

International Coordination of Macroprudential Policies with Capital Flows and Financial Asymmetries

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Abstract

Lack of coordination for prudential regulation hurts developing economies but benefits advanced economies. We consider a two-country macro model in which countries have limited ability to issue state-contingent contracts in international markets. Both countries have incentives to stabilize their economy by using prudential limits, but the emerging economy depends on the advanced economy to bear global risk. Financially developed economies are unwilling to intermediate global risk, which means bearing systemic risk, preferring financial stability over credit flows. Advanced economies prefer tighter prudential limits than would occur with coordination, giving them greater bargaining power when negotiating international agreements.

Keywords: International Capital Flows, Capital Controls, Macroeconomic Instability, Macroprudential Regulation, Policy Coordination, Spillovers, Financial Crises.

JEL classification: E44, F36, F38, F42, G15.

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1 Introduction

Following the Great Recession, economists and policy makers have debated the appropriate uses of prudential regulations to promote financial and macroeconomic stability.¹ Many hope that limiting risk-taking can decrease the frequency and duration of times of economic or financial stress. The effectiveness and implementation of these policies are complicated by interactions through international capital markets as well as the degree of international coordination. For these reasons, international coordination of prudential regulations have garnered renewed interest (Basel III was created in response to the financial crisis). This paper considers how the asymmetric nature of the global financial system is likely to affect policy coordination.

Economists have long recognized the crucial role of financial development to facilitate global intermediation.² Advanced economies (AEs) like the U.S. act as the “world’s global banker,” providing safe debt to emerging market economies (EMEs) and investing in risky foreign assets. For AEs to bear global risk on behalf of EMEs requires not only bearing fundamental risk but also exposing themselves to long-run financial instability; the frequency or duration of crises for AEs may be higher than would occur if AEs did not have leveraged aggregate balance sheets. While fundamental risk is generally compensated by excess returns (Gourinchas et al., 2010), with financial frictions or incomplete markets, sources of long-run instability need not be correctly compensated by returns (i.e., pecuniary externalities). Stability necessitates prudential policies.

In light of these concerns, our paper theoretically considers how global spillovers through international capital markets can affect countries’ capital account policy choices. We use a stochastic macroeconomic model based off Brunnermeier and Sannikov (2015a) with production of two-goods and two-countries (AE and EME) with limited ability to issue equity in international markets. The financial asymmetry is that the AE country has lower risk aversion than the EME country (see Gourinchas et al., 2010). There are no trade frictions other than (potentially) regulations imposed by each country limiting capital inflows or domestic leverage. Because international finan-

¹Common prudential policies include bank capital requirements, counterparty concentration limits, interbank exposure limits, loan-to-value ratios, and reserve requirements, as well as limitations on capital inflows.

²See for example Gourinchas and Rey (2007), Gourinchas et al. (2010), Lane and Milesi-Ferretti (2007), and Maggiori (2017).

cial markets are imperfect, global output depends endogenously on the relative share of wealth in each country, and debt imbalances (i.e., capital inflows or leverage) create volatility that increases the fraction of the time the global economy spends with misallocated capital. Higher leverage and capital inflows lead to a better *static* allocation of capital but with increased volatility that hurt *dynamic* global stability. A global social planner would impose prudential controls in both the AE and the EME because countries use too much leverage in the laissez-faire competitive equilibrium.

The asymmetric role of bearing global financial instability creates different incentives for AEs and EMEs to use prudential policies. Advanced economies have incentives to impose marginally tighter prudential regulations to “regulate away” sources of long-run financial instability (decreasing the frequency and duration of crises), which benefits AEs at the expense of EMEs. In contrast, EMEs want AEs to provide intermediation because AEs have superior risk-bearing capacities. Policy coordination must balance these opposing incentives. We show it is unlikely that advanced economies are willing to expose themselves to long-run financial instability on behalf of EMEs.

We first consider how regulation in one country positively affects outcomes and aggregate dynamics in the other (what we call “macro-level spillovers”). We find that tighter prudential regulations in the AE will decrease global volatility *when the AE is relatively poor* and improve the terms of trade for the AE *when the AE is relatively poor*. As a result, tighter regulations will decrease the frequency with which the AE is relatively poor. Hence, prudential controls by the AE increase the frequency with which the EME is relatively poor. Accordingly, the EME has strategic incentives to also tighten its policy in order to increase the frequency of being relatively rich. Incomplete markets are crucial for this result. Costinot et al. (2014) shows that countries have incentives to manipulate their terms of trade through taxes on capital flows only when their share of world output fluctuates over time. In our model, if countries could freely issue equity, then their share of world output would be constant because of (perfect) risk sharing. Thus, with complete markets, there would be no reason to manipulate the terms of trade. We numerically solve for coordinated policies chosen by a global social planner and for uncoordinated policies chosen in Nash equilibrium and find that, due to these strategic incentives, Nash policies are tighter than would be chosen by a social planner. A global social planner would (for most Pareto weights)

choose some degree of financial openness in the AE, allowing limited leverage and credit inflows to provide better international risk sharing because the AE has a higher risk tolerance. However, in Nash equilibrium the macro-level spillovers and strategic incentives lead both countries close credit flows completely (maximally regulate).

Crucially, the differences between Nash and optimal coordinated policies have asymmetric impacts on the AE and EME. When the AE does not use prudential controls but allows credit inflows and domestic leverage, the more risk averse EME benefits from better international risk sharing since the AE acts as a global intermediary, borrowing with safe debt to invest in risky capital. In Nash equilibrium, however, the AE is unwilling to act as a global intermediary, recognizing that credit flows create financial instability (longer and more frequent crises) that mostly harms its own agents to the benefit of the EME. In other words, because markets are incomplete, the so-called “exorbitant privilege” (excess return) from intermediating global risk does not sufficiently compensate the so-called “exorbitant duty” (losses in crises) of being the primary bearer of financial instability. Relative to the laissez-faire competitive equilibrium and to coordinated outcomes that place less than 80% Pareto weight on the AE, the AE prefers the Nash equilibrium because it mitigates exposure to long-run financial instability. In contrast, the EME’s welfare outcomes are worse under Nash policies.

This asymmetry means that the AE can use the Nash equilibrium as a “grim trigger” in policy coordination. AEs may have other un-modeled reasons to maintain loose controls, but our results provide a marginal incentive for tighter controls. If the EME does not agree with the policies that the AE prefers, then the AE can always credibly revert to Nash equilibrium and impose excessively tight prudential regulations. Due to these strategic interactions, any policy coordination would likely place substantial weight on the AE’s welfare. If the AE and EME try to coordinate their prudential regulations, then they would likely choose policies similar to those chosen by a social planner who places a large Pareto weight on the AE. To the extent that global recommendations for financial regulation reflect agreements reached as the outcome of (possibly complicated) negotiations among countries, our results suggest that coordination will place substantial emphasis on the welfare of AEs. Our results are of particular importance given the current focus on

developing global recommendations for financial regulation (e.g., Basel III).

Related Literature. Our paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014a, 2015a, 2016a) and He and Krishnamurthy (2012, 2013, 2014), who apply continuous-time methods to analyze the non-linear global dynamics of economies with financial frictions. Most closely related, Brunnermeier and Sannikov (2015a) demonstrate that closed capital accounts can improve welfare relative to open accounts because closing capital accounts improve global stability arising from incomplete markets. We extend the analysis of Brunnermeier and Sannikov (2015a) in two main ways: first, we allow each country to choose occasionally binding limits on capital inflows, with countries choosing potentially different policies; second, we use recursive preferences to disentangle effects driven by risk aversion and intertemporal substitution. By giving countries more flexibility in their policy choices, our model admits interesting policy spillovers and strategic considerations.

Our paper connects the literatures on the impact of capital controls on terms of trade and on macroprudential policy coordination (see below). In Costinot et al. (2014), for example, countries optimally “manipulate” terms of trade by differentially taxing capital inflows (outflows) when growth is higher (lower) than the rest of the world. We show that manipulating the terms of trade with simple leverage constraints on capital outflows proves an effective *macroprudential tool* in an economy with long-run financial instability due to incomplete markets. As in Costinot et al. (2014), the terms of trade creates individual incentives that are not necessarily aligned with social incentives. The literature on macroprudential policy coordination has studied several cases in which countries’ policy choices create spillovers. Our novel insight and main contribution is that in a financially asymmetric world, this terms-of-trade mechanism has different normative implications for the welfare of AEs and EMEs. Incentives to manipulate the terms of trade complicates macroprudential policy coordination between AEs and EMEs because it misaligns their objectives. Both countries want to mitigate exposure to systemic risk, but EMEs want AEs to provide intermediation, which requires bearing some systemic risk.

Economists have tended to focus on the use of capital controls by EMEs to induce favorable terms of trade (see e.g. Costinot et al., 2014; Farhi and Werning, 2014) or to mitigate the adverse

consequences of fickle capital flows (see e.g. Caballero and Simsek, 2016). Similar to Costinot et al. (2014), Farhi and Werning (2014), and Heathcote and Perri (2016), in our model capital controls work by changing a country’s terms of trade. However, the mechanism in our model is quite different. Rather than manipulating the risk-free interest rate and consumption behavior, capital controls primarily affect the terms of trade hedge identified by Brunnermeier and Sannikov (2015a), which has meaningful impact because of incomplete international markets. Limiting inflows alone is sufficient to benefit a country because the terms of trade hedge changes the stationary distribution of an economy, dynamically benefiting the financial position of the country in the long-run. An advantage of this approach is the simplicity of the policy instrument, as opposed to dynamically adjusting taxes on capital inflows/outflows.

The literature on macroprudential policy and pecuniary externalities includes Caballero and Krishnamurthy (2001, 2004), Aoki et al. (2009, 2010), Bianchi (2010, 2011), Mendoza (2010), and Bianchi and Mendoza (2010). Korinek (2016) provides a “first welfare theorem” when international markets lack frictions; there is role for coordination in our setting because international financial markets are imperfect. Within the continuous-time literature, Caballero and Simsek (2017) consider how macroprudential policy can mitigate damage arising from belief heterogeneity, and Phelan (2016) illustrates that leverage limits improve macroeconomic stability by endogenously increasing the frequency with which the banking sector is well-capitalized. These results contrast with Gârleanu et al. (2015), where limits on leverage reduce agents’ incentives to diversify and result in more correlated investments, increasing financial fragility.

Our model contributes to the literature on coordinated policy in international settings due to spillovers. Banerjee et al. (2016) explore the impact of spillovers from the macroeconomic policies of advanced economies to emerging market economies. Acharya and Bengui (2018) study capital flow management when AEs are at the ZLB. Jin and Shen (2019) studies macroprudential policy coordination spillovers among EMEs, who over-accumulate liquid assets, thus pushing down global rates. Sergeyev (2014) studies policy coordination in response to bank funding shocks, and global coordination works through affecting interest rate.

Similar to our analysis, Agénor et al. (2018) consider a linearized core-periphery model when

financial intermediation is subject to frictions and show that policies trade off mitigating the short-run consequences of the financial accelerator (and spillovers) and the long-run effects of higher costs of capital. Our paper is closely related to Davis and Devereux (2019), who analyze an asymmetric two-country economy where one country faces domestic financial frictions and the other does not. Our result that tighter policy improves stability is consistent with the evidence in Boar et al. (2017) who find that countries that more frequently use macroprudential tools, other things being equal, experience stronger and less volatile GDP growth. Our result that tighter policy provides strategic incentives for another country to tighten is consistent with the evidence in Agénor et al. (2018) who find that countries tend to set macroprudential policy more actively if their “neighbors” are more active.

Finally, there is a large literature studying the effectiveness of macroprudential regulation and capital controls, as well as the potential spillover effects. Analysis of early experiences with macroprudential instruments shows that some tools can reduce banks’ asset growth within countries (Claessens et al., 2013). However, effectiveness may be weakened when risky or excessive lending moves outside of the regulatory perimeter to non-covered entities or activities (Bengui and Bianchi, 2014; Aiyar et al., 2014; Reinhart and Sowerbutts, 2015). Kuttner and Shim (2016) find that debt-to-income ratios significantly affect housing credit growth. Bruno et al. (2016) document the role of macroprudential policies and capital controls in mitigating credit growth in 12 Asian economies. Berrospide et al. (2016) find that some regulatory changes spill over.³ For evidence on the limited effectiveness of partial or limited capital controls see Klein (2012) and Klein and Shambaugh (2015). Empirical evidence about the effect of capital account liberalizations are mixed, e.g., Obstfeld and Taylor (2004) and Magud et al. (2011). Economists have documented micro-level cross-border spillover effects in international lending due to domestic macroprudential policies such as capital requirements and loan-to-value ratio limits. See Obstfeld (2012, 2015); Shin (2012); Rey (2015); Buch and Goldberg (2016). Just as coordination can be complicated by micro-level spillovers from prudential controls, we argue that there are likely macro-level spillovers

³Specifically, a foreign country’s tightening of limits on loan-to-value ratios and local currency reserve requirements increase lending growth in the United States; a foreign tightening of capital requirements shifts lending by U.S. global banks away from the country where the tightening occurs to the United States; tighter U.S. capital regulation reduces lending by large U.S. global banks to foreign residents.

as well: changes in aggregate dynamics that result from global interactions in capital markets.

Outline. The remainder of the paper is organized as follows. Section 2 presents the model and the equilibrium conditions. Section 3 numerically solves for equilibrium in a calibrated economy and illustrates the effects of regulations. Section 4 solves for optimal coordinated and uncoordinated policies (constant/fixed leverage limits) and develops a simple bargaining game. Section 5 concludes.

2 The Model

This section presents a modified version of the two-country model in Brunnermeier and Sannikov (2015a). The global economy is populated by agents who live in two different countries, the AE and the EME. Variables related to the AE and the EME will be denoted by a subscript of A and E , respectively. Agents use capital to produce the two intermediate consumption goods a and b , and agents in the AE have productive advantages at producing good a (vice versa the EME and b). Capital trades in a competitive market, but agents in each country may be subject to leverage constraints owing to capital account policies in each country. Financial frictions limit international credit flows to risk-free debt (non-state contingent contracts).

2.1 Setup

Technology. Time is infinite and continuous. Capital can be used to produce either good a or good b , which are then combined into an aggregate (final) consumption good. Final consumption is given by

$$y_t = (y_{a,t})^\alpha (y_{b,t})^{1-\alpha}, \quad (1)$$

where $y_{a,t}$ is the supply of good a and $y_{b,t}$ is the supply of good b .

Agents in either country can produce goods a and b using linear production technologies, but agents in the AE have a superior technology for good a while agents in the EME have a superior technology for good b . Given k_t units of a capital, agents in the AE produce good a at rate $\bar{a}k_t$

and good b at rate $\underline{a}k_t$, where $\bar{a} > \underline{a} > 0$. The EME agents face the reverse situation, with good a produced at rate $\underline{a}k_t$ and good b at rate $\bar{a}k_t$. Because of these productivity differences, output endogenously depends on agents' risk-taking decisions.⁴

Denote the aggregate amount of world capital available at time t by K_t , and denote the share of world capital held by agents in the AE and in the EME by $\psi_{A,t}$ and $\psi_{E,t}$, respectively. Furthermore, denote the fraction of world capital devoted to production of goods a and b by $\psi_{Aa,t}$, $\psi_{Ab,t}$, $\psi_{Ea,t}$, and $\psi_{Eb,t}$, where the first superscript denotes the country employing the capital. By definition,

$$\psi_{Aa,t} + \psi_{Ab,t} + \psi_{Ea,t} + \psi_{Eb,t} = 1.$$

The supply of goods a and b per unit of capital then are given by

$$Y_{a,t} = \bar{a}\psi_{Aa,t} + \underline{a}\psi_{Ea,t}, \quad Y_{b,t} = \underline{a}\psi_{Ab,t} + \bar{a}\psi_{Eb,t},$$

yielding the total supply of the aggregate good

$$Y_t K_t = (Y_{a,t})^\alpha (Y_{b,t})^{1-\alpha} K_t. \quad (2)$$

Let the final good be the numeraire. Then the prices of a and b can be written as

$$P_{a,t} = \alpha \left(\frac{Y_{b,t}}{Y_{a,t}} \right)^{1-\alpha}, \quad P_{b,t} = (1-\alpha) \left(\frac{Y_{a,t}}{Y_{b,t}} \right)^\alpha. \quad (3)$$

There is a single type of physical capital. We model productivity shocks as shocks directly

⁴Our main insights would still hold if output followed an endowment process and countries traded claims to endowments because the incentive to manipulate terms of trade would remain (see Costinot et al. (2014)). However, we prefer a model with endogenous output for two reasons. First, a stationary distribution generally will not exist unless multiple trees and asymmetric consumption baskets are introduced, at which point the consequent model would be more complicated, not less. For intuition, suppose final output followed a single endowment process. The model would be the same as the case of no productivity differences in Brunnermeier and Sannikov (2015a), who show that the stationary distribution would be degenerate. The issue is that there are no forces in the model that cause mean reversion in the share of global wealth. Multiple endowment processes with a symmetric consumption basket would not change this problem. Second, in the real world, countries do face incomplete international markets and have to manage a fundamental trade off between static efficiency of capital allocation and dynamic stability. We want to better understand how countries behave when financial crises have consequences for real economic activity.

to capital, which can be interpreted as shocks to “effective capital.” Capital owned by agents in Country i evolves according to

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta) dt + \sigma dW_{i,t}, \quad (4)$$

where $\Phi(\cdot)$ is a standard investment technology,⁵ $\delta > 0$ is the rate of depreciation, σ is the volatility of capital growth, and $W_{i,t}$ is a standard Brownian motion. The two Brownian motions, $W_{A,t}$ and $W_{E,t}$, are independent and exogenous.

This specification is tractable and admits the interpretation of global aggregate shocks with idiosyncratic (negatively correlated) country specific shocks. Thus, the shocks can capture country-specific productivity gains as well as redistributive shocks (e.g. international law suits). Furthermore, our modeling decisions do not require the interpretation that physical capital crosses borders, but only ownership. Thus, an agent in country i buys capital, wherever it is physically located, but the productivity of capital, and shocks to productivity, are determined by the agent of ownership.⁶ It follows that aggregate capital follows the law of motion

$$\frac{dK_t}{K_t} = (\Phi(\iota_t) - \delta) dt + \psi_{A,t} \sigma dW_{A,t} + \psi_{E,t} \sigma dW_{E,t}. \quad (5)$$

To economize on notation, we will write $\mu_{K,t} = \Phi(\iota_t) - \delta$, $\Sigma_{K,t} = [\psi_{A,t} \sigma, \psi_{E,t} \sigma]^T$, and $d\mathbf{W}_t = [dW_{A,t}, dW_{E,t}]^T$.

Preferences. Agents have recursive preferences as in Duffie and Epstein (1992). Let γ_i be the risk aversion of agents in Country i . We model the AE as the global intermediary in reduced form

⁵In previous versions of this paper, capital grew according to an exogenous rate rather than with adjustment costs. The results are qualitatively and quantitatively similar. We choose this set up to stay closer to the modeling choices in Brunnermeier and Sannikov (2015a).

⁶This assumption implies that if Apple runs a factory in China, the productivity of the factory is determined by Apple, and changes in productivity are driven by changes in Apple’s characteristics. In reality shocks may depend on both the country location and the country of ownership; for tractability in our paper we assume that only the country of ownership matters. Appendix F shows that risk premia in our baseline model are similar to risk premia in a model which includes shocks dependent on both country location and country of ownership. In the case of unit elasticity of substitution and risk aversion, there exists parametrizations such that risk premia are the same.

by assuming agents in the AE have lower risk aversion ($\gamma_A \leq \gamma_E$).⁷ To focus our results on the role of the AE as the global intermediary, we assume a common discount rate ρ and unit intertemporal elasticity of substitution (EIS).⁸ The lifetime utility $V_{i,0}$ of Country i agents is given by

$$V_{i,0} = \mathbb{E} \left[\int_0^\infty f^i(c_{i,t}, V_{i,t}) dt \right], \quad (6)$$

where $f^i(\cdot)$ is the aggregator

$$f^i(c_{i,t}, V_{i,t}) = \rho(1 - \gamma_i)V_{i,t} \left(\log(c_{i,t}) - \frac{1}{1 - \gamma_i} \log((1 - \gamma_i)V_{i,t}) \right) \quad (7)$$

Markets for Physical Capital and Risk-free Bonds. Agents can trade ownership of physical capital in a competitive international market. We denote the equilibrium market price of capital per unit by q_t . Hence, capital k_t has market value $q_t k_t$. We postulate that q_t evolves endogenously according to

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{qA,t} dW_{A,t} + \sigma_{qE,t} dW_{E,t}, \quad (8)$$

where $\mu_{q,t}$, $\sigma_{qA,t}$, and $\sigma_{qE,t}$ will be determined in equilibrium. The capital price determines the rate of internal investment according to Tobin's q :

$$\Phi'(\iota_t) = \frac{1}{q_t}. \quad (9)$$

Agents can also trade a risk-free bond that is in zero net supply. We denote the endogenously determined risk-free return by dr_t^F (we implicitly assume that the risk-free interest rate has absolutely continuous paths). Agents can borrow or save in the risk-free asset, but they may face

⁷Asymmetry in the volatility of capital growth ($\sigma_A < \sigma_E$) is an alternative modeling approach that delivers the same qualitative results regarding global risk bearing. Intuitively, a lower σ_A causes agents in the AE to price capital as if their risk aversion was lower than the EME's. We prefer asymmetric risk aversion, however, because it focuses on the AE's role as a global intermediary rather than a particular micro-foundation.

⁸By using recursive preferences rather than CRRA preferences, we know that our results are driven by asymmetry in risk aversion rather than asymmetry in EIS or discount rates. Moreover, with unit EIS, consumption decisions are myopic. As a result, the policies we consider will affect countries' net worth dynamics through the link between risk taking and production decisions rather than through consumption behavior. However, our results would hold under CRRA preferences since the central mechanism is the connection between financial asymmetry and the terms of trade for intermediate goods, not the incentive to consume and save intertemporally.

borrowing limits imposed by regulation in their country. Agents cannot sell short capital.

Returns from Capital. The return from holding capital can be written as diffusion processes by summing capital gains $d(q_t k_t)/(q_t k_t)$, which has two volatility terms, and the dividend yield from using capital to produce good a or b , which has no volatility terms. We denote the return from an agent in Country i buying physical capital and using it to produce good j by dr_t^{ij} . Given equations (4) and (8) for the laws of motion for capital and the capital price and using Ito's Lemma, returns for the AE when producing good a are given by

$$dr_t^{Aa} = \left(\frac{\bar{a}P_{a,t}}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma\sigma_{qA,t} \right) dt + (\sigma + \sigma_{qA,t}) dW_{A,t} + \sigma_{qE,t} dW_{E,t}, \quad (10)$$

and analogously for the other country-goods pairs.⁹ Notably, the AE shocks affect the returns to capital used in the AE directly (k_t changes) and indirectly through the effect on the capital price q_t ; however, the EME's shocks affect returns indirectly through the effect on the capital price (analogously for how shocks affect the EME's returns). To economize on notation, we will let $\Sigma_{ij,t}$ denote the volatility vector for Country i when producing good j . For example, the volatility vector of investing in good a for the AE is $\Sigma_{Aa,t} = [\sigma + \sigma_{qA,t}, \sigma_{qE,t}]^T$. Note that $\Sigma_{ia,t} = \Sigma_{ib,t}$.

Incomplete Markets, Financial Frictions, and Prudential Regulations. The key financial friction in this model is the inability of countries to issue equity to each other. Agents can only trade risk-free bonds to purchase capital ownership, and as a result, markets are incomplete because they are limited to non-contingent financial contracts and cannot sell short capital. This reliance on risk-free debt generates financial instability that would be attenuated if agents could purchase equity in the other country. If markets were complete, then countries would not apply prudential regulations or capital controls. With free issuance of equity, countries could perfectly share aggregate risk and consume constant shares of world output. Costinot et al. (2014) shows that countries only have an incentive to manipulate capital flows only when their share of world output fluctuates over time.¹⁰

⁹e.g., $dr_t^{Eb} = \left(\frac{\bar{a}P_{b,t}}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma\sigma_{qE,t} \right) dt + \sigma_{qA,t} dW_{A,t} + (\sigma + \sigma_{qE,t}) dW_{E,t}$.

