Notes on indifference curve analysis of the choice between leisure and labor, and the deadweight loss of taxation

Jon Bakija

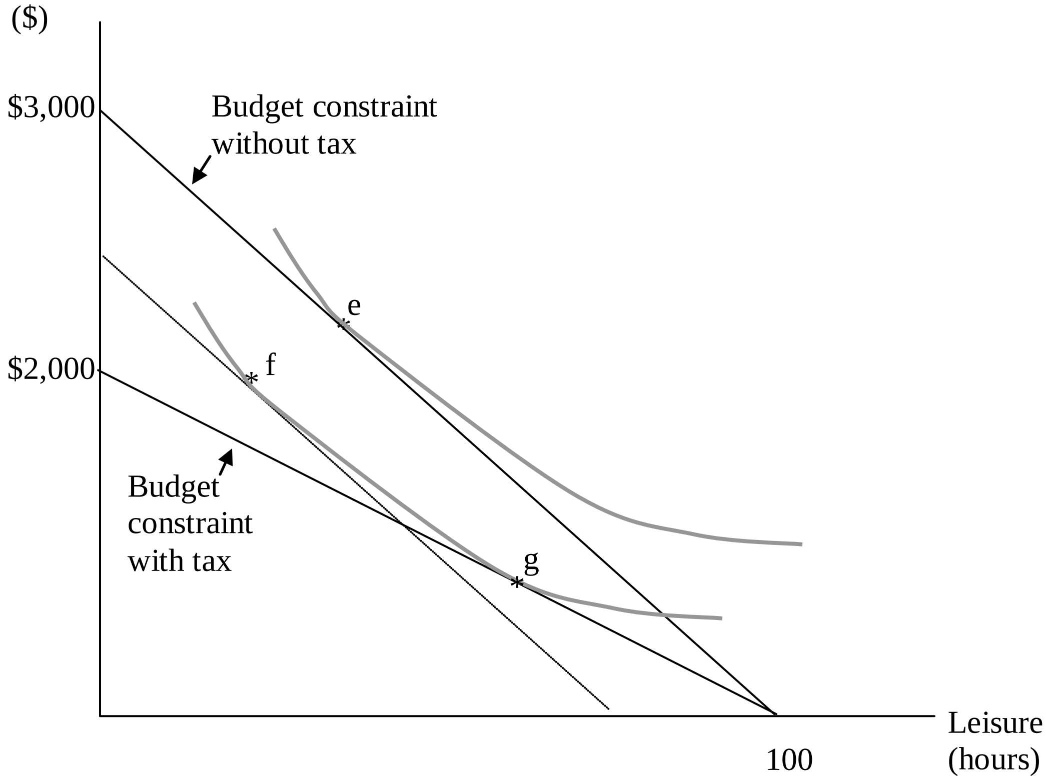
This example shows how to use a budget constraint and indifference curve diagram to analyze how a tax affects choices regarding labor supply (the number of hours worked), and illustrates more precisely what economists mean when they say a tax creates "deadweight loss."

Consider an individual's choice about how many hours to work in a week. Suppose the individual earns an hourly wage of $30. For simplicity, assume for the sake of this example that the maximum number of hours that the individual has available to allocate between work and leisure in a single week is 100 hours (for instance, suppose all other hours in the week must be spent sleeping and on basic personal needs). Then suppose that the government imposes a tax of $10 per hour worked on this individual (or equivalently, a tax of 33.3% of wage income), and the worker bears the full burden of the tax – that is, once the tax is imposed, the pre-tax wage paid by the employer stays at $30, but the after-tax wage received by the worker falls to $20 per hour. What matters to the worker is the after-tax wage, that is, the wage received after taxes are paid. Before the tax is imposed, the after-tax wage is $30 (because the tax is $0). After the tax is imposed, the after-tax wage is $20.

