_o: $\sigma_x^2 = \sigma_y^2$

H_a: $\sigma_x^2 \neq \sigma_y^2$

Since the sample standard deviation of Y is larger, we construct S_Y^2 / S_x^2. To do the test at 5% we find the value of $F_{15,15}$ such that there is 2.5% to the right. In StataQuest, choose "Inverse Statistical Tables' under the "Calculator" menu, enter the number 0.975 and choose numerator and denominator degrees of freedom 15 each. This yields the value 2.86. We will reject the null if S_Y^2 / S_x^2 exceeds 2.86.

We find $(0.6134)^2 / (0.5796)^2 = 1.12$. Therefore we do not reject the null.

(b) The assumptions are that these are two randomly selected samples from two normal populations.

Additional problem:

From tables we collect the following data

Variable	Sample mean	St.dev.	N
White males	529	101	324108
White females	526	99	387997
Total white students	528	100	712105
Total black students	499	124	119591

1.
H0: $\mu_{WM} - \mu_{WF} = 0$
H1: $\mu_{WM} - \mu_{WF} \neq 0$
We use difference in means test (see handout). The test stastic

$$\frac{\bar{x}_{WM} - \bar{x}_{WF}}{\sqrt{\frac{s_{WM}^2}{n_{WM}} + \frac{s_{WF}^2}{n_{WF}}}} = \frac{3}{0.238} = 12.59$$

is distributed normal (0,1). The critical values for this two-tailed test at 5% level of significance are-1.96 and 1.96. Our test statistic falls into the critical region and we reject the null that mean scores of white males are the same as those of white females.

2.

H0: $\mu_{WM} - \mu_{WF} = 2$

H1: $\mu_{WM} - \mu_{WF} > 2$

Again we use difference in means test (see handout). The test stastic

$$\frac{(\bar{x}_{WM} - \bar{x}_{WF}) - (\mu_{WM} - \mu_{WF})}{\sqrt{\frac{s_{WM}^2}{n_{WM}} + \frac{s_{WF}^2}{n_{WF}}}} = \frac{3-2}{0.238} = 4.2$$

is distributed normal (0,1). The critical value for this one-tailed test at 5% level of significance is 1.64. Our test statistic falls into the critical region and we reject the null hypothesis that the difference between mean scores of white males and females is equal to 2 point.

3.

H0: $\sigma_W = \sigma_B$

H1: $\sigma_W < \sigma_B$

We compare the variance of scores among black and white students the test statistic

$$F = \frac{s_B^2}{s_W^2} = \frac{124^2}{100^2} = 1.5376$$

is distributed F with ∞ and ∞ degrees of freedom. The critical region for this one-tailed test at 5% level of significance is (1, ∞). Our statistic falls into the critical region and we reject the null hypothesis that the differences among white students are the same as differences among black students. Thus the dispersion of scores is greater among black than among white students.

<u>Computer Exercise</u>

1.

```
generate ex = arsp-tbill
```

2.

```
. ttest ex = 0

  Variable |       Obs        Mean   Std. Dev.
  ---------+---------------------------------
        ex |        46    16.05542    17.15566

           Ho:  mean = 0
                   t = 6.35 with 45 d.f.
             Pr > |t| = 0.0000
               95% CI = (10.960824,21.150024)
```

With 45 d.f. the critical values on this 2-tailed test are -2.021 and 2.021. Our test statistic t is greater than critical value, hence we reject the null hypothesis that excess returns are zero.

Note 1: The computer output above gives the null hypothesis the value of the test statistic, the minimum significance level at which we could reject H0 (p-value), and the confidence interval for the population mean.

The p-value in this case is 0 which means that we could reject H0 at the significance level as low as 0%. The lower the p-value the more likely we are to reject H0.

3.

```
. ttest edji = esp, unpaired unequal

  Variable |      Obs        Mean   Std. Dev.
  ---------+---------------------------------
      edji |       46    15.32878    18.27266
       esp |       46    16.05542    17.15566
  ---------+---------------------------------
  combined |       92     15.6921

           Ho:  mean(x) - mean(y) = 0  (assuming unequal variances)
                      t = -0.20 with 89.64 d.f.
               Pr > |t| = 0.8446
                 95% CI = (-8.0687791,6.6155009)
```

The difference in mean returns of DJI and SP is statistically insignificant.

4.

```
. sdtest ardji=arsp

  Variable |      Obs        Mean   Std. Dev.
  ---------+---------------------------------
     ardji |       46    20.93877    18.03724
      arsp |       46    21.66541    17.05596
  ---------+---------------------------------
  combined |       92    21.30209    17.55346

           Ho:  sd(ardji) = sd(arsp)  (two-sided test)
   Lower tail:  F1(45,45) =  0.89
   Upper tail:  F2(45,45) =  1.12
    (Pr < F1) + (Pr > F2) =  0.7091
```

H0: sd(ardji)=sd(arsp)
H1: sd(arsp)=sd(ardji)
The test statistic is 18^2/17^2=290/325=0.89 and is distributed F with 45 and 45 d.f. In tables we find that the critical value for this one sided test is 1.69. Our test statistic is less than the critical value and we can not reject the null hypothesis that the variance of returns on both indexes are the same.

Note 1: The F table in the book does not have percentiles for exactly 45 d.f. only for 40 and 60 d.f. You can get precise critical values for all tables using stataquest, click on "Calculator," and choose "Inverse Statistical Tables." Then choose the appropriate distribution and levels of probability. In our case, go to inverse F table, enter 45 and 45 +degrees of freedom and 0.95 for the size of the left tail and you get the precise 5% critical value: 1.6415. The "Inverse Statistical Tables" submenu is also discussed on pages 221-222 of the STATAQUEST reference manual.

Note 2: The above computer output shows results for a two-sided test. The last line on the output shows the minimum significance level at which we could reject H0. Thus, in the case of the two sided test we could reject

H0 at 71% level of significance which is beyond the conventional level of 1% and 5% and we thus accept H0.