¹⁰Consider the example in Section IIIC. When there are aggregate fluctuations, countries have incentives to tax capital *inflows* when they expect to earn a larger share of world output tomorrow and tax capital *outflows* in the

In addition, agents cannot sell short investment in the production of goods a and b . Incomplete international markets can be motivated by home bias in equity holdings and micro-founded by agency problems and asymmetric information.¹¹

The heart of our analysis involves prudential regulations in each country. We model prudential regulations as borrowing limits requiring that leverage from capital inflows not exceed a country-specific threshold. Within the context of our model, leverage limits have the effect of limiting aggregate borrowing and therefore aggregate inflows. Maintaining the interpretation that capital ownership could be across borders, leverage limits would therefore apply to total balance sheets (holdings of domestic and foreign capital assets), and so limits on borrowing via debt will limit total asset holdings. In reality, macroprudential policies such as leverage limits are typically designed to target domestic excesses, not to address capital flow imbalances, and capital controls are envisioned as separate sets of tools. But the distinction is not always so clear. One example in which macroprudential policies were used to affect capital inflows is the case of Peru, which limited banks' derivatives positions as a way of stemming capital flows (see Keller, 2018). If the reader prefers a single interpretation of prudential limits in our model, then limits on capital inflows is the more robust interpretation, since heterogeneity within each country would imply that domestic leverage limits could have different effects from limits on aggregate inflows.

Consumption and Portfolio Choice. Each agent in Country i decides her consumption rate $c_{i,t}$ as well as how to allocate remaining wealth. Agents face a portfolio choice problem of how much capital to invest in the production of goods a and b , and how much to invest in the risk-free bond. For Country i , we denote the portfolio weights by $(x_{ia,t}, x_{ib,t}, 1 - x_{ia,t} - x_{ib,t})$, where $x_{ia,t}$ is the fraction of wealth invested in capital used to produce good a , $x_{ib,t}$ is the fraction of wealth invested

opposite case. If they expect to earn the same share of world output tomorrow, however, then they have no incentive to manipulate the terms of trade. This example is the proper mapping of their paper to our model, where output may be endogenous but is effectively exogenous under perfect risk sharing.

¹¹Heathcote and Perri (2013) provide an explanation for the empirically observed bias toward domestic asset holdings. See Jensen and Meckling (1976), Bolton and Scharfstein (1990), and Holmström and Tirole (1997) for theories of agency problems and asymmetric information limiting state-contingent contracts. More broadly, a home equity bias can be rooted in a fundamental trade-off between the diversification benefits and participation costs of investing in international markets. Gârleanu et al. (2015) show that limited market integration endogenously arises because of this trade-off, and since portfolios comprise more correlated securities, risk premia are higher relative to the frictionless benchmark.

in capital used to produce good b , and $1 - x_{ia,t} - x_{ib,t}$ is the fraction of wealth invested in the risk-free asset. Portfolio weights $x_{ia,t}$ and $x_{ib,t}$ must be non-negative.

Denote $L_{A,t}$ and $L_{E,t}$ as the leverage constraints imposed on the AE and EME, respectively (we restrict leverage constraints to be deterministic functions of the state variable). These constraints impose that the market value of an agent's borrowing (debt) cannot exceed $L_{i,t}$ times their wealth (leverage of 0 implies no borrowing). In other words, for an agent in Country i

$$x_{ia,t} + x_{ib,t} \leq L_{i,t} + 1. \quad (11)$$

Given a consumption rate $c_{i,t}$ and portfolio weights $(x_{ia,t}, x_{ib,t}, 1 - x_{ia,t} - x_{ib,t})$, the net worth $n_{i,t}$ of an agent evolves according to

$$\frac{dn_{i,t}}{n_{i,t}} = x_{ia,t} dr_t^{ia} + x_{ib,t} dr_t^{ib} + (1 - x_{ia,t} - x_{ib,t}) dr_t^F - \frac{c_{i,t}}{n_{i,t}} dt. \quad (12)$$

Thus, agents' problems can be summarized as maximizing utility (6) subject to the budget constraint (12) together with the solvency constraint $n_t \geq 0$ and the borrowing constraint (11). Since agents have unit intertemporal elasticity of substitution, optimal consumption is to consume a fraction ρ of net wealth, implying $c_t = \rho n_t$ for all agents.

Definition 1 (Equilibrium). *For any initial allocation of wealth, an equilibrium is a map from histories of shocks $\{W_s^A, W_s^E, s \in [0, t]\}$ to the allocation of capital $(\psi_{Aa,t}, \psi_{Ab,t}, \psi_{Ea,t}, \psi_{Eb,t})$ and the aggregate consumption good $(C_{A,t}, C_{E,t})$ as well as price q_t and risk-free rate dr_t^F such that*

1. *All agents solve their optimal consumption and portfolio choice problems, subject to the solvency constraint on their net worth and leverage constraints.*
2. *Capital, consumption, and debt markets clear.*

2.2 Solving for Equilibrium

Since countries cannot issue equity internationally but can only trade in capital and risk-free assets, an agent's portfolio decisions depend on her level of wealth, so equilibrium depends on the aggre-

gate level of wealth in each country. For example, as the AE's wealth increases, its aggregate capital holdings will increase, which will increase the fraction of global capital used to produce good a . Thus, capital allocations and final good production will depend on country's relative wealth, which will vary in response to global shocks. We use stochastic continuous-time methods to solve for global equilibrium dynamics. We solve for a recursive (or Markov), rational-expectations equilibrium in which the single state variable is the relative wealth of the two countries.

Denote the aggregate net worth of agents in the AE at time t by $N_{A,t}$. Then the relative share of net wealth held by the AE is defined to be $\eta_t \equiv \frac{N_{A,t}}{q_t K_t}$. Thus, η_t represents the share of global wealth held by the AE. The aggregate portfolio choice of the AE and EME can be written respectively as

$$\left(\frac{\psi_{Aa,t}}{\eta_t}, \frac{\psi_{Ab,t}}{\eta_t}, 1 - \frac{\psi_{A,t}}{\eta_t} \right), \quad \left(\frac{\psi_{Ea,t}}{1 - \eta_t}, \frac{\psi_{Eb,t}}{1 - \eta_t}, 1 - \frac{\psi_{E,t}}{1 - \eta_t} \right),$$

where $\psi_{A,t} = \psi_{Aa,t} + \psi_{Ab,t}$ and $\psi_{E,t} = \psi_{Ea,t} + \psi_{Eb,t}$. Equilibrium, therefore, consists of an endogenous law of motion for η_t and capital allocations and prices which are functions of the state variable η_t .

The Value Function. Our choice of the aggregator function (7) implies agents in the AE and the EME have the value functions, respectively,

$$V_{A,t} = \frac{(\xi_t n_{A,t})^{1-\gamma_A}}{1-\gamma_A}, \quad V_{E,t} = \frac{(\zeta_t n_{E,t})^{1-\gamma_E}}{1-\gamma_E}, \quad (13)$$

where ξ_t and ζ_t capture the forward-looking value of an additional dollar of net worth. We postulate that in equilibrium

$$\frac{d\xi_t}{\xi_t} = \mu_{\xi,t} dt + \Sigma_{\xi,t} \cdot d\mathbf{W}_t, \quad (14)$$

$$\frac{d\zeta_t}{\zeta_t} = \mu_{\zeta,t} dt + \Sigma_{\zeta,t} \cdot d\mathbf{W}_t. \quad (15)$$

Asset Pricing Equations. When leverage constraints do not bind, they choose portfolios such that excess returns equal the covariance of the returns with their stochastic discount factor (SDF).

We can separate the covariance into two terms. For AE agents, the first term is denoted by $\varsigma_{Aa,t}$, and the second term is denoted by $\varsigma_{\xi A,t}$. The term $\varsigma_{Aa,t}$ captures the “fundamental” variance of returns from holding capital whereas the term $\varsigma_{\xi A,t}$ captures the covariance between the marginal value of AE agents’ net worth ξ_t and returns from holding capital. We can analogously define $\varsigma_{Eb,t}$ and $\varsigma_{\zeta E,t}$. However, when leverage constraints bind, the excess returns can exceed the risk premium. We prove these claims by examining agents’ Hamilton-Jacobi-Bellman (HJB) equations. For details, see Appendix B.

The leverage constraint will bind for agents in the AE when, if they were subject to no constraints, $\psi_{A,t}/\eta_t \geq L_{A,t} + 1$ and in the EME when, if they were subject to no constraints, $\psi_{E,t}/(1 - \eta_t) \geq L_{E,t} + 1$. Thus $\psi_{A,t}/\eta_t \leq L_{A,t} + 1$ and $\psi_{E,t}/(1 - \eta_t) \leq L_{E,t} + 1$ over the state space.

Characterizing Equilibrium. Using the returns equations and our conjectures on agents’ value functions, together with market clearing for capital and consumption, we can characterize equilibrium as a system of differential equations in the capital price q_t , the AE’s value of additional net worth ξ_t , and the EME’s value of additional net worth ζ_t . This system depends on the endogenous state variable η_t . Here, we report the law of motion for η_t and characterize the state space. We derive the law of motion and the system of differential equations in Appendix B.

First, we characterize equilibrium when leverage constraints do not bind.

Proposition 1. *When leverage constraints do not bind, the equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_{\eta,t} dt + \sigma_{\eta A,t} dW_{A,t} + \sigma_{\eta E,t} dW_{E,t}, \quad (16)$$

where

$$\begin{aligned}\mu_{\eta,t} &= \frac{\bar{a}P_{a,t} - \iota_t}{q_t} - \rho + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) (\gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}) \\ &\quad - \left(\frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} - (\Sigma_{q,t} + \Sigma_{K,t}) \right) \cdot (\Sigma_{q,t} + \Sigma_{K,t}) - \sigma \sigma_{qE,t}, \\ \sigma_{\eta A,t} &= \frac{1 - \eta_t}{\eta_t} \psi_{A,t} \sigma + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qA,t}, \\ \sigma_{\eta E,t} &= -\psi_{E,t} \sigma + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qE,t}.\end{aligned}$$

The state space is divided into 3 regions. For $\eta < \eta^a$, both countries produce good a and the EME produces good b . For $\eta > \eta^b$ both countries produce good b and the AE produces good a . For $\eta \in [\eta^a, \eta^b]$ countries specialize, using only their most productive technology. Goods prices satisfy

$$\frac{\underline{a}}{\bar{a}} \leq \frac{P_{b,t}}{P_{a,t}} \leq \frac{\bar{a}}{\underline{a}},$$

where the first (second) inequality becomes equality in the left (right) region of the state space.

Equilibrium is slightly modified when leverage constraints bind. In this case, the constraint prevents agents from taking as much risk as they demand. As a result, their excess returns will not equal the covariance of their returns with their SDF. We deal with this issue by observing that in equilibrium either the AE or the EME is a net debtor. Suppose, for concreteness, that leverage constraints bind for the AE. Then the EME must be a net lender, so its agents cannot be constrained, hence their excess returns still equal the covariance of their returns with their SDF. We use this observation to re-express $d\eta_t$. The state variable η_t will follow this new law of motion until the leverage constraint no longer binds, at which point the law of motion reverts to (16).¹²

Proposition 2. *If the EME's leverage constraint binds, then η_t continues to follow the law of motion in Proposition 1. When the AE's leverage constraint binds, $\psi_{A,t} = (1 + L_{A,t})\eta_t$, and the*

¹²Even though η_t follows the same law of motion when leverage constraints do not bind, it does not necessarily imply that, as functions of η , the equilibrium prices, volatilities, and capital allocations are the same in regions of the state space where constraints do not bind. For example, if $L_{A,t}$ does not bind when $\eta > 0.3$, this does not mean that ψ_{Aa} in the equilibrium with an occasionally binding leverage constraint agrees with ψ_{Aa} in the equilibrium with no leverage constraints.

drift of η_t becomes

$$\begin{aligned}\mu_{\eta_t} = & \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \left(\gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} - \frac{\bar{a}P_{b,t}}{q_t} \right) + \frac{1}{\eta_t q_t} (\psi_{Aa,t} \bar{a}P_{a,t} + \psi_{Ab,t} \underline{a}P_{b,t}) \\ & - \frac{l_t}{q_t} - \rho + \left(\frac{\psi_{A,t}}{\eta_t} - \psi_{A,t} - 1 \right) \sigma(\sigma_{qA,t} - \sigma_{qE,t}) \\ & - \left(\frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} - (\Sigma_{q,t} + \Sigma_{K,t}) \right) \cdot (\Sigma_{q,t} + \Sigma_{K,t}).\end{aligned}\tag{17}$$

The volatility vector $\Sigma_{\eta,t}$ and the division of the state space remain the same as in (16).

3 Numerical Results

We numerically solve our model using the method of the false transient. For details, we refer the reader to Appendix D. For the remainder of this paper, we will suppress time subscripts.

3.1 Calibration

Table 1 lists the parameter values we use. While our goal is not a quantitative model, we calibrate our model parameters to be reasonable. We set the risk aversions of the AE and EME so that they are not far from 1 and 2, respectively. The discount rate ρ is set at 4% following the continuous-time macrofinance literature. We normalize $\bar{a} = 1$ and let $\underline{a} = 0.85$ to reflect gains from specialization in trade of 18%, but we also check our results are robust to other values of \underline{a} in Appendix E. For computational tractability, we choose the following functional form for $\Phi(\cdot)$:

$$\Phi(\iota) = \frac{\lambda}{\kappa} \log(\kappa \iota + 1).\tag{18}$$

We calibrate the parameters λ and κ to match a 2% growth rate of aggregate U.S. consumption and an investment to output ratio of 20%, following Di Tella (2017). The depreciation rate δ is set at 5% following Brunnermeier and Sannikov (2015b) and Di Tella (2017). Finally, we set $\alpha = 1/2$ so that overall demand for intermediate goods a or b does not influence our results, following Brunnermeier and Sannikov (2015b).

Table 1: This table lists the model’s parameters, our chosen values, the quantities to match, the target numbers, and the model’s calculations. If the quantities to match are taken from the literature or chosen by another rationale, we do not report the target and the model’s calculation.

Description	Parameter	Value	Quantity to Match	Target	Model
Risk aversion of the AE	γ_A	1.05	—	—	—
Risk aversion of the EME	γ_E	2	—	—	—
Discount rate	ρ	4%	Literature	—	—
First-Best Productivity	\bar{a}	1	Normalized	—	—
Second-Best Productivity	\underline{a}	0.85	Specialization gains	—	—
Investment Tech Productivity	λ	57	Consumption growth	2%	1.9%
Investment Tech Illiquidity	κ	5200	Investment/output ratio	20%	21%
Depreciation rate	δ	5%	Literature	—	—
Capital Growth Volatility	σ	2%	Consumption volatility	2%	2%
Cobb-Douglas Weight	α	1/2	Symmetry	—	—

For simplicity, we suppose countries choose a fixed, state-independent leverage constraint L_A and L_E . We first consider how varying constraints affect the stationary distribution (stability) of the global economy and then consider how constraints in each country affect welfare in the AE.

3.2 Stability

In the absence of any policy, the economy features a stochastic steady state around $\eta = 0.41$, near where capital is best allocated at $\eta = 0.35$ (since the AE has lower risk aversion), and the economy tends to drift toward the stochastic steady state after shocks move the system away. At the stochastic steady state, the AE’s share of world capital is only slightly greater than 1/2, so misallocation is minimal (At $\eta = 0.41$, capital is almost efficiently allocated, with $\psi_A \approx 0.501$). Hence the distribution of η is centered around when flow output is relatively high (i.e., the economy is typically stable).

Figure 1 plots the stationary distribution of η varying L_A for two cases. Panel (a) considers

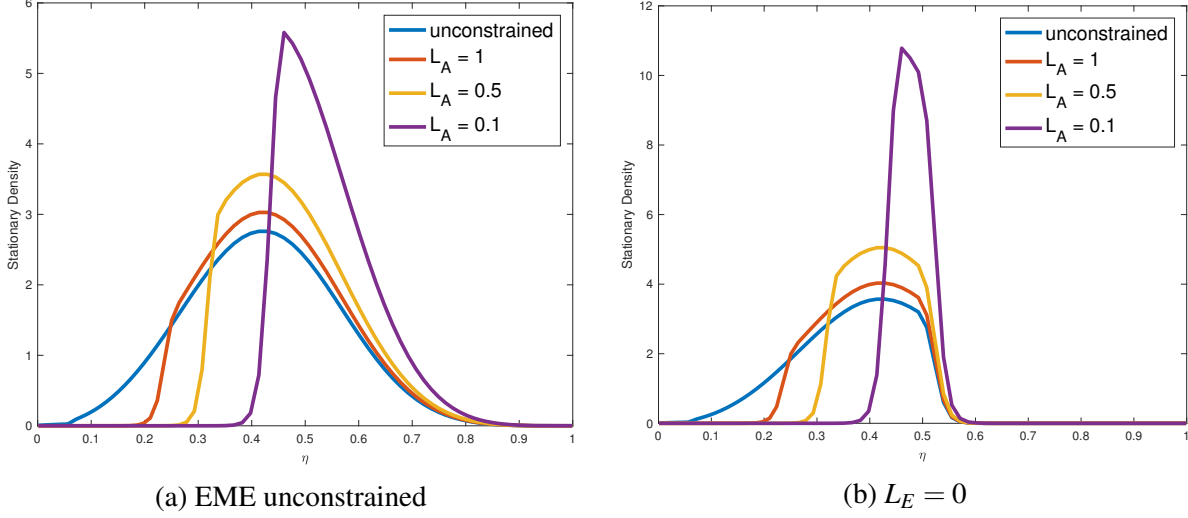


Figure 1: Stationary distribution of η (stability). Tighter constraints in the AE shift mass toward higher η (good for the AE), and tighter constraints in the EME shift mass toward lower η (good for the EME).

when the EME does not impose leverage constraints, and Panel (b) considers the other extreme when the EME sets leverage to zero (closed capital account). The blue distribution in Panel (a) thus shows the distribution in the laissez-faire economy without constraints. Notably, it is the most dispersed distribution. This is precisely the notion of long-run financial instability: the distribution of η in the competitive equilibrium is more dispersed than any country would like. Moreover, this dispersion does not occur solely because of the level of endogenous volatility Σ_η .

Tighter regulation can shift mass in the direction a country prefers. Of course, the flip-side is that when the AE tightens, the distribution must worsen from the perspective of the EME. Contrasting Panels (a) and (b) clearly illustrates how tightening by the EME has shifted the distribution toward low η (bad for the AE).

Tighter leverage constraints increase the stability of the economy and reduce long-run financial stability because of two reasons. First, leverage decreases the volatility of η . Second, it improves the terms of trade after bad shocks, thus raising the rate at which net worth recovers. This “terms of trade hedge” increases the drift of η near the stochastic steady state. These two forces together mean that following bad shocks which push the economy away from the steady state, the economy recovers at a faster rate with leverage constraints. In this way, the effect of capital controls in

our model is quite complementary to the effect in Costinot et al. (2014). Because in our model taxing/limiting inflows improves the distribution of the economy, taxes on inflows need not be coupled with taxes on outflows. Figure 2 plots changes in the drift and volatility of η_t (we define $\sigma_\eta = \sqrt{\sigma_{\eta A}^2 + \sigma_{\eta E}^2}$). Panel (a) in Figure 3 illustrates the terms of trade hedge. However, tighter regulation is not without cost. When constraints bind, the flow allocations are worsened and output suffers. These costs are reflected by lower asset prices, as Figure 3(b) shows.

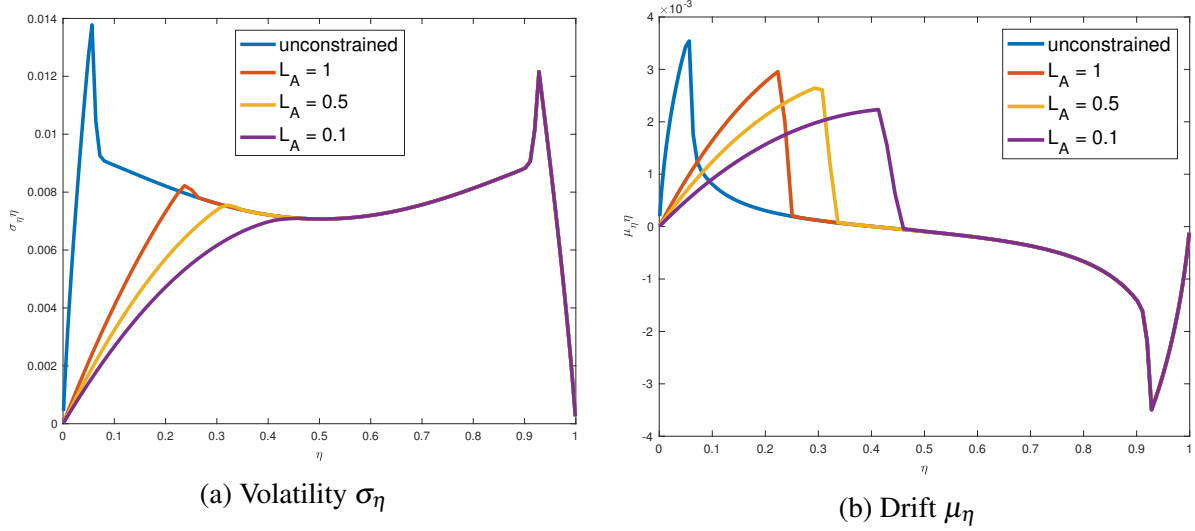


Figure 2: Evolution of η . Tighter constraints in the AE decrease volatility and (generally) increase drift, creating a more stable economy. (EME unconstrained)

3.3 Welfare

The main goal of our analysis is to understand how countries choose regulation in order to maximize the welfare of its agents. In general, leverage constraints will not improve welfare almost everywhere. Consider the AE as an example. When η is close to zero, restricting leverage slows down the rate at which agents rebuild their net worth. The reason is that expected returns will be very high but need to borrow to finance their investment. Limits on their leverage reduces their ability to capitalize on the high returns. As a result, the value function near $\eta = 0$ will decrease when the leverage constraint tightens. However, if η is further away from zero, then restrictions

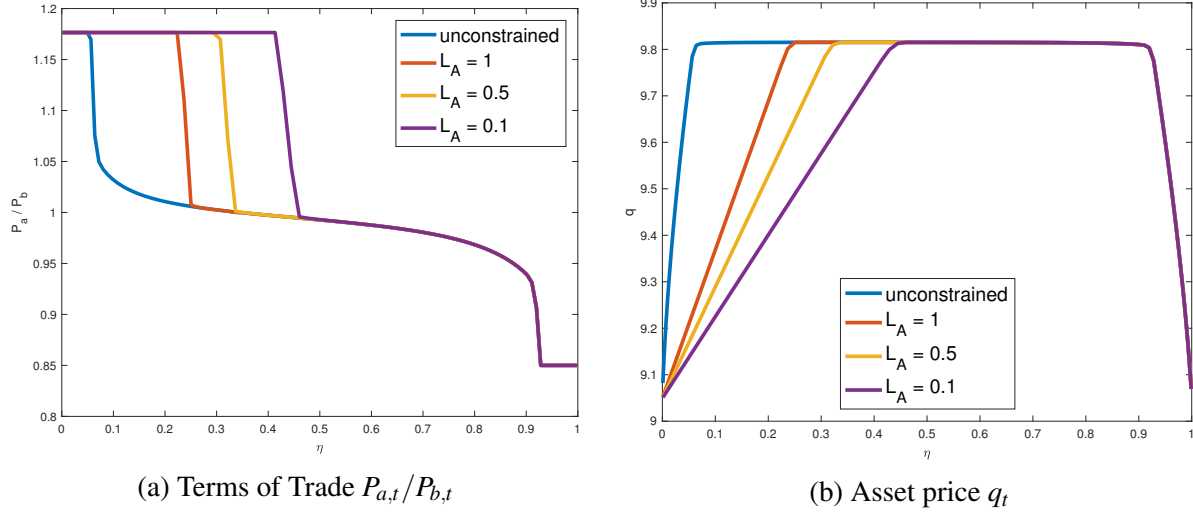


Figure 3: Equilibrium Asset Price and Terms of Trade. (EME unconstrained)

on leverage can promote financial stability enough that the value function increases with tighter constraints.

This non-monotonic response to leverage constraints across the state space complicates the welfare analysis of different regulatory regimes. Since countries are asymmetric, it does not make sense to simply assume equal initial wealth (i.e. $\eta_0 = 1/2$). The stochastic steady state may not be a reasonable initial condition either because the stationary distributions in Figure 1 indicate that the economy exhibits substantial long-run variance in η .¹³ Ideally, we want to evaluate the welfare of the AE and EME in such a way that we capture the impact of leverage constraints on the long-run financial stability of the economy—the key motivation for these policy tools in the first place.

To address these issues, we apply an ex-ante measure of welfare. Rather than choose an arbitrary initial condition, we suppose that countries adopt a “prior” about the location of η at time $t = 0$. Since neither country’s information set would include a signal of η_0 , the best “prior” is the stationary distribution of η . We therefore measure welfare by computing the expectation of Country i ’s value function with respect to the stationary distribution of η . We label this measure of welfare by $\mathbb{E}[V_i]$ and refer to it interchangeably as ex-ante welfare.