We can illustrate this situation on a budget constraint and indifference curve diagram. The individual's choice is simplified into a choice between two goods: leisure (L), measured in hours, and market consumption (C), measured in dollars. On a diagram of the budget constraint, we'll put L on the horizontal axis and C on the vertical axis. The maximum number of hours available in the week does not change, so the budget constraint always intercepts the horizontal (L) axis at 100. The number of hours worked equals 100-L. The vertical-axis intercept represents the amount of consumption that could be achieved if you worked all 100 hours, so it equals $3,000 when the after-tax wage is $30, and $2,000 when the after-tax wage is $20. The slope of the budget constraint is equal to the (negative of the) after-tax wage. Intuitively, if you want one more hour of leisure, you have to give up an amount of consumption equal to your after-tax wage. When the tax is imposed, it makes the budget constraint flatter, as the slope changes from -30 to -20. You could also think of this as an increase in the price of consumption. The opportunity cost, or price, of $1 of consumption has increased from $1/30^{th}$ of an hour to $1/20^{th}$ of an hour.

Figure 1 illustrates an example of how the tax might affect the choice between leisure and consumption, and breaks the response to the tax down into income and substitution effects. Without the tax, the individual chooses point *e*, where the indifference curve is tangent to the no-tax budget constraint. When the tax is imposed, the budget constraint pivots down as illustrated below, and the individual chooses a point like *g*, where the new budget constraint is tangent to an indifference curve. The choice can be de-composed into the income effect, shown by the movement from point *e* to point *f*, and the substitution effect, the movement from point *f* to point *g*. The dashed line is an imaginary line that is parallel to the old indifference curve, and tangent to the new budget constraint. Point *f* represents the combination of C and L that would have been chosen if income had been reduced by an amount that left the individual at the same level of utility (on the same indifference curve) as the actual tax, but if there had been no change in the relative price of leisure and consumption. The change from *f* to *g* then represents the effect of changing the relative price of leisure vs. consumption, while holding utility constant, which is the substitution effect.

