Solution Set, Assignment # 1, Econ 253, Spring 2000

<u>Textbook Problems</u>

2.2. No. If they are independent events, the occurrence of one tells you nothing about whether or not the other has occurred. If they are mutually exclusive, the occurrence of one rules out the other. If A and B are independent, $P(A|B) = P(A)$. If A and B are mutually exclusive, $P(A|B) = 0$.

2.14 (a) Because the probabilities must add up to 1, $b = 1/15$.

(b) $P(X \le 2) = 6/15$; $P(X \le 3) = 10/15$; $P(2 \le X \le 3) = 7/15$.

2.16 (d) For the cdf, assign to each value of the random variable the probability that we will get that value or less.

X	F(X)
-20	0.10
-10	0.25
10	0.70
25	0.95
30	1

$F(10) = 0.70$.

2.17 (a)

X	P (X = x)
1	0.20
2	0.40
3	0.40

Y	P (Y = y)
1	0.15
2	0.10
3	0.45
4	0.30

(f) If X and Y are independent, the conditional and unconditional probabilities should be equal. We can check this.

$P(X = 1|Y = 1) = P (X = 1 \text{ and } Y = 1) / P (Y = 1) = 0.03/0.15 = 0.20$. This is the same as $P (X = 1)$.

$P (X = 2| Y = 1) = P (X = 2 \text{ and } Y = 1) / P (Y = 1) = 0.06/0.15 = 0.40$. This is the same as $P (X = 2)$.

2. 23 Let R = Republican, D = Democrat, I = Independent, WSJ = reads Wall Street Journal.

Start by collecting the conditional and unconditional probabilities provided. We have $P (R) = 0.4$, $P (D) = 0.5$, and $P (I) = 0.1$ (These are the marginal or unconditional probabilities). Then write down the conditional probabilities: $P (WSJ|R) = 0.6$, $P (WSJ|D) = 0.3$, $P(WSJ|I) = 0.4$.

We want $P (R|WSJ) = P (R \text{ and } WSJ)/ P (WSJ)$. (1)

$P (WSJ) = P (R \text{ and } WSJ) + P(D \text{ and } WSJ) + P(I \text{ and } WSJ)$ (2)

Note that $P (R \text{ and } WSJ) = P (WSJ|R) P(R) = (0.6)(0.4) = 0.24$

Similarly, P (D and WSJ) = (0.3)(0.5) = 0.15

P (I and WSJ) = (0.4)(0.1) = 0.04

Using equation 2 we have P (WSJ) = 0.43

Using (1) we have P (WSJ|R) = 0.24/0.43 = 0.558

Additional Problems

1. (a) We know that X can take values 0, 1, 2. For the pdf, we need to find the probability that it takes each of these values.

There are 36 possible outcomes. Count the number of outcomes corresponding to each value of X and divide by 36. Thus, P (X = 0) = 25/36, P (X = 1) = 10/36, P (X = 2) = 1/36.

(b) For the cdf, assign to each value the probability of getting that value or less.

F (0) = 25/36, F (1) = 35/36, F (2) = 36/36.

2. Let A = watches network A, B = watches network C, AB = watches both.

(a) P (A + B) = P (A) + P (B) – P(AB) = 0.15 + 0.18 – 0.07 = 0.26.

(b) P (A and not B) = P (A) – P (AB) = 0.15 – 0.07 = 0.08. (The logic is easy to see if you use a Venn diagram.)

3. Let S = subscribes to the *Times*, E = previously subscribed, N = did not subscribe previously. Again, begin by writing down the conditional and unconditional probabilities that are provided in the problem.

P (E) = 0.75, P (N) = 0.25, P (S|E) = 0.8, P (S|N) = 0.1.

(a) P(S) = P(SE) + P (SN) = P (S|E)P(E) + P(S|N)P(N) = (0.8)(0.75) + (0.1)(0.25) = 0.6 + 0.025 = 0.625

(b) P(E|S) = P (ES)/ (P(S) = 0.6/0.625 = 0.96.

4. The number of cars bought has to lie between 0 and 4, and the probabilities must add up to 1.

P (at least one buys a car) = 1 – P (none buys a car)

Since each consumer's decision is independent of the others,

P (none buys a car) = (0.95)(0.95)(0.95)(0.95)

Then P (at least one buys a car) = 1 - (0.95)(0.95)(0.95)(0.95) = 0.1855