¹³Another complication is the fact that welfare is non-monotonic with η . Consider the AE. For most of the state space, welfare increases with η . For high enough η , however, aggregate output is so low that the AE would actually prefer if η was lower since the EME can produce good b at a superior rate.

To make this notation more precise, recall that the value function for Country i $V_{i,0}$ is subject to an initial condition η_0 . We can reformulate this value function as the conditional expectation

$$V_{i,0} = \mathbb{E} \left[\int_0^\infty f^i(C_{i,t}, V_{i,t}) dt \mid \eta_0 \right].$$

Ex-ante welfare then is equivalent to integrating out the initial condition η_0 using some prior on its distribution.

We prefer this measure of welfare rather than evaluating the value functions at a single point because it captures the impact of regulation on long-run financial stability in two ways. First, conditional on η_0 , leverage constraints influence the future investment opportunities which agents expect to have. For example, the stochastic steady state becomes more stable, so they expect capital to be optimally allocated more often. Second, they alter countries' uncertainty or beliefs about the economy's long-run stability. Evaluating countries' value functions only captures the first effect. The second effect, however, is important because a key benefit of greater long-run financial stability is increased certainty about the economy's health in its current state.

Welfare Spillovers. In this economy the dynamic benefits of a more stable economy generally outweigh the flow costs, and as a result tightening constraints in one country generally improves welfare for that country. However, there are important cross-country spillovers. Figure 4 plots ex-ante welfare for the AE as a function of the leverage constraint chosen by the EME, for three levels of constraints in the AE.

There are two important patterns to notice. First, given the policy in the EME, a tighter regulation in the AE (lower L_A) improves welfare in the AE. This is because the stability trade-off outweighs the flow cost, as discussed earlier. Second, fixing L_A in the AE, welfare in the AE is increasing with L_E . In other words, tightening in the EME decreases welfare in the AE. Hence, macro-level spillovers from capital account policies are reflected by changes in the stationary distribution of η and in welfare.

There are two complementary intuitions for this result, and it is important to consider both. First, when the EME tightens, that shifts mass toward states when the AE is relatively poor. As we

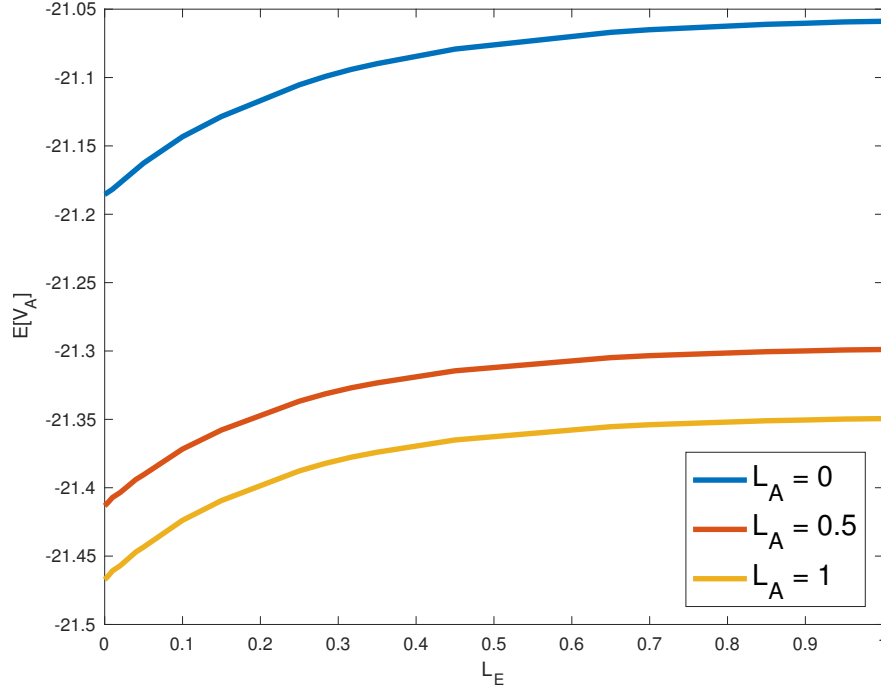


Figure 4: the AE's ex-ante welfare varying leverage constraints in the EME along the x-axis and in the AE across the different lines.

have seen, if the EME tightens and shifts mass toward states when the AE is poor, the AE can shift mass away from those states by tightening as well. Hence, uncoordinated regulation to improve stability has a “zero-sum” element: a country cannot shift mass away from $\eta = 1$ without shifting it toward $\eta = 0$. As we will see later, this mechanism is precisely why uncoordinated (Nash) policy choices will be tighter than policies coordinated by a social planner, since each country will have an incentive to tighten in order to improve the distribution of outcomes (from its perspective).

Second, more intermediation requires dealing with greater long-run financial instability. When a country takes on leverage (intermediating risk), it must also bear *systemic* risk (endogenous volatility) and a weakened terms of trade hedge, which produce greater financial instability. By restricting leverage, countries can reduce their exposure to sources of instability. These effects are reflected in Figures 2 and 3. In the first figure, the volatility of η decreases when the AE tightens regulation—and importantly, the volatility of η decreases *only* at times when the AE is relatively poor, which is precisely when the AE does not want systemic risk. In the second figure, the terms of trade hedge applies much closer to the stochastic steady state, precisely when the AE needs it

to maintain capital levels close to optimal. But as we’ve seen, limiting leverage in order to reduce instability is costly to the other country.

4 Financial Asymmetry and Strategic Incentives

In this section we let countries choose policies to maximize welfare in each country. Before diving into the results, we briefly clarify the policy game being played by countries. There are two players, the AE and the EME. They play a one-shot game. Before time $t = 0$, each country chooses and commits to a fixed leverage constraint. They maximize their ex-ante welfare subject to constraints stated in Section 2, taking as given the other country’s policy choice. We solve the coordinated equilibrium and the uncoordinated Nash equilibrium numerically.

4.1 Coordinated and Uncoordinated Policies

Our numerical results indicate that closing capital accounts is not only a Nash equilibrium but a dominant strategy. Figure 5 plots ex-ante welfare in the AE, varying L_A for a set of L_E .¹⁴ What is clear is that for each L_E , welfare in the AE is maximized by $L_A = 0$, and it monotonically increases as L_A decreases. The same behavior holds for the EME.

Table 2 compares outcomes in the competitive equilibrium (no constraints) to the coordinated and uncoordinated equilibria using the ex-ante measure. Our first main result—that uncoordinated constraints are tighter than coordinated constraints—holds broadly across parameters. For all parameters we have considered the Nash equilibrium is zero. While prudential regulations in our model are highly stylized, the key insight should apply across a wide range of potential environments. The key mechanism of our model is that, when international credit markets are imperfect, a policy that stabilizes the AE will lead the AE to be relatively richer more frequently.¹⁵ This is

¹⁴This figure only displays a few cases. We determine that closed capital accounts is a dominant strategy in two ways. First, we compute a much finer grid of pairs of leverage constraints, which again suggests closed accounts are dominant strategies. Second, we conduct a fixed point iteration where we fix one player’s constraint, compute the other player’s best response, fix that best response, compute the first player’s best response, and so on, considering many initial conditions. See Appendix D for a more thorough description.

¹⁵While in our model leverage limits are controls on capital inflows, in reality these instruments can have very different roles. Korinek and Sandri (2016) quantitatively find the optimal capital control and macroprudential regulation

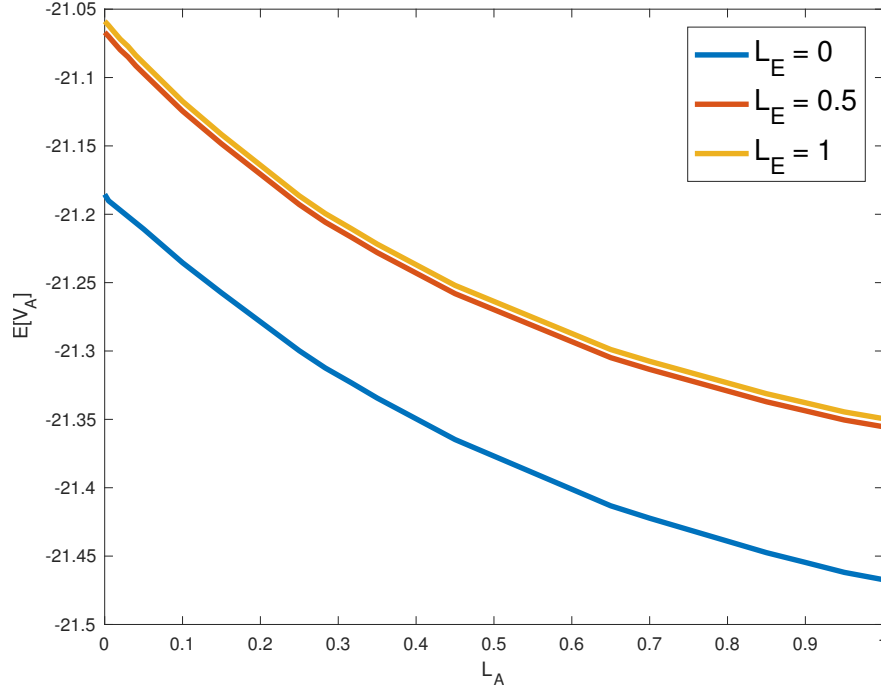


Figure 5: Welfare in the AE varying L_A for given L_E . Setting $L_A = 0$ appears to be a dominant strategy (i.e., best response to any L_E).

a negative spillover to the EME, which increases the incentive for the EME to enact stabilizing policies. While a richer setup is likely to generate positive spillovers through other sets of pecuniary externalities, we expect that the negative externality present in our model would continue to be present. Thus, while the quantitative importance of our mechanism will depend on the full set of global interactions, *on the margin* the mechanism we highlight will lead to tighter uncoordinated policies relative to global coordination.

The second critical result which emerges in this analysis is that the EME is most hurt by a lack of policy coordination. The EME experiences a 13.1% welfare increase in consumption equivalent units in the social optimum relative to the competitive equilibrium but suffers a 7.62% consumption equivalent decrease in Nash equilibrium. In contrast, the low risk aversion AE *benefits* in Nash equilibrium compared to coordination (with equal Pareto weights) and especially compared to laissez faire. With our calibration, the AE gains 26.8% in consumption equivalents relative to the

for emerging economies to mitigate contractionary exchange rate depreciations and reduce the amount and riskiness of financial liabilities. Nonetheless, the authors find it is optimal for emerging economies to employ both instruments in order to improve stability.

Table 2: Consumption equivalent ex-ante welfare in percentage points relative to the competitive equilibrium with coordinated and uncoordinated leverage limits.

	L_A	L_E	$\Delta\text{Welfare for AE}$	$\Delta\text{Welfare for EME}$
Social optimum	177.4%	0%	-7.88%	13.1%
Nash	0	0	26.8%	-7.62%

competitive equilibrium. If countries coordinate, then the AE actually suffers a 7.8% decrease in consumption equivalents. Hence, the AE benefits from lack of coordination (choosing tighter policy than the coordinated optimum) but the EME is hurt from lack of coordination (choosing essentially the same policy in each case).¹⁶

Since countries are asymmetric, equal Pareto weights are not necessarily the correct choice. Moreover, it is instructive to consider the change in the AE's and EME's ex-ante welfare as a social planner varies Pareto weights. Toward this end, Figure 6 plots welfare pairs $(\mathbb{E}[V_A], \mathbb{E}[V_E])$ for each country in different regulation regimes. The black square is the welfare pair in competitive equilibrium (no constraints), the black asterisk is the welfare pair in Nash equilibrium (zero leverage), the red line is the Pareto frontier when the Pareto weight α is less than 1/2, and the blue line is the Pareto Frontier when α is greater than or equal to 1/2.¹⁷

Figure 7 plots the optimal leverage chosen by a global social planner as we vary the weight put on the AE. When all weight is put on the EME, the social planner calls for very high leverage by the AE and zero leverage for the EME. This is sensible: the social planner wants the AE to bear all risk (fundamental and systemic) and to provide intermediation to the EME. As more weight is put on the AE, the social planner imposes tighter leverage in the AE, thus protecting the AE from systemic risk. If the social planner cares only about the AE, then the EME will be called on to

¹⁶These welfare changes are implausibly large because we assume a high degree of market incompleteness (no international equity issuance). Adding more realistic risk sharing would decrease the quantitative significance of our analysis but would not change the qualitative story.

¹⁷We construct the Pareto Frontier by computing optimal policies across 30 Pareto weights ranging equidistantly from near zero to near one. The hashes in the red and blue lines indicate particular Pareto weights.

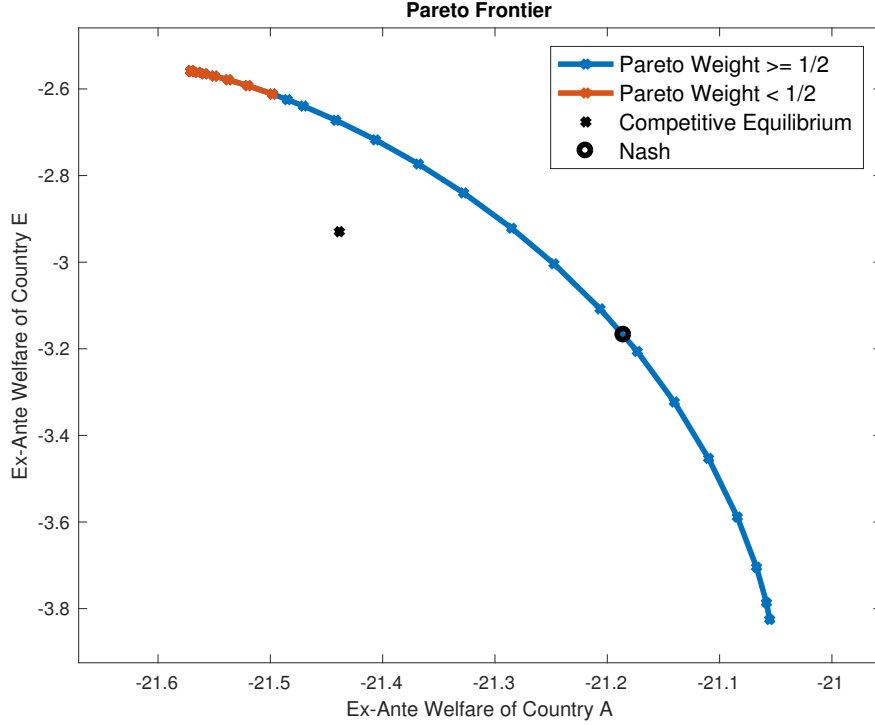


Figure 6: $\mathbb{E}[V_E]$ against $\mathbb{E}[V_A]$ as the Pareto weight α on the AE's ex-ante welfare varies from 0 to 1. The values from the competitive and Nash equilibria are also plotted.

provide intermediation.

These results show the stark contrast in welfare relative to the competitive equilibrium when countries choose policies according to Nash: the AE benefits substantially while the EME is hurt. With weights approximately in the interval $[0.6, 0.7]$, the policy chosen by a planner will *Pareto dominate* the allocation in the competitive equilibrium. The Nash equilibrium appears to be on or near the Pareto frontier when α is sufficiently large. When the planner puts non-trivial weight on the EME, then it is optimal for the AE to intermediate risk (i.e., to bear financial instability) for the sake of the EME. However, this requires the AE to suffer worse welfare outcomes. Without coordination the AE is not willing to bear the financial risk that benefits the rest of the world.

Moreover, it is likely that any policy coordination would result in policies similar to those chosen by a social planner who places a large Pareto weight on the AE. The location of Nash equilibrium in Figure 6 indicates that Pareto-improving policy coordination would produce very small gains, and the outcome would resemble that of a social planner who chooses a fairly large α . Even if the

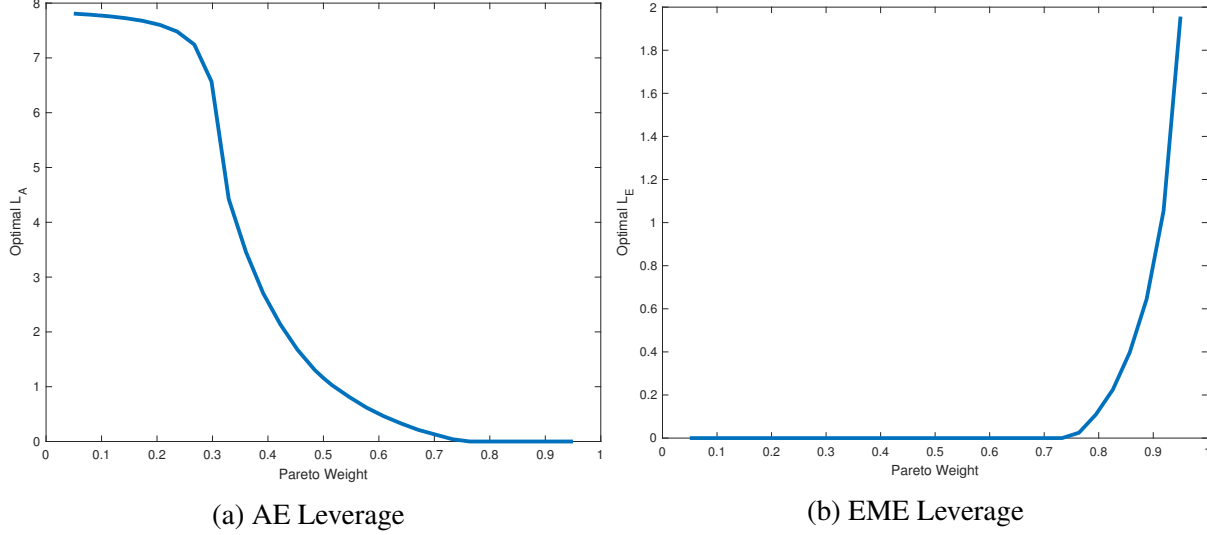
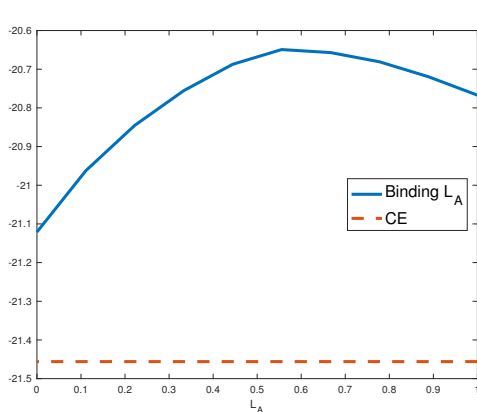


Figure 7: Optimal Leverage Varying the Pareto Weight on the AE.

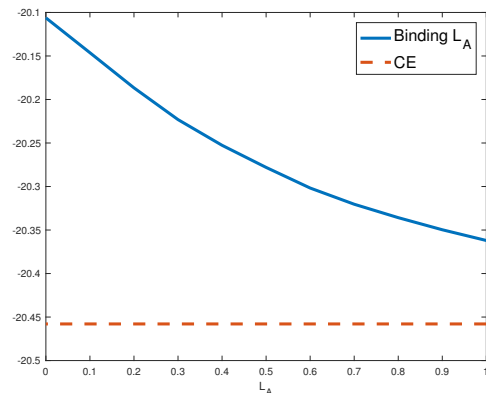
AE is willing to accept worse welfare outcomes, a priori one would expect the consequent policies to still be fairly close to Nash equilibrium. If coordination requires the AE to give up too much welfare, then the AE can always unilaterally set $L_A = 0$. Coordinating prudential regulations will necessitate policies that are strongly determined by the AE's welfare.

4.2 Mechanisms

Here, we investigate the mechanisms driving our results. First, we consider a benchmark case where we shut off the terms of trade hedge by assuming countries directly produce the final consumption good. This case illustrates the effect of leverage constraints on the pricing of risk in equilibrium. Second, we consider another benchmark case where we shut off amplification through asset prices by assuming countries have access to a linear investment technology, which causes the price of capital to be constant. This case focuses attention on the terms of trade hedge. Finally, we construct counterfactual laws of motion to compare the relative importance of risk pricing and the terms of trade in our baseline model. Due to space constraints, we only briefly describe the benchmark model variants and refer the reader to Appendix C for more details.



(a) One Good (No ToT hedge)



(b) Linear Investment Technology (Constant asset price q)

Figure 8: Panels (a) and (b) plot the ex-ante welfare of the AE in the one good model and in the linear investment technology model, respectively. The EME imposes $L_E = 0$. The blue line plots the AE's ex-ante welfare with a binding but varying leverage constraint while the dashed red line plots the AE's ex-ante welfare if it does not impose a binding leverage constraint.

Risk Pricing Only (no Terms of Trade). Intermediate goods a and b no longer exist in this benchmark. Instead, agents in the AE produce the final consumption good at rate $\bar{a}k_t$ while agents in the EME produce the final consumption good at rate $\underline{a}k_t$, where $\bar{a} > \underline{a}$.¹⁸ This modification removes the terms-of-trade hedge that promotes the rate at which net worth recovers precisely when a country becomes relatively poor. Therefore, if leverage constraints still affect welfare, then the reason is that they alter the pricing of risk in equilibrium and influence the buildup of systemic risk. Figure 8(a) shows how tighter leverage constraints affect the AE's ex-ante welfare, assuming the EME imposes $L_E = 0$. While restricting leverage improves ex-ante welfare relative to the competitive equilibrium, it is no longer monotonic. Instead, it is optimal to permit some leverage. This means that while leverage constraints have a meaningful impact on welfare through their effect on equilibrium risk pricing alone, their benefits do not justify completely closed capital accounts.

¹⁸In principle, the ideal comparison is setting $\bar{a} = \underline{a}$, but it turns out that in this equilibrium asset prices become constant, and neither country borrows. In addition, Brunnermeier and Sannikov (2015a) shows the stationary distribution is degenerate. By assuming one country is permanently more productive, we can obtain a stationary equilibrium with fire sales but without the terms of trade providing a hedge in bad times.

Intuitively, the static benefit of efficiently allocating capital is more pronounced when no terms of trade hedge applies. Without a terms of trade in intermediate goods, countries only compare the gains from higher output against the gains of lower asset price volatility. In the full model, tighter leverage constraints provide an additional benefit through a better terms-of-trade hedge. The economy becomes stabler in terms of its volatility and the rate at which countries rebuild their net worth.

Terms of Trade Only (constant asset price). We choose a new functional form for the internal investment technology:

$$\Phi(\iota) = \lambda \iota. \quad (19)$$

This linear technology implies a constant price of capital $q = 1/\lambda$, so there is no volatility in the price of capital. We choose λ to match the average price of capital in our original model. The only source of endogenous risk now is from countries' leverage choices. Figure 8(b) shows that it remains optimal for the AE to close its capital accounts, and ex-ante welfare continues to monotonically increase as L_A decreases. Moreover, we verify that, fixing $L_A = 0$, the EME's welfare monotonically increases when L_E decreases, hence $(L_A, L_E) = (0, 0)$ is still a Nash equilibrium. The fact that we obtain these results pinpoints the terms of trade hedge as the key mechanism driving our results.

For intuition, recall that the log of a geometric Brownian motion G_t with drift μ and volatility σ can be solved in closed form as

$$\log(G_t) = \log(G_0) + \left(\mu - \frac{\sigma^2}{2} \right) t + \sigma W_t$$

A change in μ has a “first-order” effect on $\log(G_t)$, but a change in σ only has a “second-order” effect since it is the variance that directly affects $\log(G_t)$. For example, if μ and σ both decreased by one percentage point, the change in μ would have a larger effect (assuming $\sigma < 1$). The

behavior of η_t is similar. We can heuristically rewrite (16), the law of motion for η , as

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = & (\text{Dividend Yield} + \text{Covariance Terms}) dt \\ & + \text{Leverage Ratio} \times (\text{Fundamental} + \text{Endogenous Vol.}) d\mathbf{W}_t. \end{aligned}$$

An increase in the terms of trade hedge increases the dividend yield and thus has a “first-order” effect because it directly affects the drift of η . Although a decrease in asset price volatility does influence the drift of η as well as the volatility, the effect occurs through covariances and is therefore still “second-order.”

In Appendix C.3 we examine the relative contributions risk pricing and the terms of trade hedge from another perspective by considering “counterfactual” scenarios in which we replace any prices of intermediate goods in the expression for μ_η in (16) by the prices from the constrained or unconstrained equilibria (thus we suppose the only effect of prudential limits is on terms of trade and not also on endogenous volatility). Our results show that the terms of trade hedge is the more important mechanism through which leverage constraints provide financial stability.