Figure 1

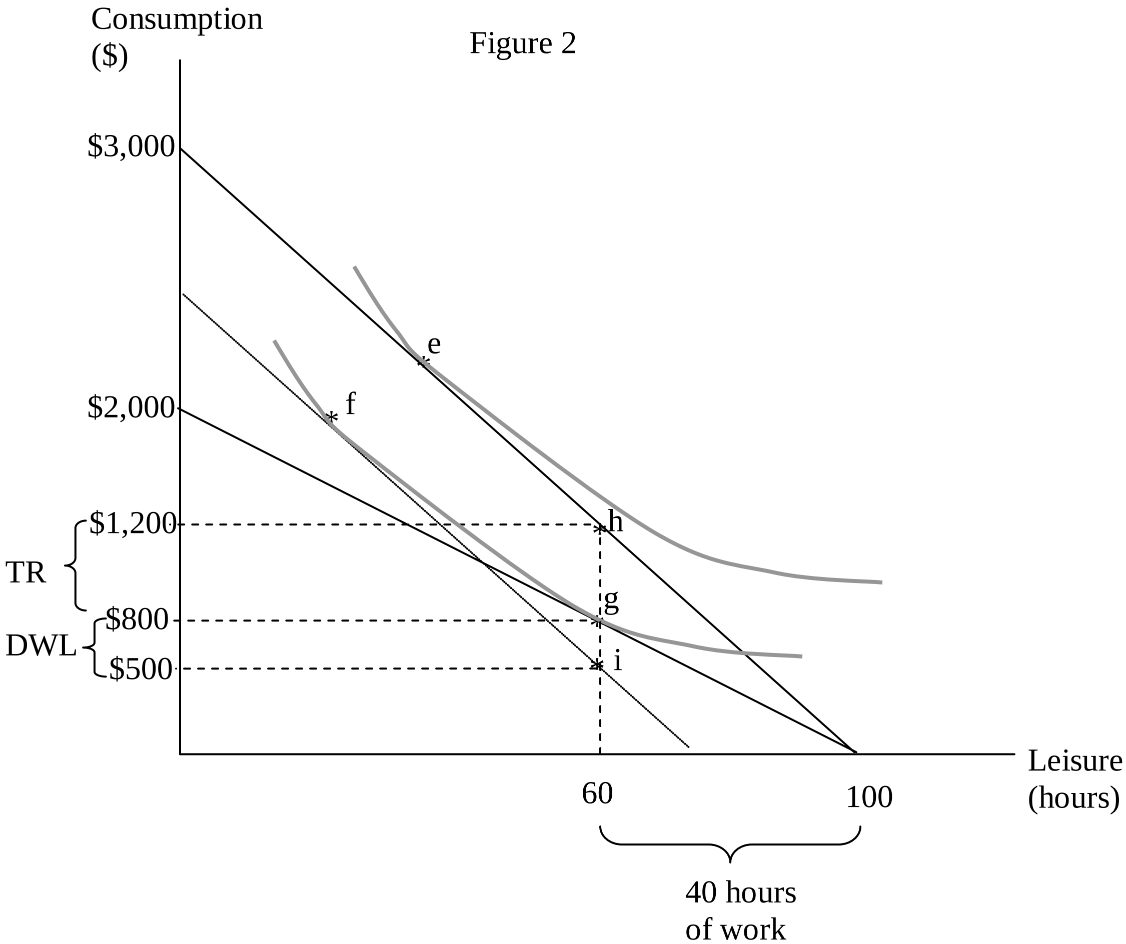


In this particular example, the substitution effect happens to be larger than the income effect, and as a result, the individual responds to the tax by increasing the amount of leisure (which is now relatively cheaper compared to consumption), or in other words, by working less. If this person had different preferences (differently shaped indifference curves), it could have been the case that the income effect was larger than the substitution effect, in which case the tax would cause the individual to work *more* (illustrating this is left as an exercise for you).

We can use this same framework to illustrate the deadweight loss from the tax. The deadweight loss from a tax is the amount by which the decline in well-being of the taxpayer, measured in dollars, exceeds the amount of revenue paid to the government. The reason the taxpayer is worse off by more than the amount of money paid to the government is that the taxpayer undertakes actions in an effort to avoid some of the tax, and these actions involve a hidden cost. In this case, the hidden cost is that the taxpayer substituted some extra leisure for less market consumption, when that market consumption was more valuable to the taxpayer than the leisure at the margin. The taxpayer switched from something more valuable to something less valuable solely in order to reduce the amount of tax payment. This made sense from the individual's point of view, because the tax savings from doing this were greater than the size of the hidden cost from switching away from more-highly-valued consumption to lower-valued leisure. But there is nonetheless a hidden cost. In order to quantify this hidden cost, we would need to put a dollar value on the amount by which the individual's well-being has declined because of the tax, and then subtract off the amount of revenue received by the government.

To make things concrete, suppose that after the tax is imposed, the individual chooses to work 40 hours, which also means taking 60 hours of leisure. First, consider how to show the amount of government revenue on the diagram.

Figure 2



If the individual does work 40 hours, then pre-tax income is $30 × 40 hours = $1,200. Pre-tax income when working 40 hours is equal to the height of point *h*. After-tax income when working 40 hours is $20 × 40 hours = $800. This is the height of point *g* in the diagram above. The difference between pre-tax income and after-tax income is the amount of tax revenue paid to the government. This equals the vertical distance between point *h* and point *g*, labeled "TR" in Figure 2, or $400 (as the example stated, the tax is $10 per hour, so the revenue is $10 times 40 hours worked).[1] It is important to note that on this diagram, unlike on a supply and demand diagram, the tax revenue is measured as a *distance*, not as the area of a rectangle. The vertical distance between point *g* and point *h* on the diagram above represents the difference between pre-tax income and after-tax income, and that difference is the entire amount of revenue that goes to the government.