Incomplete Markets and the Terms of Trade Although the effect from the terms of trade dominates the effect from endogenous risk, incomplete markets are still the key reason why the terms of trade matter. As we noted in Section 2, if markets were complete, then countries would not strategically manipulate capital flows to influence the terms of trade. Incomplete markets, however, force countries to hold portfolios where their production as a share of world output fluctuates. In equilibrium, their production is more valuable due to higher excess returns precisely when their net worth is low. This feature creates an incentive to dynamically manipulate the terms of trade between goods a and b . In other words, incomplete markets cause time variation in a country’s income which creates similar incentives for terms-of-trade manipulation to those studied in Costinot et al. (2014).

However, in contrast to previous work on terms-of-trade manipulation, prudential regulations affect the terms of trade through portfolio choice and production decisions, not through consumption behavior. With unit EIS, countries always consume a constant fraction of their share of world

wealth (e.g., for the AE, $C_A = \rho q \eta$). Limiting leverage instead affects the scale of a country's production, which influences the returns a country receives from holding capital. Because individual agents take prices as given, they do not internalize how their leverage decisions change their terms of trade. By limiting leverage, countries earn better returns on capital, which speeds up the rate at which their net worth recovers after bad shocks.

4.3 Discussion

While our model is highly stylized, these results should still hold in some form across a wide range of potential environments. For example, our results are robust to considering other ways of modeling AEs as the financial center. We obtain qualitatively identical results if AEs and EMEs have identical preferences but AEs face lower fundamental volatility for their investments. What matters for our results is that AEs have an incentive to bear global risk but financial instability caused by market incompleteness encourages them to, at least in part or on the margin, regulate that risk away.

We have abstracted away from channels for positive macro-level spillovers, and richer set-ups would likely incorporate them. However, the key insight of our model is that a stabilizing policy in one country means that country becomes relatively richer more frequently, and we expect this mechanism, on the margin, to remain in models with positive externalities. A more realistic Nash outcome would likely involve tighter leverage constraints relative to the coordinated equilibrium, but constraints would perhaps be positive rather than zero. However, since the EME would still want the AE to provide financial intermediation, any policy coordination will continue to place substantial weight on the AE's welfare.

We can apply these insights to the case of international financial regulations. Global recommendations for financial regulation reflect agreements reached as the outcome of (possibly complicated) negotiations among countries. Because countries can always revert to Nash policies unilaterally, our results suggest that AEs (especially the U.S.) have strong bargaining power when negotiating international agreements. Crucially, the incentives for coordination differ between AEs and EMEs. AEs prefer Nash policies because they mitigate exposure to long-run financial insta-

bility. In contrast, EMEs want AEs to provide intermediation, which destabilizes AEs. To obtain intermediation, EMEs need to compensate AEs, so during negotiations, greater attention will be paid toward the concerns of AEs. In other words, the Pareto weight assigned to the welfare of AEs will be relatively large.

While we do not claim that our model exactly explains the policy decisions of AEs and EMEs, our model does predict recent empirical facts. Barth et al. (2013) show that after the financial crisis the U.S. has tightened regulations (e.g. on bank capital) more relative to many of its peers, reflecting the unilateral incentives which lead to the excessively stringent policies in our model's Nash equilibrium. In the 1990s, AEs held the "Washington Consensus" view and pressured EMEs to adopt free capital accounts. Presumably, AEs perceived economic benefits from these actions. This outcome reflects our model's prediction that a country's welfare increases with the laxness of the other country's prudential controls because it increases the probability of being relatively rich. Indeed, market liberalization in the 1990s precipitated financial crises in several EMEs and likely caused wealth redistribution toward AEs.

To be fair, prior to the financial crisis, AEs increased their international financial integration more quickly than EMEs. Over the period 1990-2004, for example, Lane and Milesi-Ferretti (2007) show that the sum of foreign assets and liabilities to GDP rose faster in AEs. This evidence appears to contradict our model in the sense that in older historical periods, AEs have developed freer capital accounts, not tighter ones.

However, this argument assumes that world outcomes must be an uncoordinated Nash equilibrium. We have also shown that financial asymmetries generate asymmetric objectives. EMEs want AEs to provide intermediation, but AEs want to avoid bearing systemic risk. This asymmetry gives AEs, on the margin, more bargaining power in international policy coordination. It is reasonable that AEs may open their capital accounts to EMEs in exchange for economic benefits from EMEs, although we do not explicitly model this bargaining process inside our model. As a proof of concept, however, in Appendix A, we consider a simple example where the AE in our model adopts open an capital account and allows the EME to maintain closed a capital account if the EME pays the AE a wealth transfer.

The experience of EMEs in the design of international financial regulations also corroborates our prediction. The Basel standards are a key component of international financial regulations, but only after the financial crisis were EMEs included in discussions to construct these standards. Walter (2015) and Shimizu (2013) observe that this narrow membership led to a focus during Basel III on regulations most relevant to banks residing in developed countries. Similar concerns have been expressed about the previous Basel accords as well. For a more extensive discussion, we refer the reader to Appendix A.

Several dimensions of our model can be further extended to provide greater insight as to how global macroprudential and capital account regulation should be conducted. Our analysis ignores any important heterogeneity within countries. Indeed, Buch and Goldberg (2016) find that heterogeneity among banks lead to different responses to cross-border spillovers. Thus, a homogenous constraint may not be the best choice, and further research should be conducted to determine how agent heterogeneity influences the optimal constraint. Second, including richer heterogeneity across countries (besides asymmetric risk aversions), and particularly across financial sectors, would likely provide additional forces for strategic interactions between countries.

5 Conclusion

Emerging market economies depend on “center” countries in the global financial system to provide intermediation and risk-bearing, but to do so requires that AEs bear systemic risk. When international credit markets are imperfect, more stable countries are more likely to be relatively wealthy compared to less stable countries. Prudential regulations, when effective, increases economic stability, creating a macro-level spillover through international capital flows. As a result, tight prudential controls in one country provides strategic incentives for tight policy in the other. Because countries can unilaterally choose their prudential regulations, AEs have an incentive to provide less intermediation than demanded by EMEs. Therefore, coordination will likely result in policies that place relatively more emphasis on the outcomes of AEs than the outcomes of EMEs.

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Appendices for Online Publication

A International Negotiations and Bargaining Power

We can apply these insights to the case of international financial regulations. Global recommendations for financial regulation reflect agreements reached as the outcome of (possibly complicated) negotiations among countries. Because countries can always revert to Nash policies unilaterally, our results suggest that AEs (especially the U.S.) have strong bargaining power when negotiating international agreements. Crucially, the incentives for coordination differ between AEs and

EMEs. AEs prefer Nash policies because they mitigate exposure to long-run financial instability. In contrast, EMEs want AEs to provide intermediation, which destabilizes AEs. To obtain intermediation, EMEs need to compensate AEs, so during negotiations, greater attention will be paid toward the concerns of AEs. In other words, the Pareto weight assigned to the welfare of AEs will be relatively large. In principle, policy coordination leads to outcomes at least as good as the uncoordinated equilibrium, but achieving successful coordination is nontrivial, and the benefits may accrue unequally to participants. Understanding the incentives for coordination and the incentives that cause Nash equilibrium to differ from the coordinated equilibrium offer important insight for the conduct and creation of new international regulations. First, we present a simple example showing how financial asymmetry could lead to policies that primarily benefit the AE. Second, we discuss how our model's predictions are consistent with commentary about the ways in which AEs both control and benefit the most from recent international financial regulations.

A.1 Bargaining Power Example

This section expands the space of permissible actions by the EME to model this bargaining power in a simple reduced form. The EME can impose a wealth tax on its agents at a rate τ and use those taxes to pay the AE when η is below some threshold η_τ . At time $t = 0$, the EME and AE can also implement a one-time wealth transfer to achieve an initial wealth allocation. It is in the EME's interest to pay this tax and agree to the initial wealth redistribution because a sufficiently generous tax and redistribution can make the AE willing to raise its leverage constraint above zero. One could interpret these policies as insurance paid to the AE by the EME in exchange for the AE's providing intermediation. The tax modifies the laws of motion for η (16) and (17) by adding the term $\tau(1 - \eta)/\eta$ to μ_η when $\eta < \eta_\tau$. We do not interpret the wealth tax literally but instead utilize it as a modeling device that represents the capacity of AEs to negotiate international policies to their benefit.

Since we aim mainly to provide an example, we do not conduct an exhaustive search for the optimal policy the AE and EME can obtain through redistributive policies. We choose the parameters $\eta_\tau = 0.5$, $\tau = 40\text{bp}$, $\eta_0 = 0.4934$, L_A never binding, and $L_E = 0$. With these choices, the AE

obtains a consumption equivalent welfare gain of 55bp while the EME acquires a 5.8bp increase. When this option is available, both countries strictly prefer to cooperate on this bargaining solution instead of the Nash equilibrium. However, whereas these policies improve the AE's welfare outcomes even more, they do not help the EME very much. Relative to the competitive equilibrium, the EME still suffers a welfare decrease of 19.1% in consumption equivalents.

A.2 Negotiations of International Agreements

Advanced economies (AEs) hold substantial influence over the governance and direction of many international economic organizations, and only recently have emerging market economies (EMEs) begun to take significant leadership in the writing of international regulatory standards. As of September 2018, the U.S., Japan, Germany, France, and the U.K. alone hold 36% of the allotted votes on the IMF Executive Board. Moreover, every president of the World Bank was American, every managing director of the IMF has been an European, and every chairman of the Bank for International Settlements (BIS) has been from an AE. Out of 20 members on BIS's board of directors, 16 come from AEs. Furthermore, it was not until after the Financial Crisis that EMEs were included in regulatory discussions by the Basel Committee on Banking Supervision (BCBS). Similarly, the G20 replaced the G8 as the main body for international economic cooperation only in 2009.

Despite greater formal representation for EMEs, AEs continue to dominate the activities of these organizations when it comes to financial regulation. Walter (2015) finds evidence that EMEs still do not participate very much in the creation of financial rules like Basel III, which may be explained by mismatch between the agendas of these regulatory bodies and EME interests as well as EME's comparative lack of resources and experiences navigating international financial regulation. Bair (2012) relates an account of the Basel III discussions about leverage ratios, portraying the debate as two-sided between an opposing group (Germany, France, and Japan), and a supporting group (the U.S., U.K., Canada, Switzerland, the Netherlands, Sweden, and most Basel Committee members). Notably, the account made no mention of EMEs. These claims are corroborated by Watanagase (2013), who alleges that "most proposed regulations [by Basel III] are formulated

and calibrated mainly based on advanced markets’ contexts and data, which can be significantly different from [EMEs].”

Existing explanations for this outcome can be broadly characterized as follows: AEs hold significant bargaining power during the negotiation of new rules because they have greater market size and more expertise with financial systems and regulation.¹⁹ As a result, AEs are able to shape the international financial system to better fit their preferences. Our paper has proposed an additional channel through which AEs have greater bargaining power over EMEs: the key role AEs play in bearing financial instability. AEs with developed financial systems can better intermediate risky investments through the issuance of debt liabilities, but doing so exposes AEs to a greater risk of financial crisis.

Our results corroborate and supplement commentary about the ways in which AEs both control and benefit the most from recent international financial regulations. Indeed, there is some concern that AEs design global macroprudential and capital account policies that are not well-suited, at a micro level, for EMEs.²⁰ When creating international regulatory standards in response to the financial crises of the 1990s, developed countries either vested new powers in organizations with predominantly AE membership or constructed new institutions like the Financial Stability Forum rather than use established organizations like the IMF. The latter choice reflected the fact that on the IMF Executive Board, the precedent is to make a decision only when consensus has been achieved. If AEs had chosen to vest the IMF with these powers, then they would have had to consider the concerns of EMEs when writing new international financial regulations. Through the creation of new institutions and the use of existing ones with membership dominated by their own representatives—whether this was the desired intention or not—AEs could write regulations based primarily on their own preferences. For more details, see chapter 5 in Drezner (2007).

Despite these circumstances, the standards written by these bodies still became de facto interna-

¹⁹For example, in a speech on behalf of the BIS, White (2001) justified the sparse membership of the BCBS during Basel II on the grounds that representatives from AEs had greater expertise due to their more developed financial systems. Walter (2015) discusses evidence for why EMEs may have limited knowledge on constructing and implementing international financial regulations. See chapter 2 of Drezner (2007) for a game-theoretic model demonstrating how AEs can take advantage of their economic power and market size to influence international rule-setting.

²⁰In the popular press, Taylor (2010) writing in the *Financial Times* emphasizes that Basel III regulations are too complex for emerging markets to implement well (e.g. risk management offices are just not sophisticated enough).

tional standards for EMEs. AEs linked these standards to participation in international institutions and to market mechanisms. Access to resources from organizations like the IMF began to require compliance with financial regulatory standards written by AE-dominated organizations like the Financial Stability Forum. Even though developing countries did not participate in the creation of those standards, they nevertheless had to follow them to acquire needed resources. Beck et al. (2018) finds evidence that EMEs, when seeking advice from the IMF and World Bank, have been advised or encouraged to adopt parts of Basel II and III, despite these standards not being official requirements. Markets also reinforced the need to comply with these regulations. Failure to demonstrate compliance would generate negative signals in financial markets. To facilitate expansion by both EME banks into foreign markets and AE banks into EME markets, many EMEs adopted Basel II and Basel III because they are de facto international regulatory standards, as Beck et al. (2018) notes.

Because EMEs were included in Basel discussions only after the Financial Crisis, Basel III still tended to focus on regulations most relevant to banks residing in AEs. Shimizu (2013) notes that some of the regulations, like the more stringent capital adequacy regulation, may impair economic growth in Asian economies without substantially improving financial stability because of the differences in the structure of their financial systems. Historically, the introduction of Basel regulatory standards sharply contracted the supply and weakened the demand for credit in Japan, contributing to the country's "lost decade." Substantial portions of Shimizu (2013) also criticize Basel's capital adequacy regulation as ineffective for addressing systemic risk and as a major cause of Japan's credit crunch during the 1990s.

The implementation of Basel III's leverage ratio may similarly penalize EMEs. Most banks in Asian EMEs focus on supplying traditional financial products and especially trade finance. For example, Watanagase (2013) reports that at the time of the Financial Crisis, only 0.1 percent of total assets held by the Thai banking sector were invested in products related to collateralized debt obligations. Basel III's leverage ratio, however, could render traditional financial products like trade finance more costly than holding off-balance-sheet items like derivatives. On the margin, the comparatively higher cost of traditional products may incentivize financial institutions to invest

more of their portfolio in sophisticated and riskier products.

Basel III and unilateral regulatory reforms in response to the financial crisis (e.g., ring-fencing in the UK and the Volcker rule in the US) may also reduce the volume of cross-border transactions with EME banks due to higher costs, adversely impacting EMEs. Alford (2013) worries that, because of additional supervision, systemically important financial institutions (SIFI) may reduce their investment in emerging markets. Beck (2018) discusses some of the channels through which fund flows may be reduced.

While Basel III focused on many rules-based regulations, EMEs tend to favor more discretionary regulations because their financial systems are different from both each others' and the financial systems of AEs. Shimizu (2013) pushes Basel III to allow flexibility in the national implementation of its standards because rules targeting the causes of financial crises in Western AEs like the U.S. may not be effective in Asian economies and could constrict the flow of funds needed for growth. Other central banks, notably the Reserve Bank of India, have also seen success using dynamic sector risk-weight adjustment, where regulators use their discretion to identify the systemic risk of different asset classes and re-weight their supervision accordingly (see Acharya (2013)). Watanagase (2013) advocates discretionary risk-based supervision as an essential component of financial regulations because exceedingly rules-based regulations may not adapt to new technologies and changing sources of systemic risk. Knowledge of the linkages between the financial sector and the real economy is also limited, so regulatory rules may not be complete in their scope, making discretion a necessary dimension for policymakers.

B Proofs

B.1 Asset Pricing Equations

First, consider the Hamilton-Jacobi-Bellman (HJB) equation for the AE agents:

$$0 = \max_{c_{A,t}, x_{Aa,t}, x_{Ab,t}} f(c_{A,t}, V_{A,t}) + \mathbb{E}_t[dV_{A,t}]$$

subject to the leverage constraint $L_{A,t}$ and the law of motion for net worth (12). For now, suppress the max operator for economy of presentation. By Ito's product rule and Ito's lemma,

$$\begin{aligned}\mathbb{E}[d(\xi_t n_{A,t})] &= \xi_t n_{A,t} (\mu_{\xi,t} + \mu_{nA,t} + \Sigma_{\xi,t} \cdot \Sigma_{nA,t}) \\ \mathbb{E}\left[\frac{d(\xi_t n_{A,t})^{1-\gamma_A}}{1-\gamma_A}\right] &= (\xi_t n_{A,t})^{-\gamma_A} \mathbb{E}[d(\xi_t n_{A,t})] - \frac{\gamma_A}{2} (\xi_t n_{A,t})^{-1-\gamma_A} (d(\xi_t n_{A,t}))^2 \\ &= (\xi_t n_{A,t})^{1-\gamma_A} (\mu_{\xi,t} + \mu_{nA,t} + \Sigma_{\xi,t} \cdot \Sigma_{nA,t}) - \frac{\gamma_A}{2} (\xi_t n_{A,t})^{1-\gamma_A} \|\Sigma_{\xi,t} + \Sigma_{nA,t}\|^2\end{aligned}$$

Plugging this quantity into the HJB yields

$$\begin{aligned}0 &= \rho(1-\gamma_A) \frac{(\xi_t n_{A,t})^{1-\gamma_A}}{1-\gamma_A} \left[\log(c_{A,t}) - \frac{1}{1-\gamma_A} \log((\xi_t n_{A,t})^{1-\gamma_A}) \right] \\ &\quad + (\xi_t n_{A,t})^{1-\gamma_A} (\mu_{\xi,t} + \mu_{nA,t} + \Sigma_{\xi,t} \cdot \Sigma_{nA,t}) - \frac{\gamma_A}{2} (\xi_t n_{A,t})^{1-\gamma_A} \|\Sigma_{\xi,t} + \Sigma_{nA,t}\|^2\end{aligned}$$

Cancelling by $(\xi_t n_{A,t})^{1-\gamma_A}$ and plugging in for the drift of the AE's net worth yields

$$\begin{aligned}0 &= \rho \left[\log(c_{A,t}) - \frac{1}{1-\gamma_A} \log((\xi_t n_{A,t})^{1-\gamma_A}) \right] + \mu_{\xi,t} + \Sigma_{\xi,t} \cdot \Sigma_{nA,t} \\ &\quad + \left[dr_t^F + \mathbb{E}[x_{Aa,t}(dr_t^{Aa} - dr_t^F) + x_{Ab,t}(dr_t^{Ab} - dr_t^F)] - \frac{c_{A,t}}{n_{A,t}} \right] \\ &\quad - \frac{\gamma_A}{2} (\|\Sigma_{\xi,t}\|^2 + 2\Sigma_{\xi,t} \cdot \Sigma_{nA,t} + \|\Sigma_{nA,t}\|^2).\end{aligned}$$

Additionally plugging in for $\Sigma_{nA,t}$ obtains

$$\begin{aligned}0 &= \rho \left[\log(c_{A,t}) - \frac{1}{1-\gamma_A} \log((\xi_t n_{A,t})^{1-\gamma_A}) \right] + \mu_{\xi,t} \\ &\quad + \left[dr_t^F + \mathbb{E}[x_{Aa,t}(dr_t^{Aa} - dr_t^F) + x_{Ab,t}(dr_t^{Ab} - dr_t^F)] - \frac{c_{A,t}}{n_{A,t}} \right] \\ &\quad + (1-\gamma_A) \Sigma_{\xi,t} \cdot [(x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t}] \\ &\quad - \frac{\gamma_A}{2} (\|\Sigma_{\xi,t}\|^2 + \|(x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t}\|^2).\end{aligned}$$

Differentiating with respect to $c_{A,t}$ yields

$$\rho(\xi_t n_{A,t})^{1-\gamma_A} c_{A,t}^{-1} - (\xi_t n_{A,t})^{1-\gamma_A} n_{A,t}^{-1} = 0 \Rightarrow \frac{c_{A,t}}{n_{A,t}} = \rho.$$

Differentiating w.r.t. to the portfolio shares on capital funded by inside equity yields, when leverage constraints are not binding,

$$\begin{aligned}\mathbb{E}[dr_t^{Aa}] - dr_t^F &= \gamma_A(x_{Aa,t} + x_{Ab,t})\Sigma_{Aa,t} \cdot \Sigma_{Aa,t} + (\gamma_A - 1)\Sigma_{\xi,t} \cdot \Sigma_{Aa,t}, \\ \mathbb{E}[dr_t^{Ab}] - dr_t^F &\leq \gamma_A(x_{Aa,t} + x_{Ab,t})\Sigma_{Aa,t} \cdot \Sigma_{Aa,t} + (\gamma_A - 1)\Sigma_{\xi,t} \cdot \Sigma_{Aa,t}.\end{aligned}$$

Note that the second inequality arises because the AE specializes in the production of good a .

Let $\varsigma_{Aa,t}$ denote the covariance between the AE's volatility of holding capital and its volatility of net worth, and let $\varsigma_{\xi A,t}$ denote the covariance between the volatility of ξ and the AE's volatility of holding capital. These quantities are similarly defined for the EME. Using this short-hand, we have

$$\mathbb{E}[dr_t^{Aa}] - dr_t^F \leq \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1)\varsigma_{\xi A,t}.$$

To summarize, agents in the AE consume at rate ρ per unit of wealth, and they price risky capital according to

$$\mathbb{E}[dr_t^{Aa}] - dr_t^F = \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1)\varsigma_{\xi A,t}, \quad (20)$$

$$\mathbb{E}[dr_t^{Ab}] - dr_t^F \leq \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1)\varsigma_{\xi A,t}, \quad (21)$$

$$\mathbb{E}[dr_t^{Eb}] - dr_t^F \leq \gamma_A \varsigma_{Ae,t} + (\gamma_A - 1)\varsigma_{\xi E,t}, \quad (22)$$

where

$$\varsigma_{Aa,t} = (x_{Aa,t} + x_{Ab,t})[(\sigma + \sigma_{qA,t})^2 + \sigma_{qE,t}^2], \quad (23)$$

$$\varsigma_{\xi A,t} = (\sigma + \sigma_{qA,t})\sigma_{\xi A,t} + \sigma_{qE,t}\sigma_{\xi E,t}. \quad (24)$$

B.2 Proof of Proposition 1: Law of Motion for η

We conjecture that all equilibrium variables depend endogenously on the share of global wealth held by the AE:

$$\eta_t \equiv \frac{N_{A,t}}{q_t K_t}.$$

In the subsequent sections, we verify this conjecture, but first we derive the equilibrium law of motion for η_t . Aggregating (12) over all agents in the AE and substituting equations (20), (21), and (22) obtains

$$\begin{aligned} \frac{dN_{A,t}}{N_{A,t}} = & dr_t + \left\{ \gamma_A [(x_{Aa,t} + x_{Ab,t}) \zeta_{Aa,t} + (\gamma_A - 1) [(x_{Aa,t} + x_{Ab,t}) \zeta_{\xi A,t} - \frac{C_{A,t}}{N_{A,t}}] \right\} dt \\ & + (x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t} \cdot d\mathbf{W}_t. \end{aligned} \quad (25)$$

By Ito's product rule, global wealth evolves according to

$$\frac{d(q_t K_t)}{q_t K_t} = (\mu_{q,t} + \Phi(\iota_t) - \delta + \Sigma_{q,t} \cdot \Sigma_{K,t}) dt + (\Sigma_{q,t} + \Sigma_{K,t}) \cdot d\mathbf{W}_t. \quad (26)$$

Expanding the returns equation in (20) allows us to write

$$\mu_{q,t} + \Phi(\iota_t) - \delta = dr_t + \gamma_A \zeta_{Aa,t} + (\gamma_A - 1) \zeta_{\xi A,t} - \frac{\bar{a} P_{a,t} - \iota_t}{q_t} - \sigma \sigma_{qA,t}.$$

Substituting this and applying Ito's quotient rule yields

$$\begin{aligned} \frac{d(1/(q_t K_t))}{1/(q_t K_t)} = & \left(\|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - dr_t - \gamma_A \zeta_{Aa,t} - (\gamma_A - 1) \zeta_{\xi A,t} + \frac{\bar{a} P_{a,t} - \iota_t}{q_t} + \sigma \sigma_{qA,t} - \Sigma_{q,t} \cdot \Sigma_{K,t} \right) dt \\ & - (\Sigma_{q,t} + \Sigma_{K,t}) \cdot d\mathbf{W}_t. \end{aligned}$$

By Ito's product rule,

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = & \left\{ \gamma_A [(x_{Aa,t} + x_{Ab,t}) \varsigma_{Aa,t} - \varsigma_{Aa,t}] + (\gamma_A - 1) [(x_{Aa,t} + x_{Ab,t}) \varsigma_{\xi A,t} - \varsigma_{\xi A,t}] - \frac{C_{A,t}}{N_{A,t}} - \Sigma_{q,t} \cdot \Sigma_{K,t} \right\} dt \\ & + \left\{ \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 + \frac{\bar{a}P_{a,t} - \iota_t}{q_t} + \sigma \sigma_{qA,t} - (x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t}) \right\} dt \\ & + \left\{ (x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t} - \Sigma_{q,t} - \Sigma_{K,t} \right\} \cdot d\mathbf{W}_t. \end{aligned}$$

This simplifies to

$$\begin{aligned} \frac{d\eta_t}{\eta_t} = & \left\{ \frac{\bar{a}P_{a,t} - \iota_t}{q_t} - \frac{C_{A,t}}{N_{A,t}} + (x_{Aa,t} + x_{Ab,t} - 1) (\gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}) \right\} dt \\ & + \left\{ \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 + \sigma \sigma_{qA,t} - \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t} - (x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t}) \right\} dt \\ & + \left\{ (x_{Aa,t} + x_{Ab,t}) \Sigma_{Aa,t} - \Sigma_{q,t} - \Sigma_{K,t} \right\} \cdot d\mathbf{W}_t \end{aligned}$$

We can replace portfolio shares and dot products by observing

$$\begin{aligned} x_{Aa,t} + x_{Ab,t} &= \frac{\psi_{A,t} q_t K_t}{N_{A,t}} = \frac{\psi_{A,t}}{\eta_t} \\ \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 &= (\psi_{A,t} \sigma + \sigma_{qA,t})^2 + (\psi_{E,t} \sigma + \sigma_{qE,t})^2. \end{aligned}$$

We can also simplify the volatility vector to become

$$\begin{aligned} \Sigma_{\eta t} &= \begin{bmatrix} \frac{\psi_{A,t}}{\eta_t} (\sigma + \sigma_{qA,t}) - \sigma_{qA,t} - \psi_{A,t} \sigma \\ \frac{\psi_{A,t}}{\eta_t} \sigma_{qE,t} - \sigma_{qE,t} - \psi_{E,t} \sigma \end{bmatrix} \\ &= \begin{bmatrix} \frac{1 - \eta_t}{\eta_t} \psi_{A,t} \sigma + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qA,t} \\ -\psi_{E,t} \sigma + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qE,t} \end{bmatrix} \end{aligned}$$

Re-arranging and substitutions yield Proposition 1.

B.3 Proof of Proposition 1: Asset Pricing Condition and Division of the State Space

In this section, we drop time subscripts.

Asset Pricing Condition. To obtain the global asset pricing condition, we difference the AE's required return on good a and the EME's required return on good b , which yields

$$\frac{\bar{a}(P_a - P_b)}{q} + \sigma\sigma_{qA} - \sigma\sigma_{qE} = \gamma_A \zeta_{Aa} + (\gamma_A - 1)\zeta_{\xi A} - \gamma_E \zeta_{Eb} - (\gamma_E - 1)\zeta_{\zeta E} \quad (27)$$

The AE Does Not Specialize. In this case, the AE chooses to produce good b . Then (21) binds with equality, yielding the equality

$$\frac{P_a}{P_b} = \frac{a}{\bar{a}},$$

which leads to the equations

$$\begin{aligned} \psi_{Eb} &= \frac{1 - (1 + \frac{1-\alpha}{\alpha})\psi_{Ab}}{1 + \frac{\bar{a}}{a}\frac{1-\alpha}{\alpha}} \\ \psi_{Ab} &= \frac{1 - (1 + \frac{\bar{a}}{a}\frac{1-\alpha}{\alpha})\psi_{Eb}}{1 + \frac{1-\alpha}{\alpha}}. \end{aligned}$$

Depending on how the algorithm is coded, one of these forms may be more convenient. To pin down ψ_{Aa} or ψ_{Ea} , we use the market clearing condition for consumption, which reduces to

$$\rho q = (\bar{a}\psi_{Aa} + \underline{a}\psi_{Ea})^\alpha (\underline{a}\psi_{Ab} + \bar{a}\psi_{Eb})^{1-\alpha} - \iota(q), \quad (28)$$

where $\iota(\cdot)$ is defined as a function of q by Tobin's q (9). Note that this condition holds even if leverage constraints bind

The EME Does Not Specialize. In this case, the EME chooses to produce good a . Then we have the equality

$$\frac{P_a}{P_b} = \frac{\bar{a}}{\underline{a}},$$

which leads to the equations

$$\begin{aligned}\psi_{Aa} &= \frac{1 - (1 + \frac{1-\alpha}{\alpha})\psi_{Ea}}{1 + \frac{\bar{a}}{\underline{a}} \frac{1-\alpha}{\alpha}} \\ \psi_{Ea} &= \frac{1 - (1 + \frac{\bar{a}}{\underline{a}} \frac{1-\alpha}{\alpha})\psi_{Aa}}{1 + \frac{1-\alpha}{\alpha}}.\end{aligned}$$

Depending on how the algorithm is coded, one of these forms may be more convenient. Note that this condition holds even if leverage constraints bind.

Only One Country Does Not Specialize. The previous two sections show that the price ratio of good a to good b satisfies

$$\frac{\underline{a}}{\bar{a}} \leq \frac{P_a}{P_b} \leq \frac{\bar{a}}{\underline{a}}.$$

In order for the AE to not specialize, the left inequality must bind with equality, and in order for the EME to not specialize, the right inequality must bind with inequality. Since $\bar{a} > \underline{a}$, only one inequality can bind at a time. Therefore, there are three regions of capital allocations.

1. The AE specializes, the EME does not specialize.
2. Both countries specialize.
3. The AE does not specialize, the EME specializes.

B.4 Proof of Proposition 2

Suppose that the leverage constraint binds for the AE agents. Then (20) will not bind with equality. However, if the AE agents are levered, then the EME agents are not levered, so their asset-pricing condition must still hold. Writing out returns for the EME, we have

$$\begin{aligned}
\mathbb{E}[dr_t^{Ea}] - dr_t &\leq \gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} \\
\mathbb{E}[dr_t^{Eb}] - dr_t &= \gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} \\
\mathbb{E}[dr_t^{Aa}] - dr_t &\leq \gamma_E \varsigma_{Ee,t} + (\gamma_E - 1) \varsigma_{\zeta A,t} \\
\varsigma_{Eb,t} &= \frac{\psi_{E,t}}{1 - \eta_t} [\sigma_{qA,t}^2 + (\sigma + \sigma_{qE,t})^2] \\
\varsigma_{Ee,t} &= \frac{\psi_{E,t}}{1 - \eta_t} [(\sigma + \sigma_{qA,t})\sigma_{qA,t} + (\sigma + \sigma_{qE,t})\sigma_{qE,t}] + \sigma_{qE,t}^2 \\
\varsigma_{\zeta E,t} &= \sigma_{qA,t}\sigma_{\zeta A,t} + (\sigma + \sigma_{qE,t})\sigma_{\zeta E,t} \\
\varsigma_{\zeta A,t} &= (\sigma + \sigma_{qA,t})\sigma_{\zeta A,t} + \sigma_{qE,t}\sigma_{\zeta E,t}.
\end{aligned}$$

Expanding the excess returns for production of intermediate goods yields

$$\begin{aligned}
\mathbb{E}[dr_t^{Aa}] &= \frac{\bar{a}P_{a,t} - \iota_t}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma\sigma_{qA,t}, \\
\mathbb{E}[dr_t^{Eb}] &= \frac{\bar{a}P_{b,t} - \iota_t}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma\sigma_{qE,t}.
\end{aligned}$$

Observe the similar terms in these two expressions. By adding and subtracting the appropriate terms, we can write the excess returns from producing good a for the AE in terms of the excess returns from producing good b for the EME:

$$\mathbb{E}[dr_t^{Aa}] - dr_t = \mathbb{E}[dr_t^{Eb}] - dr_t + \frac{\bar{a}(P_{a,t} - P_{b,t})}{q_t} + \sigma\sigma_{qA,t} - \sigma\sigma_{qE,t}.$$

Since the EME's asset-pricing condition still holds when the AE is constrained, we have

$$\mathbb{E}[dr_t^{Aa}] - dr_t = \gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} + \frac{\bar{a}(P_{a,t} - P_{b,t})}{q_t} + \sigma\sigma_{qA,t} - \sigma\sigma_{qE,t},$$

where we continue to use the notation that $\varsigma_{ij,t}$ denotes the risk premium required by Country i when investing in returns on good j or equity as if their risk aversion was one. We may similarly write the excess returns for the other expressions:

$$\begin{aligned}\mathbb{E}[dr_t^{Ab}] - dr_t &\leq \gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\xi E,t} - \frac{(\bar{a} - \underline{a})P_{b,t}}{q_t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t}, \\ \mathbb{E}[dr_t^{Eb}] - dr_t &= \gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\xi E,t}.\end{aligned}$$

We can repeat this same exercise for the EME. If leverage constraints bind for E , then excess returns satisfy

$$\begin{aligned}\mathbb{E}[dr_t^{Ea}] - dr_t &\leq \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t} - \frac{(\bar{a} - \underline{a})P_{a,t}}{q_t} - \sigma \sigma_{qA,t} + \sigma \sigma_{qE,t}, \\ \mathbb{E}[dr_t^{Eb}] - dr_t &= \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t} - \frac{\bar{a}(P_{a,t} - P_{b,t})}{q_t} - \sigma \sigma_{qA,t} + \sigma \sigma_{qE,t}, \\ \mathbb{E}[dr_t^{Aa}] - dr_t &\leq \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}.\end{aligned}$$

To derive the appropriate law of motion, observe that the AE's asset-pricing conditions always hold true as long as it is not constrained. Because we only used the AE's conditions in the original derivation of η_t without leverage constraints, when the EME is constrained, the same law of motion still applies. The difference will be the set of equations that pin down capital allocations.

However, if constraints bind for the AE, then our stated law of motion is incorrect because, for example,

$$\mathbb{E}[dr_t^{Aa}] - dr_t \neq \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}.$$

To rectify this, we re-write

$$\begin{aligned}
\frac{dN_{At}}{N_{At}} &= dr_t - \frac{c_{At}}{n_{At}} dt + \frac{\psi_{Aa,t}}{\eta_t} \left(\gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} + \frac{\bar{a}(P_{a,t} - P_{b,t})}{q_t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t} \right) dt \\
&\quad + \frac{\psi_{Ab,t}}{\eta_t} \left(\gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} - \frac{(\bar{a} - \underline{a})P_{b,t}}{q_t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t} \right) dt \\
&\quad + (\gamma_E - 1) \varsigma_{\zeta E,t} dt + \frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} \cdot d\mathbf{W}_t.
\end{aligned}$$

Adjusting the law of motion for aggregate wealth yields

$$\begin{aligned}
\frac{d(1/(q_t K_t))}{1/(q_t K_t)} &= \left(\|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - dr_t - \gamma_E \varsigma_{Eb,t} - (\gamma_E - 1) \varsigma_{\zeta E,t} + \frac{\bar{a}P_{b,t} - \iota_t}{q_t} + \sigma \sigma_{qE,t} - \Sigma_{q,t} \cdot \Sigma_{K,t} \right) dt \\
&\quad - (\Sigma_{q,t} + \Sigma_{K,t}) \cdot d\mathbf{W}_t.
\end{aligned}$$

By Ito's product rule,

$$\begin{aligned}
\frac{d\eta_t}{\eta_t} &= -\frac{c_{At}}{n_{At}} dt + \frac{\psi_{Aa,t}}{\eta_t} \left(\gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} + \frac{\bar{a}(P_{a,t} - P_{b,t})}{q_t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t} \right) dt \\
&\quad + \frac{\psi_{Ab,t}}{\eta_t} \left(\gamma_E \varsigma_{Eb,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} - \frac{(\bar{a} - \underline{a})P_{b,t}}{q_t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t} + (\gamma_E - 1) \varsigma_{\zeta E,t} \right) dt \\
&\quad + \left(\|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - \gamma_E \varsigma_{Eb,t} - (\gamma_E - 1) \varsigma_{\zeta E,t} + \frac{\bar{a}P_{b,t} - \iota_t}{q_t} + \sigma \sigma_{qE,t} - \Sigma_{q,t} \cdot \Sigma_{K,t} \right) dt \\
&\quad - \frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t}) dt + \left[\frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} - \Sigma_{q,t} - \Sigma_{K,t} \right] \cdot d\mathbf{W}_t.
\end{aligned}$$

Focusing on the drift, we have the simplification

$$\begin{aligned}
\mu_{\eta_t} &= \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) (\gamma_E \zeta_{Eb,t} + (\gamma_E - 1) \zeta_{\zeta E,t} + \sigma \sigma_{qA,t} - \sigma \sigma_{qE,t}) + (\gamma_E - 1) \zeta_{\zeta E,t} \\
&\quad + \frac{\psi_{Aa,t}}{\eta_t} \frac{\bar{a}P_{a,t}}{q_t} - \frac{\psi_{A,t}}{\eta_t} \frac{\bar{a}P_{b,t}}{q_t} + \frac{\psi_{Ab,t}}{\eta_t} \frac{aP_{b,t}}{q_t} - \frac{C_{A,t}}{N_{A,t}} + \frac{\bar{a}P_{b,t} - l_t}{q_t} \\
&\quad + \sigma \sigma_{qE,t} - \psi_{A,t} \sigma \sigma_{qA,t} - \psi_{E,t} \sigma \sigma_{qE,t} \\
&\quad + \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - \frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t}) \\
&= \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \left(\gamma_E \zeta_{Eb,t} + (\gamma_E - 1) \zeta_{\zeta E,t} - \frac{\bar{a}P_{b,t}}{q_t} \right) - \frac{l_t}{q_t} + (\gamma_E - 1) \zeta_{\zeta E,t} \\
&\quad + \frac{\psi_{Aa,t}}{\eta_t} \frac{\bar{a}P_{a,t}}{q_t} + \frac{\psi_{Ab,t}}{\eta_t} \frac{aP_{b,t}}{q_t} - \frac{C_{A,t}}{N_{A,t}} + \left[\left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) - \psi_{A,t} \right] (\sigma \sigma_{qA,t} - \sigma \sigma_{qE,t}) \\
&\quad + \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - \frac{\psi_{A,t}}{\eta_t} \Sigma_{Aa,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t})
\end{aligned}$$

Simplification yields Proposition 2.

B.5 Equilibrium ODEs

To solve for equilibrium, we construct a system of ODEs in three dependent variables: q , ξ , and ζ . It is convenient to work with a change of variables for ξ and ζ because it allows us to reduce the system to a first-order differential equation in q and two other second order ODEs. If we directly constructed ODEs for ξ and ζ from the HJBs of Countries the AE and EME, then we would need to solve a system of three second-order differential equations, as in Di Tella (2017).

Change of Variables. Define v_A and v_E by

$$v_A = \xi \eta q, \quad v_E = \zeta (1 - \eta) q$$

Then

$$V_A = \frac{(\xi \eta q K)^{1-\gamma_A}}{1-\gamma_A} = \frac{(v_A K)^{1-\gamma_A}}{1-\gamma_A}$$

$$V_E = \frac{(\zeta(1-\eta)qK)^{1-\gamma_E}}{1-\gamma_E} = \frac{(v_E K)^{1-\gamma_E}}{1-\gamma_E}.$$

We can therefore write

$$\mathbb{E}[dV_A] = (v_A K)^{1-\gamma_A} (\mu_{v_A} + \Phi(\iota) - \delta + \Sigma_{v_A} \cdot \Sigma_K) - \frac{\gamma_A}{2} (v_A K)^{1-\gamma_A} \|\Sigma_{v_A} + \Sigma_K\|^2$$

$$\mathbb{E}[dV_E] = (v_E K)^{1-\gamma_E} (\mu_{v_E} + \Phi(\iota) - \delta + \Sigma_{v_E} \cdot \Sigma_K) - \frac{\gamma_E}{2} (v_E K)^{1-\gamma_E} \|\Sigma_{v_E} + \Sigma_K\|^2.$$

Plugging back into the HJBs for the AE and EME yields

$$0 = \rho (v_A K)^{1-\gamma_A} (\log(\rho \eta q K) - \log(v_A K))$$

$$+ (v_A K)^{1-\gamma_A} (\mu_{v_A} + \Phi(\iota) - \delta + \Sigma_{v_A} \cdot \Sigma_K) - \frac{\gamma_A}{2} (v_A K)^{1-\gamma_A} \|\Sigma_{v_A} + \Sigma_K\|^2$$

$$\mu_{v_A} = \frac{\gamma_A}{2} [(\psi_A \sigma_A + \sigma_{v_A A})^2 + (\psi_E \sigma_E + \sigma_{v_A E})^2]$$

$$+ \rho (\log(v_A) - \log(\rho q \eta)) - (\Phi(\iota) - \delta) - \psi_A \sigma_A \sigma_{v_A A} - \psi_E \sigma_E \sigma_{v_A E}$$

and

$$\mu_{v_E} = \frac{\gamma_E}{2} [(\psi_A \sigma_A + \sigma_{v_E A})^2 + (\psi_E \sigma_E + \sigma_{v_E E})^2]$$

$$+ \rho (\log(v_E) - \log(\rho q (1-\eta))) - (\Phi(\iota) - \delta) - \psi_A \sigma_A \sigma_{v_E A} - \psi_E \sigma_E \sigma_{v_E E}.$$

By Ito's lemma, we also have

$$\mu_{v_A} = \frac{\partial_{\eta} v_A}{v_A} \mu_{\eta} \eta + \frac{1}{2} \frac{\partial_{\eta \eta} v_A}{v_A} \eta^2 \|\Sigma_{\eta}\|^2$$

$$\mu_{v_E} = \frac{\partial_{\eta} v_E}{v_E} \mu_{\eta} \eta + \frac{1}{2} \frac{\partial_{\eta \eta} v_E}{v_E} \eta^2 \|\Sigma_{\eta}\|^2.$$

Equating these two expressions for the drifts of v_A and v_E , respectively, yields two second-order differential equations for v_A and v_E .

This formulation of the HJB is particularly convenient since we have the equalities

$$\sigma_{v_A A} = \sigma_{\xi A} + \sigma_{\eta A} + \sigma_{qA},$$

$$\sigma_{v_A E} = \sigma_{\xi E} + \sigma_{\eta E} + \sigma_{qE},$$

hence we may write

$$\sigma_{\xi A} = \sigma_{v_A A} - (\sigma_{\eta A} + \sigma_{qA}),$$

$$\sigma_{\xi E} = \sigma_{v_A E} - (\sigma_{\eta E} + \sigma_{qE}).$$

For v_E , observe that by Ito's lemma,

$$\frac{d(1-\eta)}{1-\eta} = -\frac{1}{1-\eta}d\eta = -\frac{\eta}{1-\eta}\mu_\eta dt - \frac{\eta}{1-\eta}\Sigma_\eta \cdot d\mathbf{W}_t,$$

hence

$$\sigma_{\zeta A} = \sigma_{v_E A} + \frac{\eta}{1-\eta}\sigma_{\eta A} - \sigma_{qA},$$

$$\sigma_{\zeta E} = \sigma_{v_E E} + \frac{\eta}{1-\eta}\sigma_{\eta E} - \sigma_{qE}.$$

ODE System. The first-order ODE pinning down q is equation (27), which is an implicit first-order differential equation in q , ξ , and ζ . Note that we do not have to actually compute the derivatives of ξ and ζ . Instead, we may use the identities relating the volatilities of v_A and v_B with

ξ and ζ , respectively. All together, we have the following system of ODEs:

$$\frac{\bar{a}(P_a - P_b)}{q} + \sigma \sigma_{qA} - \sigma \sigma_{qE} = \gamma_A \varsigma_{Aa} + (\gamma_A - 1) \varsigma_{\xi A} - \gamma_E \varsigma_{Eb} - (\gamma_E - 1) \varsigma_{\zeta E} \quad (29)$$

$$\begin{aligned} \frac{\partial_{\eta} v_A}{v_A} \mu_{\eta} \eta + \frac{1}{2} \frac{\partial_{\eta} \eta v_A}{v_A} \eta^2 \|\Sigma_{\eta}\|^2 &= \frac{\gamma_A}{2} [(\psi_A \sigma_A + \sigma_{v_A A})^2 + (\psi_E \sigma_E + \sigma_{v_A E})^2] \\ &+ \rho(\log(v_A) - \log(\rho q \eta)) - (\Phi(\iota) - \delta) \\ &- \psi_A \sigma_A \sigma_{v_A A} - \psi_E \sigma_E \sigma_{v_A E} \end{aligned} \quad (30)$$

$$\begin{aligned} \frac{\partial_{\eta} v_E}{v_E} \mu_{\eta} \eta + \frac{1}{2} \frac{\partial_{\eta} \eta v_E}{v_E} \eta^2 \|\Sigma_{\eta}\|^2 &= \frac{\gamma_E}{2} [(\psi_A \sigma_A + \sigma_{v_E A})^2 + (\psi_E \sigma_E + \sigma_{v_E E})^2] \\ &+ \rho(\log(v_E) - \log(\rho q(1 - \eta))) - (\Phi(\iota) - \delta) \\ &- \psi_A \sigma_A \sigma_{v_E A} - \psi_E \sigma_E \sigma_{v_E E}. \end{aligned} \quad (31)$$

B.6 Consumption-Equivalent Welfare Gains and Losses

We suppress time subscripts here. Because we know the form of agents' value functions, we can sharply quantify a country's consumption-equivalent welfare gains or losses in different equilibria. Let a superscript of (0) denote objects from some initial equilibrium and a superscript of (1) denote objects from an alternative equilibrium. Define ϕ_g as the percentage gain in consumption-equivalent welfare. Consider the AE. With unit intertemporal elasticity of substitution, $\rho n_A = c_A$, so a percentage increase in the AE's consumption stream is equivalent to a percentage increase in the AE's net worth. Holding fixed the AE's marginal value of wealth in the initial equilibrium, we have that

$$V_A^{(1)} = \frac{(\xi^{(0)}(1 + \phi_g)c_A^{(0)}/\rho)^{1-\gamma_A}}{1-\gamma_A} = (1 + \phi_g)^{1-\gamma_A} \frac{(\xi^{(0)}c_A^{(0)}/\rho)^{1-\gamma_A}}{1-\gamma_A} = (1 + \phi_g)^{1-\gamma_A} V_A^{(0)},$$

or equivalently,

$$\phi_g = \left(\frac{V_A^{(1)}}{V_A^{(0)}} \right)^{1/(1-\gamma_A)} - 1.$$

For the consumption-equivalent welfare loss, the percentage loss ϕ_c is given by

$$\phi_c = 1 - \left(\frac{V_A^{(1)}}{V_A^{(0)}} \right)^{1/(1-\gamma_A)}.$$

C Benchmark Cases

Here we analyze in more detail the benchmark cases we use to discern the mechanisms underlying our results.

C.1 Endogenous Risk

Brunnermeier and Sannikov (2014b) highlight the role of endogenous risk from Σ_q in generating systemic risk. Because Σ_η increases with the level of asset price volatility, endogenous risk affects the long-run stability of the economy. To get a better idea of the role endogenous risk plays in our model, we shut off the terms of trade hedge by assuming there are no intermediate goods. Instead, countries directly produce the final consumption good with a linear production technology as in Brunnermeier and Sannikov (2014b). We assume the AE is more productive with capital than the EME because if both countries are equally productive with capital, then q is constant in equilibrium. Let \bar{a} be the AE's productivity and \underline{a} be the EME's productivity, with $\bar{a} > \underline{a}$.

The model becomes similar to the model in Brunnermeier and Sannikov (2014b).²¹ The economy is solved in two regions. In the first region, the more productive country does not hold all capital. Countries choose their capital holdings so that holds

$$\bar{a} - \underline{a} = \gamma_A \varsigma_{Aa} - (\gamma_A - 1) \varsigma_{\xi A} - \gamma_E \varsigma_{Eb} - (\gamma_E - 1) \varsigma_{\zeta E}$$

in equilibrium. In the second region, $\psi_A = 1$. By market clearing for consumption, the price of

²¹The primary difference is that we obtain a nondegenerate stationary distribution by implementing redistributive taxes. The AE pays a fraction ν of their net worth in taxes to the EME, while the EME pays a fraction τ of their net worth in taxes to the AE. The former guarantees the AE does not accumulate all wealth in the world economy, while the latter guarantees the AE's net worth does not get stuck near zero due to the first tax.

capital is²²

$$q = \frac{\bar{a}\psi_A + \underline{a}(1 - \psi_A) + 1/\kappa}{\rho + \lambda/\kappa}.$$

Figure 9 illustrates equilibrium in this special case. Because the countries are no longer symmetric in productivity, the drift and volatility of η do not exhibit the same symmetric shape as the original model. Because countries directly produce the final consumption good rather than intermediate goods, there is no terms of trade hedge present. Despite this, leverage constraints still have a stabilizing effect. Moreover, their impact on equilibrium also resembles the original model's. Figure 9(a) shows that the drift of η again starts to peak sooner and at a lower maximum as leverage constraints tighten. Similarly, Figure 9(b) indicates that lower leverage constraints decrease the volatility of η . For constraints $L_A = 1$ and $L_A = 0.5$, these effects cause the stationary distribution to shift toward the right. Therefore, the terms of trade hedge does not entirely explain the improvement in long-run financial stability.