Now, we need a measure of how much worse off the tax makes the individual, in dollars. One way to measure this would be to figure out the size of the "lump-sum" tax that we would have to take away from the individual in order to leave him or her at the same level of utility as the actual wage tax does. A lump-sum tax is a fixed amount of money that does not depend on anything that you do – for example, a head tax of $1,000 per person. Since nothing you do can change the amount of the lump-sum tax, it does not change any relative prices or incentives. Because there is no incentive to change your behavior in an effort to avoid the tax, a lump-sum tax involves no hidden costs. The harm to you from a lump-sum tax is exactly equal to the revenue raised by the government.

A lump-sum tax causes only a parallel shift in the budget constraint, without changing the slope. In Figure 2, the lump-sum tax that would leave the individual at the same level of utility as the actual tax is depicted by the dashed line – it is the "imaginary" budget constraint (parallel to the original budget constraint) that we used to illustrate the income effect. The dollar amount of the lump-sum tax is the same no matter how many hours of leisure are chosen – it equals the vertical distance between point *h* and point *i*. The exact size of this lump-sum tax will depend on the shape of the individual's indifference curves, but as should be apparent from the diagram, it will always be at least as large as the revenue raised by the actual tax. For example's sake, let's say the size of the lump-sum tax is $700. We call the amount of this lump-sum tax the "equivalent variation" – it is the equivalent variation in your income that would leave you on the same indifference curve as the actual tax.

The deadweight loss from the wage tax equals the equivalent variation minus the tax revenue raised by the government. In Figure 2, the deadweight loss is the vertical distance between point *i* and point *g*, and is labeled "DWL." There are two ways of looking at why there is deadweight loss or waste here. First, if the government had

[1] To verify that the height of point *h* is $1,200, the height of point *g* is $800, and the difference is government tax revenue, note that the equation for the before-tax budget constraint is: C = 3000 – 30L, and the equation for the after-tax budget constraint is C = 2000 – 20L. Before taxes, if L = 60, then C = 3000 – 30(60) = $1,200. After taxes, if L = 60, then C = 2000 – 20(60) = $800. The vertical distance between the two budget constraints at any point is equal to the amount of tax paid to the government, or $10 times the number of hours of work.

instead imposed the lump-sum tax, it would have raised $700 instead of $400, and the taxpayer would have been just as happy as under the wage tax. So in a sense, $300 is being wasted by using a tax that distorts the individual's incentives. Another way to look at it is that the wage tax makes the taxpayer worse off by $700, the government only raises $400 in revenue, and the $300 difference represents the hidden cost that arises because the taxpayer switched from something he liked more (consumption) to something he liked less (leisure) purely to avoid taxes.
Some implications of all this are as follows.

- In general, deadweight loss depends entirely on the substitution effect, not the income effect. If there is no substitution effect, there is no deadweight loss. A tax or other policy that only changes income in a lump-sum fashion, without changing any relative prices, does not cause any deadweight loss, because it only has an income effect. Or, if preferences are such that there is no substitution effect (for example, if indifference curves are "L" shaped, which occurs when goods are perfect complements to each other), then even a policy that changes relative prices will not cause deadweight loss.

- If there is a substitution effect, then a policy such a tax or subsidy that changes relative prices does cause deadweight loss, regardless of what happens with the income effect. (Exceptions can occur when there is a market failure, which we'll learn about later – when there is market failure, then the market prices are no longer necessarily the efficient prices).

- In general, conventional supply and demand diagrams only provide an approximation to the true deadweight loss associated with any distortionary policy or market failure. This is because conventional supply and demand curves include both income and substitution effects. To be perfectly accurate in our depiction of deadweight loss, we would need to use *compensated* demand and supply curves, that is, demand and supply curves that only include substitution effects, and remove all income effects. For an example of how such a curve would be constructed, consider Figure 2 above. We could derive a compensated labor supply curve for this individual by holding utility constant at the new indifference curve that applies after the tax, and then just changing the slope of the budget constraint around this indifference curve to determine the effects of different after-tax wages on the supply of labor. Points *f* and *g* would be two points on the individual's labor supply curve. In most cases, the deadweight loss depicted on a conventional supply-demand diagram is a pretty good approximation to the actual deadweight loss, but you should at least be aware that it is only an approximation.