Recall that μ_η also depends on leverage choices, the volatility of asset prices, and the volatility of ξ , which is the marginal value of the AE's net worth (see the drift in (16)). Panels (a) and (b) in Figure 10 show that leverage constraints dampen the average volatilities of asset prices and ξ but shift the peak volatility away from $\eta = 0$. By altering equilibrium volatility, the AE indirectly affects the returns earned from risk taking and causes the behavior in the drift of η shown in Figure 9(a). Panels (c) and (d) in Figure 10 confirm that the same changes in volatilities occur in the original model with two intermediate goods. Even without a terms of trade hedge, leverage constraints can still stabilize the stationary distribution by influencing equilibrium volatility.

²² When numerically solving for equilibrium, it is more convenient to write ψ_A as a function of q :

$$\psi_A = \frac{(\rho\kappa + \lambda)q - 1 - \kappa\underline{a}}{\kappa(\bar{a} - \underline{a})}.$$

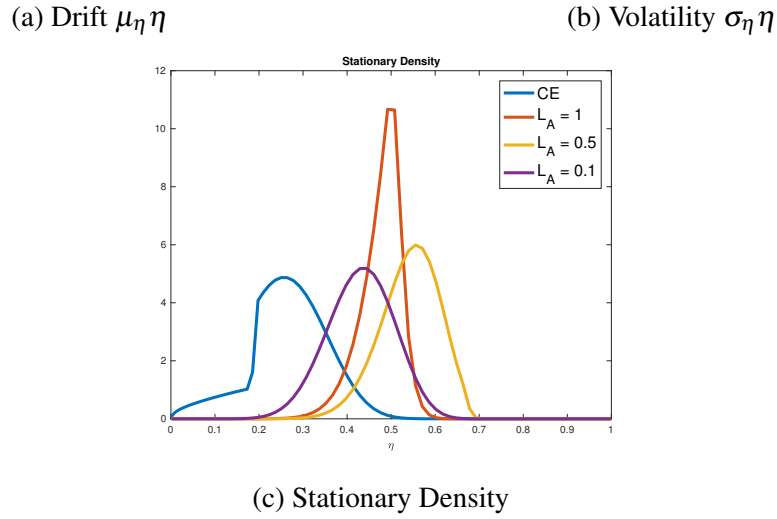
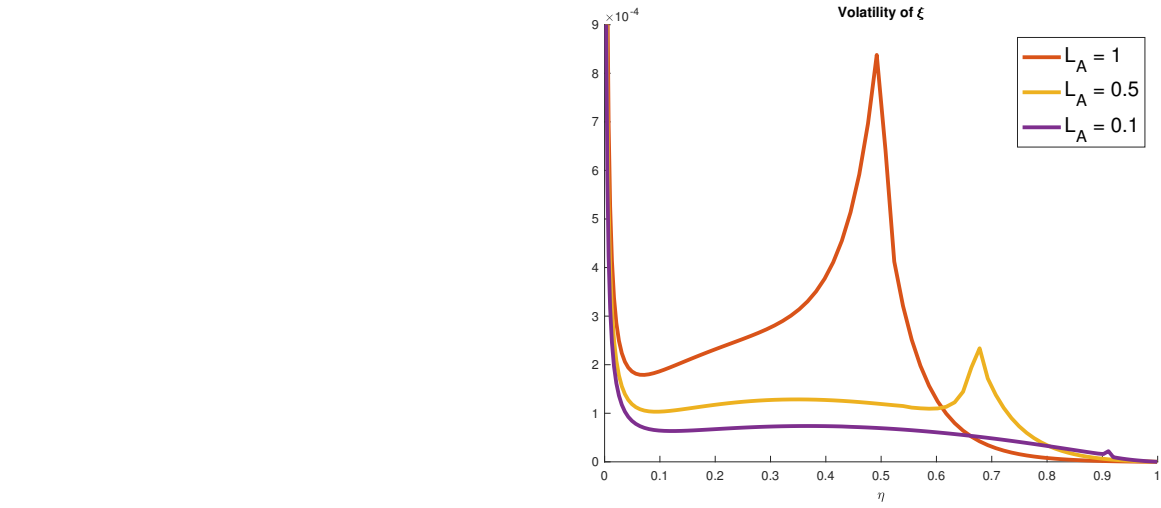
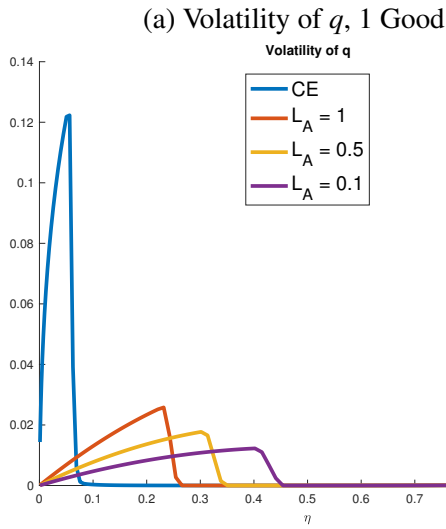


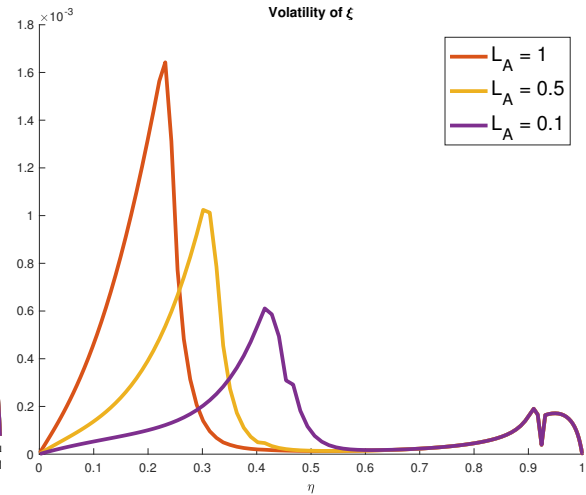
Figure 9: Panels (a) - (c) illustrate equilibrium with a linear investment technology while varying leverage constraints. Panel (a) plots the terms of trade, Panel (b) plots the drift of η , Panel (c) plots the volatility of η , and Panel (d) plots the stationary distributions. In all cases, the EME does not use a leverage constraint. The competitive equilibrium is in blue, $L_A = 1$ is in red, $L_A = 0.5$ is in yellow, and $L_A = 0.1$ is in purple.



(b) Volatility of ξ , 1 Good



(c) Volatility of q , 2 Goods



(d) Volatility of ξ , 2 Goods

Figure 10: Panels (a) and (b) plot the volatilities of q and ξ , respectively, when there is only one good produced by capital. Panels (c) and (d) plot the same volatilities but from the original model. In all cases, the EME does not use a leverage constraint. The competitive equilibrium is in blue, $L_A = 1$ is in red, $L_A = 0.5$ is in yellow, and $L_A = 0.1$ is in purple.

C.2 Terms of Trade Hedge

As a counterpart to the previous section's analysis, we now focus on the terms of trade hedge and shut off amplification through asset prices by assuming $\Phi(\iota)$ is a linear investment technology. Without asset price amplification, endogenous volatility only emerges from countries' choice of leverage. For simplicity, assume $\Phi(\iota) = \lambda \iota$. Tobin's q implies

$$\Phi'(\iota) = \lambda = \frac{1}{q} \Rightarrow q = \frac{1}{\lambda},$$

hence the price of capital is constant, so asset prices are not volatile.

Figure 11 demonstrates that equilibrium in this special case closely resembles our original model. Tighter leverage constraints still cause the terms of trade hedge to apply sooner when ψ_A deviates from its optimal value, as Panel (a) shows. Panel (b) indicates that this effect on the terms of trade is associated with the drift of η peaking faster but at a smaller maximum, just like in the original model. Because leverage constraints restrict the leverage ratio ψ_A/η , the volatility of η also decreases. These changes in the drift and volatility of η cause the stationary distribution to shift its mass to the right. Although we have shut off asset price amplification, the same phenomena that characterized equilibrium in our original model still appear.

Although the previous and current sections indicate that leverage constraints promote financial stability through both endogenous risk and the terms of trade hedge, the latter mechanism is the stronger factor. In the one good model, tighter leverage constraints do not always generate a stabler stationary distribution. Figure 9(c) shows that the stationary distribution when $L_A = 0.1$ is substantially left of the stationary distribution when $L_A = 0.5$. Tightening $L_A = 0.5$ to $L_A = 0.1$ actually decreases long-run stability. In contrast, Figure 11(d) suggests that tighter constraints always shift the distribution to the right like in the original model. This difference points to the terms of trade hedge as the more important mechanism for why leverage constraints improve outcomes for the AE.

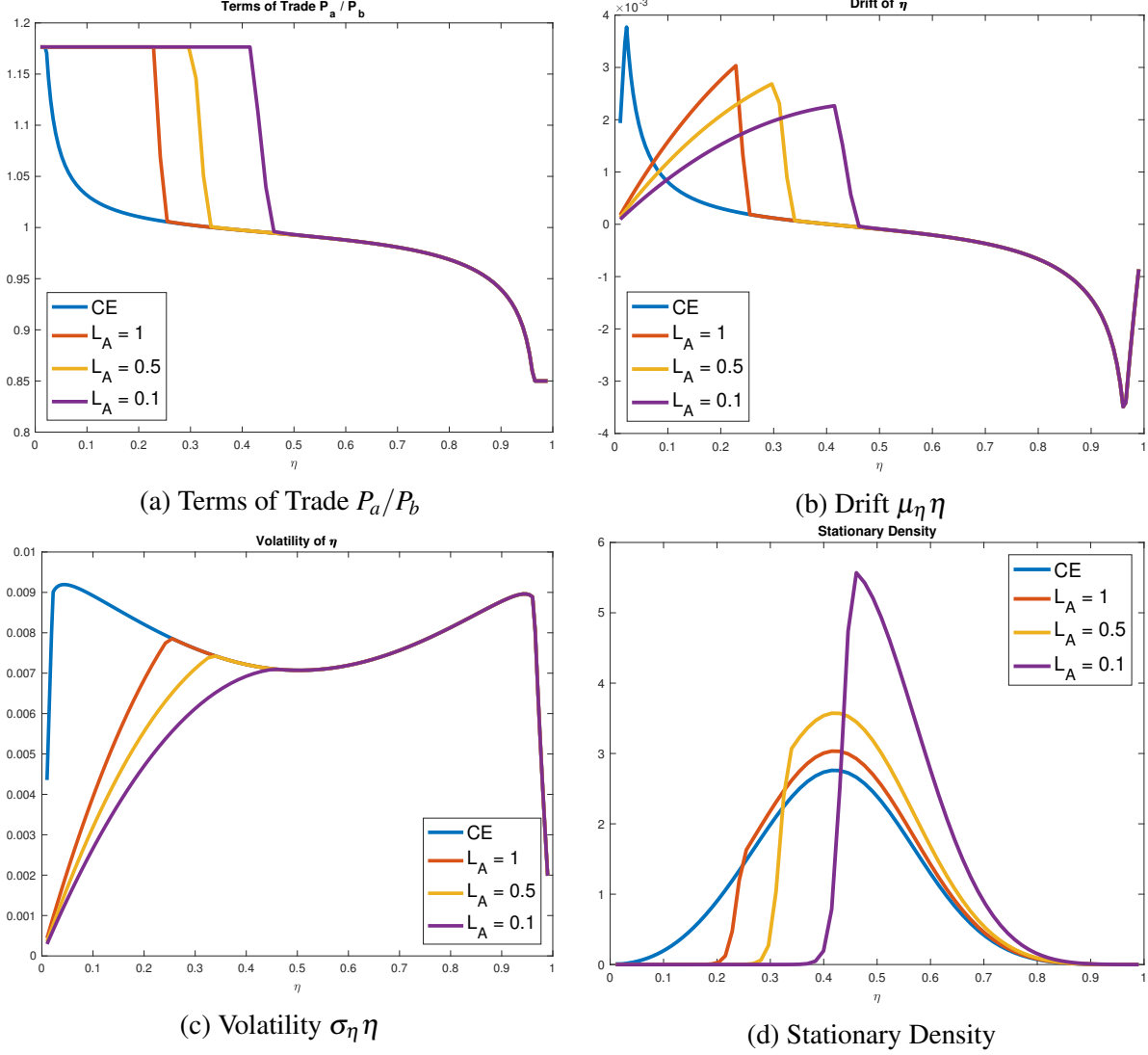


Figure 11: Panels (a) - (d) illustrate equilibrium with a linear investment technology while varying leverage constraints. Panel (a) plots the terms of trade, Panel (b) plots the drift of η , Panel (c) plots the volatility of η , and Panel (d) plots the stationary distributions. In all cases, the EME does not use a leverage constraint. The competitive equilibrium is in blue, $L_A = 1$ is in red, $L_A = 0.5$ is in yellow, and $L_A = 0.1$ is in purple.

C.3 Counterfactual Laws of Motion for η .

Here, we examine the relative contributions of equilibrium risk pricing and the terms of trade hedge from another perspective. Figure 12 presents the drifts of η and stationary distributions in “counterfactual” scenarios. The solid blue and red lines in Figure 12(a) and (b) are μ_η from

the competitive equilibrium and the $L_A = 1$ equilibrium. The yellow dashed line in Panel (a) is a “counterfactual” μ_η from the competitive equilibrium where we replace any prices of intermediate goods in the expression for μ_η in (16) by the prices from the $L_A = 1$ equilibrium. For example, in the counterfactual drift, the first term $\bar{a}P_{a,t}/q_t$ is replaced by $\bar{a}P_{a,t}^{(1)}/q_t$, where the (1) superscript indicates that this $P_{a,t}^{(1)}$ is the price of good a in the $L_A = 1$ equilibrium. Similarly, the yellow dashed line in Panel (b) is μ_η from the $L_A = 1$ equilibrium but with any prices of intermediate goods in (16) and (17) replaced by the prices from the competitive equilibrium.

These counterfactual drifts of η demonstrate how the terms of trade hedge affects the rate of recovery to the stochastic steady state. In Panel (a), the change in the terms of trade caused by the AE’s leverage constraint substantially increases μ_η when $\eta \in [0.1, 0.2]$. However, the better terms of trade for the AE does not explain all of the change in the drift of η . The difference between the solid red line and dashed yellow line results from changes in equilibrium risk pricing caused by leverage constraints. Accounting for those changes approximately doubles μ_η for $\eta \in [0.1, 0.2]$. Panel (b) reinforces this point. If one replaces the intermediate good price terms in μ_η from the $L_A = 1$ equilibrium with prices from the competitive equilibrium, μ_η drops sharply but not to zero.

Panel (c) shows how these counterfactual drifts affect long-run stability. The solid blue and red lines are the stationary distributions from the competitive equilibrium and $L_A = 1$ with the solid blue and red lines, respectively. The stationary distribution in the dashed yellow line results from using the counterfactual drift in Panel (a) and the volatility of η in the competitive equilibrium. By just using the terms of trade from the $L_A = 1$ equilibrium, the implied stationary distribution (dashed yellow) is almost the same as the $L_A = 1$ distribution (solid red). In contrast, the stationary distribution in the dashed purple line results from using the counterfactual drift in Panel (b) and the volatility of η in the $L_A = 1$ equilibrium. While the drift generating the dashed purple distribution has the same volatility as the $L_A = 1$ equilibrium, the distribution is only slightly more stable than the competitive equilibrium’s, with a little less mass near $\eta = 0$ and a little more mass for $\eta \in [0.1, 0.2]$. The difference between these two distributions shows that the terms of trade hedge is the more important mechanism through which leverage constraints provide financial stability. Changing the terms of trade produces almost as much stability as the full $L_A = 1$ equilibrium,

but changing the volatility of equilibrium objects barely improves stability. The reason is that the terms of trade hedge make the stochastic steady state more stable. Given an adverse shock, tighter leverage constraints cause the terms of trade hedge to apply much closer to the optimal allocation of capital. This encourages a quicker return to “good times.”

As further evidence, Panel (d) constructs two more counterfactual stationary distributions. To generate the dashed yellow distribution, we pair the drift of η from the competitive equilibrium with the volatility of η from the $L_A = 1$ equilibrium, and to generate the dashed purple distribution, we pair the drift of η from the $L_A = 1$ equilibrium with the volatility of η from the competitive equilibrium. The difference between the dashed yellow and purple lines demonstrates that the stabilizing force of leverage constraints operates through their effect on the terms of trade hedge. When we use the drift of η from the competitive equilibrium, where the terms of trade hedge is relatively weak, the AE’s long-run financial stability becomes worse. In contrast, when we only use the drift from the constrained equilibrium, the consequent stationary distribution is almost the same as the true constrained equilibrium.

D Numerical Method

Here, we provide a description of the algorithm used to compute dynamics and welfare.

D.1 Equations

Rather than solve the ODEs implied by the drifts of v_A and v_E directly, we will utilize the false transient method and convert the system of ODEs into a system of PDEs with one time derivative and one “spatial” derivative. A natural way to add false time derivatives is the time-dependent version of Ito’s lemma, which yields the new drifts

$$\begin{aligned}\mu_{v_A} &= \frac{\partial_t v_A}{v_A} + \frac{\partial_\eta v_A}{v_A} \mu_\eta \eta + \frac{1}{2} \frac{\partial_{\eta\eta} v_A}{v_A} \eta^2 \|\Sigma_\eta\|^2 \\ \mu_{v_E} &= \frac{\partial_t v_E}{v_E} + \frac{\partial_\eta v_E}{v_E} \mu_\eta \eta + \frac{1}{2} \frac{\partial_{\eta\eta} v_E}{v_E} \eta^2 \|\Sigma_\eta\|^2.\end{aligned}$$

The equilibrium PDEs then are given by

$$\begin{aligned}
\frac{\bar{a}(P_a - P_b)}{q} + \sigma \sigma_{qA} - \sigma \sigma_{qE} &= \gamma_A \varsigma_{Aa} + (\gamma_A - 1) \varsigma_{\xi A} - \gamma_E \varsigma_{Eb} - (\gamma_E - 1) \varsigma_{\zeta E} \\
\frac{\partial_t v_A}{v_A} + \frac{\partial_\eta v_A}{v_A} \mu_\eta \eta + \frac{1}{2} \frac{\partial_{\eta\eta} v_A}{v_A} \eta^2 \|\Sigma_\eta\|^2 &= \frac{\gamma_A}{2} [(\psi_A \sigma_A + \sigma_{v_A A})^2 + (\psi_E \sigma_E + \sigma_{v_A E})^2] \\
&\quad + \rho(\log(v_A) - \log(\rho q \eta)) - (\Phi(\iota) - \delta) \\
&\quad - \psi_A \sigma_A \sigma_{v_A A} - \psi_E \sigma_E \sigma_{v_A E} \\
\frac{\partial_t v_E}{v_E} + \frac{\partial_\eta v_E}{v_E} \mu_\eta \eta + \frac{1}{2} \frac{\partial_{\eta\eta} v_E}{v_E} \eta^2 \|\Sigma_\eta\|^2 &= \frac{\gamma_E}{2} [(\psi_A \sigma_A + \sigma_{v_E A})^2 + (\psi_E \sigma_E + \sigma_{v_E E})^2] \\
&\quad + \rho(\log(v_E) - \log(\rho q(1 - \eta))) - (\Phi(\iota) - \delta) \\
&\quad - \psi_A \sigma_A \sigma_{v_E A} - \psi_E \sigma_E \sigma_{v_E E}.
\end{aligned}$$

We pin down capital allocations using the conditions

$$\begin{aligned}
q &= \frac{\kappa(\bar{a}\psi_{Aa} + \underline{a}\psi_{Ea})^\alpha (\underline{a}\psi_{Ab} + \bar{a}\psi_{Eb})^{1-\alpha} + 1}{\rho\kappa + \lambda} \\
1 &= \psi_{Aa} + \psi_{Ab} + \psi_{Ea} + \psi_{Eb} \\
\frac{\underline{a}}{\bar{a}} &\leq \frac{P_a}{P_b} \leq \frac{\bar{a}}{\underline{a}} \\
\psi_A &\leq (1 + L_A)\eta, \quad \psi_E \leq (1 + L_E)(1 - \eta),
\end{aligned}$$

where L_i is presumed infinite if Country i does not impose leverage constraints.

The volatilities for a generic endogenous object f take the form

$$\begin{aligned}
\sigma_{fA} &= \frac{\partial_\eta f}{f} \sigma_{\eta A} \eta \\
\sigma_{fE} &= \frac{\partial_\eta f}{f} \sigma_{\eta E} \eta.
\end{aligned}$$

The endogenous objects whose volatilities are required to compute equilibrium are q , v_A , and

v_E . We also compute the volatilities for ξ and ζ using the identities

$$\begin{aligned}\sigma_{\xi A} &= \sigma_{v_A A} - \sigma_{\eta A} - \sigma_{q A} \\ \sigma_{\xi E} &= \sigma_{v_A E} - \sigma_{\eta E} - \sigma_{q E} \\ \sigma_{\zeta A} &= \sigma_{v_E A} + \frac{\eta}{1 - \eta} \sigma_{\eta A} - \sigma_{q A} \\ \sigma_{\zeta E} &= \sigma_{v_E E} + \frac{\eta}{1 - \eta} \sigma_{\eta E} - \sigma_{q E}.\end{aligned}$$

D.2 Algorithms

D.2.1 Solving Equilibrium

We solve for q , v_A , and v_E using the method of false transients. We adopt the solution procedure outlined in Brunnermeier and Sannikov (2016b) and split the solution method into a static step and a time step. There are a variety of ways to reduce the dimensionality of the state space to a finite one. We choose finite differences, although other methods like Chebyshev collocation would be viable as well. In our description of the algorithm, assume all derivatives of q , v_A , and v_E with respect to η have been approximated using finite differences.²³ To start the algorithm, the user needs to specify initial guesses for v_A and v_E . We initially use similar guesses to those proposed by Brunnermeier and Sannikov (2016b) to solve at least one equilibrium. Once we have solved some equilibria, we save those value functions and use them as initial guesses for other equilibria in our numerical exercises. The static step goes first, followed by the time step. Repeat until convergence is achieved for v_A and v_E .²⁴

Static Step. Assume we know v_A and v_E .

1. Use the stationary economies when $\eta = 0$ and $\eta = 1$ to provide boundary conditions for q at $\eta = 0$ and $\eta = 1$.
2. Convert the conditions pinning down capital allocations and the first-order differential equa-

²³We apply an upwind difference scheme to v_A and v_E since the state space has only one non-time variable.

²⁴We choose a tolerance of 10^{-6} for the maximum pointwise change in v_A and v_E .

tion (by approximating q' with finite differences) into functions of q .

3. Apply a nonlinear equation solver to compute q .
4. Back out capital allocations, goods prices, μ_η , Σ_η , volatilities of equilibrium objects, the growth rate of capital, and other quantities of interest (e.g. output levels).

Time Step. Assume we are given q , capital allocations, the capital growth rate, μ_η , Σ_η , and the volatilities of v_A and v_E . Then take a small time step back from t to $t - \Delta t$ and solve the PDE backwards in time.²⁵

D.2.2 Solving for the Coordinated and Uncoordinated Equilibrium

We arrive at most of our results by calculating ex-ante welfare over a fine grid of many pairs of leverage constraints and interpolating to determine optimal and Nash policies. One of the nodes in this grid is always $(L_A, L_E) = (0, 0)$. In addition, we use an iterative procedure described below to verify that Nash is $(0, 0)$. The algorithm is essentially a fixed point iteration, and convergence occurs when both countries are playing best responses.

1. Initialize L_A .²⁶
2. Do until convergence to a Nash equilibrium
 - (i) Fixing L_A , compute ex-ante welfare using a grid of leverage constraints for the EME. Interpolate and select the optimal L_E .
 - (ii) Fixing L_E , compute ex-ante welfare using a grid of leverage constraints for the AE. Interpolate and select the optimal L_A .

²⁵Since the derivatives w.r.t. η have been approximated with finite differences, solving the PDE reduces to solving an ordinary differential equation. The size of the time step may also be iteratively updated.

²⁶To check our results do not depend on the initial L_A , we initialize many L_A .

E Robustness

Here, we check the robustness of our results. We show that zero leverage constraints still comprise a Nash equilibrium in the case of symmetric risk aversion coefficients. We also check that our main results are not sensitive to our choices of risk aversion coefficients nor the gains from specialization in trade. For all the parameters we consider, we still obtain a Nash equilibrium in which both countries impose zero leverage constraints, and the AE gains at the expense of the EME, relative to the competitive equilibrium. Below, we report consumption equivalent welfare changes from the competitive equilibrium to Nash for each country.

E.1 Symmetric Economy

We set $\gamma_A = \gamma_E = 1.5$ so that our coefficients are equally between 1 and 2, but this choice is not essential. We still obtain a Nash equilibrium of $(L_A, L_E) = (0, 0)$. However, because there is no asymmetry in risk aversion coefficients, Nash does not harm one country at the expense of the other. Relative to the competitive equilibrium, Nash policies produce welfare gains of 7.01% in consumption equivalents for both countries. Optimal policies are slightly positive at $L_A = L_E \approx 90\text{bp}$, but the gains relative to Nash are small (on the order of a couple basis points). In a symmetric world, the strategic incentives align very closely with the socially optimal ones because neither country demands the other's intermediation relatively more. It is precisely an asymmetry in risk tolerance that lies at the crux of our analysis.

E.2 Risk Aversion

Table 3 reports the consumption equivalent change from the competitive equilibrium to Nash equilibrium for each country while varying our choices for risk aversion coefficients. The more asymmetric risk aversion is, the larger the increase in welfare that the AE obtains in Nash and the larger the decrease in welfare that the EME experiences relative to the competitive equilibrium. This result reinforces our point that the EME wants the AE to bear sources of long-run financial instability because the AE has relatively higher risk tolerance. A larger asymmetry in risk aversion

Risk Aversion	Welfare Change for AE	Welfare Change for E	Debt/Output Ratio
$\gamma_A = 1.4, \gamma_E = 1.6$	11.0%	3.31%	-.271
$\gamma_A = 1.25, \gamma_E = 1.75$	16.9%	-1.38%	-1.82
$\gamma_E = 1.9$	24.8%	-6.62%	-3.37
Original	26.8%	-7.62%	-3.38
$\gamma_A = 1.01$	28.2%	-8.34%	-3.89
$\gamma_E = 2.1$	28.6%	-8.53%	-3.89
$\gamma_E = 3$	43.2%	-14.6%	-5.43

Table 3: This table displays the consumption equivalent welfare change from the competitive equilibrium to Nash while varying the degree of asymmetry in risk aversion. We report the original calibration (for comparison) and the risk aversion coefficient(s) we change from our original calibration. For example, the third row has only $\gamma_E = 1.9$, so we maintain $\gamma_A = 1.05$ for that row's calculations. The different cases are reported in order of the absolute difference in risk aversion.

leads to greater demand by the EME for intermediation by the AE. When the AE denies that intermediation, it produces larger gains for itself while worsening the welfare loss suffered by the EME.

However, if the asymmetry in risk aversion is sufficiently small, then Nash equilibrium improves outcomes for both countries. This occurs in the case of $(\gamma_A, \gamma_E) = (1.4, 1.6)$. The intuition is that when the EME's risk tolerance is close enough to the AE's, then the EME's demand for intermediation by the AE is relatively low, and the benefits from greater long-run financial stability are relatively high. Although both countries obtain better outcomes, the welfare gains are still asymmetric. the AE's welfare gain is greater than the gain in the symmetric economy while the EME's gain is lower. This difference emerges due to the mechanism highlighted by our original calibration. With an asymmetry in risk aversion, the country with lower risk aversion ends up bearing the burden of long-run instability relatively more, so zero leverage constraints are more beneficial for that country.

The final column of Table 3 reports the debt to domestic output ratio in the stochastic steady state. This column gives some sense about the degree of imbalances in the stochastic steady state

\underline{a}	Welfare Change for AE	Welfare Change for E
0.75	26.7%	-7.62%
0.85	26.8%	-7.62%
0.95	27.9%	-7.63%

Table 4: This table displays the consumption equivalent welfare change from the competitive equilibrium to Nash while varying the degree of productivity gains from specialization.

as the asymmetry in risk aversion widens. The wider the asymmetry, the more leveraged the AE becomes in the steady state. This occurs for two reasons. First, the AE becomes more leveraged in general since it has a higher risk tolerance. Second, the stochastic steady state shifts toward zero as the AE because the AE is able to lever up and achieve nearly optimal capital allocations much quicker.

E.3 Gains from Specialization

Table 4 reports the consumption equivalent change from the competitive equilibrium to Nash equilibrium for each country. Generally, the smaller the gains from specialization (i.e. the greater \underline{a} is), the larger the increase in welfare that the AE obtains in Nash and the larger the decrease in welfare that the EME experiences relative to the competitive equilibrium. The size of the changes, however, are not that different across the different parameter choices.

F Extension: Location and Ownership Shocks

Here, we show that our baseline model generates similar risk premia to a model that includes shocks dependent on the country location and country of ownership. In Section F.5, we show that with log flow utility, there exists a parametrization of the extended model such that the risk premia required by AE agents are the same as in the original model.

F.1 Set Up

For brevity, we do not mention details that do not change relative to the baseline model, such as preferences.

Technology. Without loss of generality, we assume the aggregate stock of capital at time $t = 0$ is equally distributed between the AE and the EME. Capital produced from internal investment in a given country remains located in that country. Since the rate of internal investment is determined by Tobin's q , every agent chooses the same rate, so capital grows at the same rate in the aggregate in either country. As a result, capital remains equally distributed between the AE and the EME for all time.

Define $\psi_{ijk,t}$, $\psi_{ijk,t}$, $\psi_{ijk,t}$, and $\psi_{ijk,t}$ to be the share of capital *owned* by Country $i = A, E$ used to produce good $j = a, b$ with capital *located* in Country $k = A, E$. market clearing for capital implies

$$\sum_{i=A,E} \sum_{j=a,b} \sum_{k=A,E} \psi_{ijk,t} = 1. \quad (32)$$

We additionally define

$$\begin{aligned} \psi_{i,t} &= \sum_{j=a,b} \sum_{k=A,E} \psi_{ijk,t} \\ \psi_{ik,t} &= \sum_{j=a,b} \psi_{ijk,t} \end{aligned}$$

For a single agent located in Country i using capital in Country k , the capital that they own evolves according to

$$\frac{dk_t}{k_t} = (\Phi(\iota_t) - \delta) dt + \sigma_{iO} dW_{iO,t} + \sigma_{kL} dW_{kL,t}. \quad (33)$$

In the interest of generality, we also let σ_{iO} depend on the user's country of origin. In total, there are now four Brownian motions: $W_{AO,t}$, $W_{EO,t}$, $W_{AL,t}$ and $W_{EL,t}$. We assume these Brownian motions are mutually independent. The first two shocks are shocks to capital *owned* by the AE and the EME, respectively. The latter two shocks are shocks to capital *located* in the AE and the EME,

respectively. Aggregating together, the stock of world capital K_t evolves according to

$$\begin{aligned} \frac{dK_t}{K_t} = & (\Phi(\iota_t) - \delta) dt + \psi_{A,t} \sigma_{AO} dW_{AO,t} + \psi_{E,t} \sigma_E dW_{EO,t} \\ & + (\psi_{AA,t} + \psi_{EA,t}) \sigma_{AL} dW_{AL,t} + (\psi_{AE,t} + \psi_{EE,t}) \sigma_{EL} dW_{AE,t}. \end{aligned} \quad (34)$$

We conjecture that the price of capital follows

$$\frac{dq_t}{q_t} = \mu_{q,t} dt + \sigma_{qAO,t} dW_{AO,t} + \sigma_{qBO,t} dW_{BO,t} + \sigma_{qAL,t} dW_{AL,t} + \sigma_{qEL,t} dW_{EL,t}. \quad (35)$$

Returns from Capital. The returns for the AE investing in good a using capital located in the AE and capital located in the EME are given by, respectively,

$$\begin{aligned} dr_t^{AaA} = & \left(\frac{\bar{a}P_{a,t}}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma_{AO} \sigma_{qAO,t} + \sigma_{AL} \sigma_{qAL,t} \right) dt \\ & + (\sigma_{AO} + \sigma_{qAO,t}) dW_{AO,t} + \sigma_{qEO,t} dW_{EO,t} + (\sigma_{AL} + \sigma_{qAL,t}) dW_{AL,t}. \end{aligned} \quad (36)$$

$$\begin{aligned} dr_t^{AaE} = & \left(\frac{\bar{a}P_{a,t}}{q_t} + \mu_{q,t} + \Phi(\iota_t) - \delta + \sigma_{AO} \sigma_{qAO,t} + \sigma_{EL} \sigma_{qEL,t} \right) dt \\ & + (\sigma_{AO} + \sigma_{qAO,t}) dW_{AO,t} + \sigma_{qEO,t} dW_{EO,t} + (\sigma_{AE} + \sigma_{qAE,t}) dW_{AE,t}. \end{aligned} \quad (37)$$

The AE can also invest in good a using capital located in the EME as well as in good b using capital located in the AE or the EME. Similar returns equations hold for agents in the EME. It will be convenient to let $\mathbf{W}_t = [W_{AO,t}, W_{EO,t}, W_{AL,t}, W_{EL,t}]^T$ stack all exogenous shocks and let $\Sigma_{ijk,t}$ denote the volatility vector of dr_t^{ijk} so that $\Sigma_{ijk,t} \cdot d\mathbf{W}_t$ gives the volatility of dr_t^{ijk} .

Portfolio Choice. The portfolio choice problem is symmetric for agents in either country, so we only characterize the problem of AE agents. An agent in the AE chooses a consumption rate $c_{A,t}$ and portfolio weights $(x_{AaA,t}, x_{AaE,t}, x_{AbA,t}, x_{AbE,t}, 1 - \sum_{j=a,b} \sum_{k=A,E} x_{Ajk,t})$. It will be convenient to let $x_{A,t} = \sum_{j=a,b} \sum_{k=A,E} x_{Ajk,t}$ and $x_{Ak,t} = \sum_{j=a,b} x_{Ajk,t}$ for $k = A, E$. We may then write the evolution

of net worth for AE agents as

$$\frac{dn_{A,t}}{n_{A,t}} = (1 - x_{A,t}) dr_t^F - \frac{c_{A,t}}{n_{A,t}} dt + \sum_{j=a,b} \sum_{k=A,E} x_{Ajk,t} dr_t^{Ajk}. \quad (38)$$

Denote by $\Sigma_{Ajk,t}$ the volatility for AE agents to invest in the production of good $j = a, b$ with capital located in Country $k = A, E$. The risk premium required by AE agents is the covariance of these returns with the stochastic discount factor of agents in the AE. In equilibrium, we obtain the following conditions:

$$\begin{aligned} \mathbb{E}[dr_t^{AaA}] - dr_t^F &= \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaA,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaA,t} \\ \mathbb{E}[dr_t^{AaE}] - dr_t^F &= \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaE,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaE,t} \\ \mathbb{E}[dr_t^{AbA}] - dr_t^F &\leq \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaA,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaA,t} \\ \mathbb{E}[dr_t^{AbE}] - dr_t^F &\leq \gamma_A (x_{AA,t} \Sigma_{AaE,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaE,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaE,t}. \end{aligned}$$

The inequalities are sometimes slack because there are no short sales. The first two equalities must hold since the AE has a superior production technology for good a , and there are diversification benefits to using capital in both countries.

Taking the difference between the first two equalities yields

$$\begin{aligned} \sigma_{AL} \sigma_{qAL,t} - \sigma_{EL} \sigma_{qEL,t} &= \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) (\Sigma_{AaA,t} - \Sigma_{AaE,t}) \\ &\quad + (\gamma_A - 1) \Sigma_{\xi,t} \cdot (\Sigma_{AaA,t} - \Sigma_{AaE,t}). \end{aligned}$$

Focusing on $\Sigma_{AaA,t} - \Sigma_{AaE,t}$, simplification yields the vector

$$[0, 0, \sigma_{AL} + \sigma_{qAL,t}, -\sigma_{EL} - \sigma_{qAE,t}]^T$$

We therefore obtain

$$\begin{aligned} \sigma_{AL}\sigma_{qAL,t} - \sigma_{EL}\sigma_{qEL,t} &= \gamma_A(x_{AA,t}(\sigma_{AL} + \sigma_{qAL,t})^2 - x_{AE,t}(\sigma_{EL} + \sigma_{qEL,t})^2) \\ &\quad + (\gamma_A - 1)(\sigma_{\xi AA,t}(\sigma_{AL} + \sigma_{qAL,t}) - \sigma_{\xi AE,t}(\sigma_{EL} + \sigma_{qEL,t})). \end{aligned} \quad (39)$$

This equation pins down the allocation of the AE's capital located in the AE and in the EME.

F.2 Law of Motion for η

The aggregate law of motion for net worth in the AE is

$$\frac{dN_{A,t}}{N_{A,t}} = dr_t^F - \frac{C_{A,t}}{N_{A,t}} + \sum_{j=a,b} \sum_{k=A,E} (dr_t^{Ajk} - dr_t^F).$$

Notice that the asset-pricing conditions and the volatilities of returns for producing in the AE are the same for both goods a and b . In other words, when AE agents produce both goods a and b , we must have

$$dr_t^{AaA} = dr_t^{AbA}, \quad dr_t^{AaE} = dr_t^{AbE}.$$

We can therefore write $dN_{A,t}$ as

$$\frac{dN_{A,t}}{N_{A,t}} = dr_t^F - \frac{C_{A,t}}{N_{A,t}} + x_{AA,t}(dr_t^{AaA} - dr_t^F) + x_{AE,t}(dr_t^{AaE} - dr_t^F).$$

Multiplying the returns terms by one yields

$$\begin{aligned} \frac{dN_{A,t}}{N_{A,t}} &= dr_t^F - \frac{C_{A,t}}{N_{A,t}} + x_{A,t} \frac{x_{AA,t}}{x_{A,t}} (dr_t^{AaA} - dr_t^F) + x_{A,t} \frac{x_{AE,t}}{x_{A,t}} (dr_t^{AaE} - dr_t^F) \\ &= dr_t^F - \frac{C_{A,t}}{N_{A,t}} + x_{A,t} \left(\frac{x_{AA,t}}{x_{A,t}} (dr_t^{AaA} - dr_t^F) + \frac{x_{AE,t}}{x_{A,t}} (dr_t^{AaE} - dr_t^F) \right). \end{aligned}$$

With the law of motion for $N_{A,t}$ written in this form, we can now show the similarity in risk premia between the present model and the baseline model. The risk premium for holding capital

will take the same form as in the baseline model, which had

$$\mathbb{E}[dr_t^{Aa}] - dr_t^F = \gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}.$$

Define dr_t^A to be the return earned by AE agents per unit of capital held. This quantity is given by

$$dr_t^A = \frac{x_{AA,t}}{x_{A,t}} dr_t^{AaA} + \frac{x_{AE,t}}{x_{A,t}} dr_t^{AaE}. \quad (40)$$

The return per unit of capital is an average of the returns dr_t^{AaA} and dr_t^{AaE} , weighted by the relative shares of capital located in the AE and the EME. Taking expectations of the excess returns and substituting in the asset pricing conditions for AE agents yield

$$\begin{aligned} \mathbb{E}[dr_t^A] - dr_t^F &= \frac{x_{AA,t}}{x_{A,t}} (\mathbb{E}[dr_t^{AaA}] - dr_t^F) + \frac{x_{AE,t}}{x_{A,t}} (\mathbb{E}[dr_t^{AaE}] - dr_t^F) \\ &= \frac{x_{AA,t}}{x_{A,t}} \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaA,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaA,t} \\ &\quad + \frac{x_{AE,t}}{x_{A,t}} \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaE,t} + (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaE,t} \\ &= \frac{x_{AA,t}}{x_{A,t}} \gamma_A x_{A,t} ((\sigma_{AO} + \sigma_{qAO,t})^2 + \sigma_{qEO,t}^2) \\ &\quad + \frac{x_{AA,t}}{x_{A,t}} (\gamma_A - 1) (\sigma_{\xi AO,t} (\sigma_{AO} + \sigma_{qAO,t}) + \sigma_{\xi EO,t} \sigma_{qEO,t}) \\ &\quad + \frac{x_{AA,t}}{x_{A,t}} (\gamma_A x_{AA,t} (\sigma_{AL} + \sigma_{qAL,t})^2 + (\gamma_A - 1) \sigma_{\xi AL,t} (\sigma_{AL} + \sigma_{qAL,t})) \\ &\quad + \frac{x_{AE,t}}{x_{A,t}} \gamma_A x_{A,t} ((\sigma_{AO} + \sigma_{qAO,t})^2 + \sigma_{qEO,t}^2) \\ &\quad + \frac{x_{AE,t}}{x_{A,t}} (\gamma_A - 1) (\sigma_{\xi AO,t} (\sigma_{AO} + \sigma_{qAO,t}) + \sigma_{\xi EO,t} \sigma_{qEO,t}) \\ &\quad + \frac{x_{AE,t}}{x_{A,t}} (\gamma_A x_{AE,t} (\sigma_{EL} + \sigma_{qEL,t})^2 + (\gamma_A - 1) \sigma_{\xi EL,t} (\sigma_{EL} + \sigma_{qEL,t})). \end{aligned}$$

Combining like terms obtains

$$\begin{aligned}
\mathbb{E}[dr_t^A] - dr_t^F &= \gamma_A x_{A,t} ((\sigma_{AO} + \sigma_{qAO,t})^2 + \sigma_{qEO,t}^2) \\
&\quad + (\gamma_A - 1) (\sigma_{\xi AO,t} (\sigma_{AO} + \sigma_{qAO,t}) + \sigma_{\xi EO,t} \sigma_{qEO,t}) \\
&\quad + \frac{x_{AA,t}}{x_{A,t}} (\gamma_A x_{AA,t} (\sigma_{AL} + \sigma_{qAL,t})^2 + (\gamma_A - 1) \sigma_{\xi AL,t} (\sigma_{AL} + \sigma_{qAL,t})) \\
&\quad + \frac{x_{AE,t}}{x_{A,t}} (\gamma_A x_{AE,t} (\sigma_{EL} + \sigma_{qEL,t})^2 + (\gamma_A - 1) \sigma_{\xi EL,t} (\sigma_{EL} + \sigma_{qEL,t})).
\end{aligned}$$

Re-arrange terms to acquire

$$\begin{aligned}
\mathbb{E}[dr_t^A] - dr_t^F &= \gamma_A x_{A,t} \left[(\sigma_{AO} + \sigma_{qAO,t})^2 + \frac{x_{AA,t}^2}{x_{A,t}^2} (\sigma_{AL} + \sigma_{qAL,t})^2 + \frac{x_{AE,t}^2}{x_{A,t}^2} (\sigma_{EL} + \sigma_{qEL,t})^2 + \sigma_{qEO,t}^2 \right] \\
&\quad + (\gamma_A - 1) \sigma_{\xi AO,t} (\sigma_{AO} + \sigma_{qAO,t}) \\
&\quad + (\gamma_A - 1) \left[\frac{x_{AA,t}}{x_{A,t}} \sigma_{\xi AL,t} (\sigma_{AL} + \sigma_{qAL,t}) + \frac{x_{AE,t}}{x_{A,t}} \sigma_{\xi EL,t} (\sigma_{EL} + \sigma_{qEL,t}) + \sigma_{\xi EO,t} \sigma_{qEO,t} \right].
\end{aligned}$$

If we define

$$\zeta_{Aa,t} = x_{A,t} \left[(\sigma_{AO} + \sigma_{qAO,t})^2 + \frac{x_{AA,t}^2}{x_{A,t}^2} (\sigma_{AL} + \sigma_{qAL,t})^2 + \frac{x_{AE,t}^2}{x_{A,t}^2} (\sigma_{EL} + \sigma_{qEL,t})^2 + \sigma_{qEO,t}^2 \right] \quad (41)$$

$$\begin{aligned}
\zeta_{\xi A,t} &= \sigma_{\xi AO,t} (\sigma_{AO} + \sigma_{qAO,t}) + \frac{x_{AA,t}}{x_{A,t}} \sigma_{\xi AL,t} (\sigma_{AL} + \sigma_{qAL,t}) \\
&\quad + \frac{x_{AE,t}}{x_{A,t}} \sigma_{\xi EL,t} (\sigma_{EL} + \sigma_{qEL,t}) + \sigma_{\xi EO,t} \sigma_{qEO,t}, \quad (42)
\end{aligned}$$

then the excess returns per unit of capital satisfies

$$\mathbb{E}[dr_t^A] - dr_t^F = \gamma_A \zeta_{Aa,t} + (\gamma_A - 1) \zeta_{\xi A,t}, \quad (43)$$

as in the original model. The primary difference from the baseline model is that the terms $\zeta_{Aa,t}$ and $\zeta_{\xi A,t}$ are affected by the physical location of capital owned by AE agents and portfolio rebalancing conducted in response to location-specific shocks.

Plugging (43) into the law of motion for $N_{A,t}$ yields

$$\begin{aligned}\frac{dN_{A,t}}{N_{A,t}} &= dr_t^F + \left(x_{A,t}(\gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}) - \frac{C_{A,t}}{N_{A,t}} \right) dt \\ &\quad + x_{A,t}(\sigma_{AO} + \sigma_{qAO,t}) dW_{AO,t} + x_{A,t} \sigma_{qEO,t} dW_{EO,t} \\ &\quad + x_{AA,t}(\sigma_{AL} + \sigma_{qAL,t}) dW_{AL,t} + x_{AE,t}(\sigma_{EL} + \sigma_{qEL,t}) dW_{EL,t}.\end{aligned}$$

Let $\Sigma_{NA,t}$ denote the volatility vector for this process. By Ito's product rule,

$$\frac{d(q_t K_t)}{q_t K_t} = (\mu_{q,t} + \Phi(\iota_t) - \delta + \Sigma_{q,t} \cdot \Sigma_{K,t}) dt + (\Sigma_{q,t} + \Sigma_{K,t}) \cdot d\mathbf{W}_t.$$

Using the AE's asset-pricing conditions, we have

$$\begin{aligned}-\mu_{q,t} - \Phi(\iota_t) + \delta &= \frac{\bar{a}P_{a,t}}{q_t} + \sigma_{AO} \sigma_{qAO,t} + \sigma_{AL} \sigma_{qAL,t} - dr_t^F \\ &\quad - \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaA,t} - (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaA,t} \\ -\mu_{q,t} - \Phi(\iota_t) + \delta &= \frac{\bar{a}P_{a,t}}{q_t} + \sigma_{AO} \sigma_{qAO,t} + \sigma_{EL} \sigma_{qEL,t} - dr_t^F \\ &\quad - \gamma_A (x_{AA,t} \Sigma_{AaA,t} + x_{AE,t} \Sigma_{AaE,t}) \cdot \Sigma_{AaE,t} - (\gamma_A - 1) \Sigma_{\xi,t} \cdot \Sigma_{AaE,t}.\end{aligned}$$

By Ito's lemma, we have

$$\frac{d(1/(q_t K_t))}{1/(q_t K_t)} = (\|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - \mu_{q,t} - \Phi(\iota_t) + \delta - \Sigma_{q,t} \cdot \Sigma_{K,t}) dt - (\Sigma_{q,t} + \Sigma_{K,t}) \cdot d\mathbf{W}_t.$$

By Ito's product rule,

$$\begin{aligned}\frac{d\eta_t}{\eta_t} &= \left(dr_t^F - \mu_{q,t} - \Phi(\iota_t) + \delta + \|\Sigma_{q,t} + \Sigma_{K,t}\|^2 - \Sigma_{q,t} \cdot \Sigma_{K,t} - \frac{C_{A,t}}{N_{A,t}} \right) dt \\ &\quad + (x_{A,t}(\gamma_A \varsigma_{Aa,t} + (\gamma_A - 1) \varsigma_{\xi A,t}) - \Sigma_{NA,t} \cdot (\Sigma_{q,t} + \Sigma_{K,t})) dt \\ &\quad + (\Sigma_{NA,t} - \Sigma_{q,t} - \Sigma_{K,t}) \cdot d\mathbf{W}_t.\end{aligned}$$

Notice that $x_{AA,t} + x_{AE,t} = x_{A,t}$ implies $x_{AA,t}/x_{A,t} + x_{AE,t}/x_{A,t} = 1$. Thus, we have

$$\begin{aligned}
& -\mu_{q,t} - \Phi(\iota_t) + \delta \\
&= \frac{x_{AA,t}}{x_{A,t}}(-\mu_{q,t} - \Phi(\iota_t) + \delta) + \frac{x_{AE,t}}{x_{A,t}}(-\mu_{q,t} - \Phi(\iota_t) + \delta) \\
&= \frac{\bar{a}P_{a,t}}{q_t} - dr_t^F + \sigma_{AO}\sigma_{qAO,t} \\
&\quad - \frac{x_{AA,t}}{x_{A,t}} \left[\gamma_A(x_{AA,t}\Sigma_{AaA,t} + x_{AE,t}\Sigma_{AaE,t}) \cdot \Sigma_{AaA,t} + (\gamma_A - 1)\Sigma_{\xi,t} \cdot \Sigma_{AaA,t} - \sigma_{AL}\sigma_{qAL,t} \right] \\
&\quad - \frac{x_{AE,t}}{x_{A,t}} \left[\gamma_A(x_{AA,t}\Sigma_{AaA,t} + x_{AE,t}\Sigma_{AaE,t}) \cdot \Sigma_{AaE,t} + (\gamma_A - 1)\Sigma_{\xi,t} \cdot \Sigma_{AaE,t} - \sigma_{EL}\sigma_{qEL,t} \right] \\
&= \frac{\bar{a}P_{a,t}}{q_t} - dr_t^F + \sigma_{AO}\sigma_{qAO,t} + \frac{x_{AA,t}}{x_{A,t}}\sigma_{AL}\sigma_{qAL,t} + \frac{x_{AE,t}}{x_{A,t}}\sigma_{EL}\sigma_{qEL,t} - \gamma_A\zeta_{Aa,t} - (\gamma_A - 1)\zeta_{\xi A,t}.
\end{aligned}$$

Additionally,

$$\begin{aligned}
\Sigma_{q,t} \cdot \Sigma_{K,t} &= \psi_{A,t}\sigma_{AO}\sigma_{qAO,t} + \psi_{E,t}\sigma_{EO}\sigma_{qEO,t} \\
&\quad + (\psi_{AA,t} + \psi_{EA,t})\sigma_{AL}\sigma_{qAL,t} + (\psi_{AE,t} + \psi_{EE,t})\sigma_{EL}\sigma_{qEL,t} \\
&= \psi_{A,t}\sigma_{AO}\sigma_{qAO,t} + (1 - \psi_{A,t})\sigma_{EO}\sigma_{qEO,t} \\
&\quad + (\psi_{AA,t} + \psi_{EA,t})\sigma_{AL}\sigma_{qAL,t} + (\psi_{AE,t} + \psi_{EE,t})\sigma_{EL}\sigma_{qEL,t}
\end{aligned}$$

Plugging back into the law of motion for η_t , we obtain

$$\begin{aligned}
\mu_{\eta,t} &= \frac{\bar{a}P_{a,t}}{q_t} - \frac{C_{A,t}}{N_{A,t}} + (x_{A,t} - 1)(\gamma_A\zeta_{Aa,t} + (\gamma_A - 1)\zeta_{\xi A,t}) - (\Sigma_{NA,t} - \Sigma_{q,t} - \Sigma_{K,t}) \cdot (\Sigma_{q,t} + \Sigma_{K,t}) \\
&\quad + (1 - \psi_{A,t})(\sigma_{AO}\sigma_{qAO,t} - \sigma_{EO}\sigma_{qEO,t}) + \left(\frac{x_{AA,t}}{x_{A,t}} - \psi_{AA,t} - \psi_{EA,t} \right) \sigma_{AL}\sigma_{qAL,t} \\
&\quad + \left(\frac{x_{AE,t}}{x_{A,t}} - \psi_{AE,t} - \psi_{EE,t} \right) \sigma_{EL}\sigma_{qEL,t}
\end{aligned}$$

for the drift. For the volatility vector, we have

$$\begin{aligned}\Sigma_{\eta,t} &= \begin{bmatrix} x_{A,t}(\sigma_{AO} + \sigma_{qAO,t}) - \sigma_{qA,t} - \psi_{A,t}\sigma_{AO} \\ x_{A,t}\sigma_{qEO,t} - \sigma_{qE,t} - (1 - \psi_{A,t})\sigma_{EO} \\ x_{AA,t}(\sigma_{AL} + \sigma_{qAL,t}) - \sigma_{qAL,t} - (\psi_{AA,t} + \psi_{EA,t})\sigma_{AL} \\ x_{AE,t}(\sigma_{EL} + \sigma_{qEL,t}) - \sigma_{qEL,t} - (\psi_{AE,t} + \psi_{EE,t})\sigma_{EL} \end{bmatrix} \\ &= \begin{bmatrix} (x_{A,t} - \psi_{A,t})\sigma_{AO} + (x_{A,t} - 1)\sigma_{qA,t} \\ -(1 - \psi_{A,t})\sigma_{EO} + (x_{A,t} - 1)\sigma_{qE,t} \\ (x_{AA,t} - \psi_{AA,t} - \psi_{EA,t})\sigma_{AL} + (x_{AA,t} - 1)\sigma_{qAL,t} \\ (x_{AE,t} - \psi_{AE,t} - \psi_{EE,t})\sigma_{EL} + (x_{AE,t} - 1)\sigma_{qEL,t} \end{bmatrix}\end{aligned}$$

We now write portfolio shares in terms of capital allocations by observing

$$x_{A,t} = \frac{\psi_{A,t}}{\eta_t}, \quad x_{AA,t} = \frac{\psi_{AA,t}}{\eta_t}, \quad x_{AE,t} = \frac{\psi_{AE,t}}{\eta_t}.$$

The drift of η becomes

$$\begin{aligned}\mu_{\eta,t} &= \frac{\bar{a}P_{a,t}}{q_t} - \frac{C_{A,t}}{N_{A,t}} + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) (\gamma_A \zeta_{Aa,t} + (\gamma_A - 1) \zeta_{\xi A,t}) \\ &\quad - (\Sigma_{NA,t} - \Sigma_{q,t} - \Sigma_{K,t}) \cdot (\Sigma_{q,t} + \Sigma_{K,t}) + (1 - \psi_{A,t})(\sigma_{AO}\sigma_{qAO,t} - \sigma_{EO}\sigma_{qEO,t}) \\ &\quad + \left(\frac{\psi_{AA,t}}{\psi_{A,t}} - \psi_{AA,t} - \psi_{EA,t} \right) \sigma_{AL}\sigma_{qAL,t} + \left(\frac{\psi_{AE,t}}{\psi_{A,t}} - \psi_{AE,t} - \psi_{EE,t} \right) \sigma_{EL}\sigma_{qEL,t},\end{aligned}\tag{44}$$

and the volatility of η becomes

$$\Sigma_{\eta,t} = \begin{bmatrix} \psi_{A,t} \left(\frac{1}{\eta_t} - 1 \right) \sigma_{AO} + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qAO,t} \\ -(1 - \psi_{A,t})\sigma_{EO} + \left(\frac{\psi_{A,t}}{\eta_t} - 1 \right) \sigma_{qEO,t} \\ \left(\frac{\psi_{AA,t}}{\eta_t} - \psi_{AA,t} - \psi_{EA,t} \right) \sigma_{AL} + \left(\frac{\psi_{AA,t}}{\eta_t} - 1 \right) \sigma_{qAL,t} \\ \left(\frac{\psi_{AE,t}}{\eta_t} - \psi_{AE,t} - \psi_{EE,t} \right) \sigma_{EL} + \left(\frac{\psi_{AE,t}}{\eta_t} - 1 \right) \sigma_{qEL,t} \end{bmatrix}\tag{45}$$

F.3 Asset Price Volatilities

For the remainder of this appendix section, we suppress time subscripts and regard equilibrium quantities as functions of η . By Ito's lemma, for a twice-continuously differentiable f , the volatility vector of f is

$$\Sigma_f = \frac{f'}{f} \Sigma_\eta \eta.$$

Thus, we have

$$\Sigma_q = \begin{bmatrix} \sigma_{qAO} \\ \sigma_{qEO} \\ \sigma_{qAL} \\ \sigma_{qEL} \end{bmatrix} = \frac{q'}{q} \Sigma_\eta \eta = \frac{q'}{q} \begin{bmatrix} \psi_A(1-\eta)\sigma_{AO} + (\psi_A - \eta)\sigma_{qAO} \\ -(1-\psi_A)\eta\sigma_{EO} + (\psi_A - \eta)\sigma_{qEO} \\ (\psi_{AA}(1-\eta) - \psi_{EA}\eta)\sigma_{AL} + (\psi_{AA} - \eta)\sigma_{qAL} \\ (\psi_{AE}(1-\eta) - \psi_{EE}\eta)\sigma_{EL} + (\psi_{AE} - \eta)\sigma_{qEL} \end{bmatrix}.$$

Solving for the volatilities of q yields

$$\begin{aligned} \sigma_{qAO} &= \frac{q' \psi_A(1-\eta)\sigma_{AO}}{q - q'(\psi_A - \eta)} \\ \sigma_{qEO} &= -\frac{q'(1-\psi_A)\eta\sigma_{EO}}{q - q'(\psi_A - \eta)} \\ \sigma_{qAL} &= \frac{q'(\psi_{AA}(1-\eta) - \psi_{EA}\eta)\sigma_{AL}}{q - q'(\psi_{AA} - \eta)} \\ \sigma_{qEL} &= \frac{q'(\psi_{AE}(1-\eta) - \psi_{EE}\eta)\sigma_{EL}}{q - q'(\psi_{AE} - \eta)}. \end{aligned}$$

For ξ , we similarly have

$$\begin{aligned} \sigma_{\xi AA} &= \frac{\xi'}{\xi} ((\psi_{AA}(1-\eta) - \psi_{EA}\eta)\sigma_{AL} + (\psi_{AA} - \eta)\sigma_{qAL}) \\ \sigma_{\xi AE} &= \frac{\xi'}{\xi} ((\psi_{AE}(1-\eta) - \psi_{EE}\eta)\sigma_{EL} + (\psi_{AE} - \eta)\sigma_{qEL}). \end{aligned}$$

By (39), an AE agent holds capital in both the AE and the EME if and only if

$$\begin{aligned}
& \sigma_{AL}\sigma_{qAL} - \sigma_{EL}\sigma_{qEL} - \gamma_A \left(\frac{\psi_{AA}}{\eta}(\sigma_{AL} + \sigma_{qAL})^2 - \frac{\psi_{AE}}{\eta}(\sigma_{EL} + \sigma_{qEL})^2 \right) \\
&= (\gamma_A - 1) \frac{\xi'}{\xi} \left((\psi_{AA}(1 - \eta) - \psi_{EA}\eta)\sigma_{AL} + (\psi_{AA} - \eta)\sigma_{qAL} \right) (\sigma_{AL} + \sigma_{qAL}) \\
&\quad - (\gamma_A - 1) \frac{\xi'}{\xi} \left((\psi_{AE}(1 - \eta) - \psi_{EE}\eta)\sigma_{EL} + (\psi_{AE} - \eta)\sigma_{qEL} \right) (\sigma_{EL} + \sigma_{qEL}).
\end{aligned}$$

F.4 Equal Volatilities for Location Shocks

In the case that $\sigma_{AL} = \sigma_{EL}$, it is straightforward to characterize the location of capital usage. Intuitively, since the volatility of capital shocks is the same in either country, agents will split their capital usage between each country equally. In other words, $\psi_{AA} = \psi_{AE}$ and $\psi_{EA} = \psi_{EE}$. These allocations imply $\sigma_{qAL} = \sigma_{qEL}$, which delivers the following equalities

$$\begin{aligned}
& \sigma_{AL}\sigma_{qAL} = \sigma_{EL}\sigma_{qEL} \\
& \frac{\psi_{AA}}{\eta}(\sigma_{AL} + \sigma_{qAL})^2 = \frac{\psi_{AE}}{\eta}(\sigma_{EL} + \sigma_{qEL})^2 \\
& (\psi_{AA}(1 - \eta) - \psi_{EA}\eta)\sigma_{AL}(\sigma_{AL} + \sigma_{qAL}) = (\psi_{AE}(1 - \eta) - \psi_{EE}\eta)\sigma_{EL}(\sigma_{EL} + \sigma_{qEL}) \\
& (\psi_{AA} - \eta)\sigma_{qAL}(\sigma_{AL} + \sigma_{qAL}) = (\psi_{AE} - \eta)\sigma_{qEL}(\sigma_{EL} + \sigma_{qEL}).
\end{aligned}$$

These equalities imply that the asset-pricing condition pinning down ψ_{AA} is satisfied for AE agents. Thus, with equal location volatilities and allocation of capital across countries, AE agents satisfy their asset-pricing conditions. By symmetry, the same reasoning applies to EME agents.

F.5 Isomorphism

In this section, we establish an observational equivalence between the baseline model and this extension when $\gamma_A = 1$ and $\gamma_B = 1$, i.e. both AE and EME agents have log utility. By an observational equivalence, we mean there exists parametrizations such that the models produce the same equilibrium allocations.

By symmetry of this special case, we only need to consider the problem for AE agents. Define

ζ'_{Aa} as the original model's parallel to ζ_{Aa} . Note that since $\gamma_A = 1$, we do not need to consider $\zeta_{\xi A}$. For this section, we will assume $\sigma_L \equiv \sigma_{AL} = \sigma_{EL}$ and $\sigma_O \equiv \sigma_{AO} = \sigma_{EO}$. We want to find σ_O and σ_L such that $\zeta'_{Aa} = \zeta_{Aa}$. For reference,

$$\zeta'_{Aa} = \frac{\psi_A}{\eta}((\sigma + \sigma_{qA})^2 + \sigma_{qE}^2).$$

We want

$$\begin{aligned} & \left(\sigma + \frac{q'\psi_A(1-\eta)\sigma}{q-q'(\psi_A-\eta)} \right)^2 + \left(\frac{q'(1-\psi_A)\eta\sigma}{q-q'(\psi_A-\eta)} \right)^2 \\ &= \left(\sigma_O + \frac{q'\psi_A(1-\eta)\sigma_O}{q-q'(\psi_A-\eta)} \right)^2 + \frac{1}{2} \left(\sigma_L + \frac{1}{2} \frac{q'(\psi_A(1-\eta) - \psi_E\eta)\sigma_L}{q-q'(\psi_A/2-\eta)} \right)^2 \\ & \quad + \left(\frac{q'(1-\psi_A)\eta\sigma_O}{q-q'(\psi_A-\eta)} \right)^2 \\ &= 2 \left(\sigma + \frac{q'\psi_A(1-\eta)\sigma}{q-q'(\psi_A-\eta)} \right)^2 - 2 \left(\sigma_O + \frac{q'\psi_A(1-\eta)\sigma_O}{q-q'(\psi_A-\eta)} \right)^2 \\ & \quad + 2 \left(\frac{q'(1-\psi_A)\eta\sigma}{q-q'(\psi_A-\eta)} \right)^2 - 2 \left(\frac{q'(1-\psi_A)\eta\sigma_O}{q-q'(\psi_A-\eta)} \right)^2 \\ &= \left(\sigma_L + \frac{1}{2} \frac{q'(\psi_A(1-\eta) - \psi_E\eta)\sigma_L}{q-q'(\psi_A/2-\eta)} \right)^2 \\ &= 2 \left[(\sigma^2 - \sigma_O^2) + 2 \frac{q'\psi_A(1-\eta)}{q-q'(\psi_A-\eta)} (\sigma^2 - \sigma_O)^2 + \left(\frac{q'\psi_A(1-\eta)}{q-q'(\psi_A-\eta)} \right)^2 (\sigma^2 - \sigma_O^2) \right] \\ & \quad + 2 \left(\frac{q'(1-\psi_A)\eta}{q-q'(\psi_A-\eta)} \right)^2 (\sigma^2 - \sigma_O^2) \\ &= \sigma_L^2 + \frac{q'(\psi_A(1-\eta) - (1-\psi_A)\eta)}{q-q'(\psi_A/2-\eta)} \sigma_L^2 + \frac{1}{4} \left(\frac{q'(\psi_A(1-\eta) - (1-\psi_A)\eta)}{q-q'(\psi_A/2-\eta)} \right)^2 \sigma_L^2 \\ &= 2(\sigma^2 - \sigma_O^2) \left[1 + 2 \frac{q'(\psi_A(1-\eta))}{q-q'(\psi_A-\eta)} + \left(\frac{q'(\psi_A(1-\eta))}{q-q'(\psi_A-\eta)} \right)^2 + \left(\frac{q'(1-\psi_A)\eta}{q-q'(\psi_A-\eta)} \right)^2 \right] \\ &= 2(\sigma^2 - \sigma_O^2) \left[\left(1 + \frac{q'(\psi_A(1-\eta))}{q-q'(\psi_A-\eta)} \right)^2 + \left(\frac{q'(1-\psi_A)\eta}{q-q'(\psi_A-\eta)} \right)^2 \right] \\ &= \sigma_L^2 \left(1 + \frac{1}{2} \frac{q'(\psi_A(1-\eta) - (1-\psi_A)\eta)}{q-q'(\psi_A/2-\eta)} \right)^2. \end{aligned}$$

Re-arranging obtains

$$\begin{aligned}
\sigma_L^2 &= 2(\sigma^2 - \sigma_O^2) \frac{\left(1 + \frac{q'(\psi_A(1-\eta))}{q - q'(\psi_A - \eta)}\right)^2 + \left(\frac{q'(1-\psi_A)\eta}{q - q'(\psi_A - \eta)}\right)^2}{\left(1 + \frac{1}{2} \frac{q'(\psi_A(1-\eta) - (1-\psi_A)\eta)}{q - q'(\psi_A/2 - \eta)}\right)^2} \\
\sigma_L^2 &= 2(\sigma^2 - \sigma_O^2) \frac{\left(\frac{q - q'(1-\psi_A)\eta}{q - q'(\psi_A - \eta)}\right)^2 + \left(\frac{q'(1-\psi_A)\eta}{q - q'(\psi_A - \eta)}\right)^2}{\left(1 + \frac{1}{2} \frac{q'(\psi_A - \eta)}{q - q'(\psi_A/2 - \eta)}\right)^2} \\
\sigma_L^2 &= 2(\sigma^2 - \sigma_O^2) \frac{\frac{q^2 - 2qq'(1-\psi_A)\eta + (q'(1-\psi_A)\eta)^2 + (q'(1-\psi_A)\eta)^2}{(q - q'(\psi_A - \eta))^2}}{\left(1 + \frac{1}{2} \frac{q'(\psi_A - \eta)}{q - q'(\psi_A/2 - \eta)}\right)^2} \\
\sigma_L^2 &= 2(\sigma^2 - \sigma_O^2) \frac{q^2 - 2qq'(1-\psi_A)\eta + 2(q'(1-\psi_A)\eta)^2}{(q - q'(\psi_A - \eta))^2 \left(1 + \frac{1}{2} \frac{q'(\psi_A - \eta)}{q - q'(\psi_A/2 - \eta)}\right)^2}.
\end{aligned}$$

Thus, for a given q , q' , and ψ_A , we can obtain $\zeta'_{Aa} = \zeta_{Aa}$ if we choose σ_L and σ_O such that $\sigma_O < \sigma$ and they satisfy

$$\sigma_L = \sqrt{2(\sigma^2 - \sigma_O^2)} \frac{\sqrt{q^2 - 2qq'(1-\psi_A)\eta + 2(q'(1-\psi_A)\eta)^2}}{\left| (q - q'(\psi_A - \eta)) \left(1 + \frac{1}{2} \frac{q'(\psi_A - \eta)}{q - q'(\psi_A/2 - \eta)}\right) \right|}.$$

We now have equivalent risk premia in the baseline model and in this extension for AE and EME agents. Since agents have unit EIS in the baseline model as well, the two equations pinning down the behavior of q and ψ_A are market clearing for consumption and the difference between the AE's asset-pricing condition and the EME's asset-pricing condition. Because risk premia are the same when σ_O and σ_L satisfy the above conditions, q and ψ_A solve the same equations as in the baseline model. Therefore, the extension of the baseline model is observationally equivalent in the sense that capital allocations and prices are equivalent.

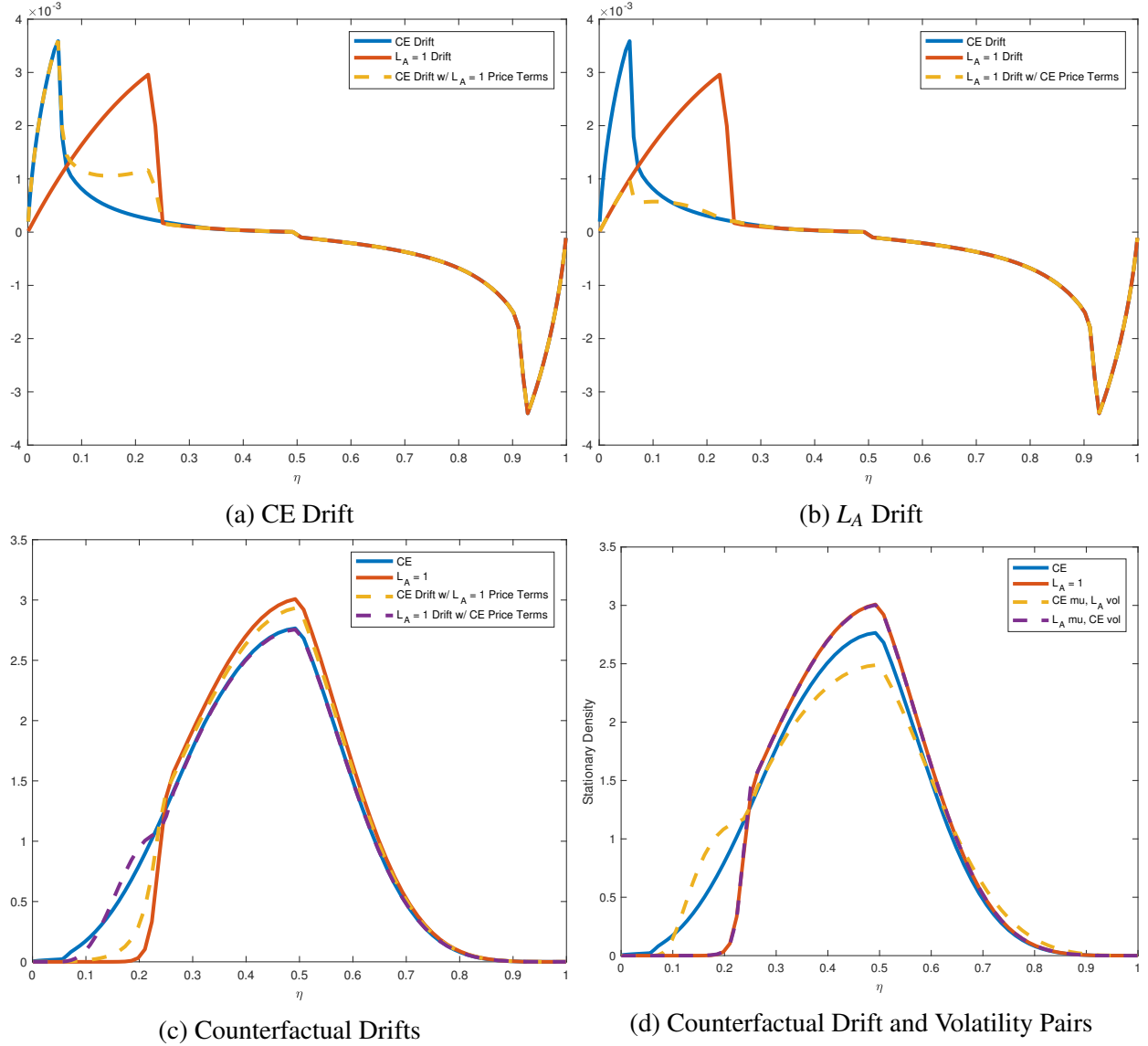


Figure 12: This figure examines counterfactual μ_η and stationary distribution. In all cases, L_E does not bind. Panels (a) and (b) plot μ_η from the competitive equilibrium with the solid blue line and from the $L_A = 1$ equilibrium with the solid red line. The yellow dashed line in (a) plots μ_η from the competitive equilibrium but with the prices of intermediate goods that appear in the μ_η term replaced by the prices from the $L_A = 1$ equilibrium. The yellow dashed line in (b) plots μ_η from the $L_A = 1$ equilibrium but with the prices of intermediate goods replaced by the prices from the competitive equilibrium. Panels (c) and (d) plots the stationary distributions from the competitive equilibrium and the $L_A = 1$ equilibrium in solid blue and red lines, respectively. Panel (c) also plots the distributions implied by the counterfactual drifts from Panels (a) and (b). The dashed yellow line in Panel (d) is the counterfactual stationary distribution if we use the drift from the competitive equilibrium paired with the volatility from the $L_A = 1$ equilibrium. The dashed purple line in Panel (d) is the stationary distribution if we use the drift from the $L_A = 1$ equilibrium paired with the volatility from the competitive equilibrium.