International Coordination of Macroprudential Policies with Capital Flows and Financial Asymmetries

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Abstract

We consider a two-country macro model in which countries have limited ability to issue state-contingent contracts in international markets. Both countries have incentives to stabilize their economy by using macroprudential policy (limiting leverage or capital inflows), but the emerging economy depends on the advanced economy to bear global risk. Lack of coordination hurts developing economies but benefits advanced economies. Financially developed economies are unwilling to intermediate global risk, which means bearing systemic risk, preferring financial stability over credit flows. Advanced economies prefer tighter macroprudential policies than would occur with coordination, giving them greater bargaining power when negotiating international agreements.

Keywords: International Capital Flows, Capital Controls, Macroeconomic Instability, Macroprudential Regulation, Policy Coordination, Spillovers, Financial Crises.


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1 Introduction

Advanced economies (AEs) hold substantial influence over the governance and direction of many international economic organizations, and only recently have emerging market economies (EMEs) begun to take significant leadership in the writing of international regulatory standards. Despite greater formal representation for EMEs, AEs continue to dominate the activities of these organizations when it comes to financial regulation. The existing explanations for this outcome can be broadly characterized as follows: AEs hold significant bargaining power during the negotiation of new rules because they have greater market size and more expertise with financial systems and regulation. As a result, AEs are able to shape the international financial system to better fit their preferences. We propose an additional channel through which AEs have greater bargaining power over EMEs: the key role AEs play in bearing global systemic risk. AEs with developed financial systems can better intermediate risky investments through the issuance of debt liabilities, but doing so exposes AEs to a greater risk of financial crisis. Accordingly, AEs have an incentive to impose tight macroprudential policies to “regulate away” systemic risks, which benefits AEs at the expense of EMEs. Because AEs can unilaterally choose their own domestic macroprudential policies, they may hold a “grim trigger” that can be used to coerce EMEs to agree to regulations preferred by

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1 As of September 2018, the U.S., Japan, Germany, France, and the U.K. alone hold 36% of the allotted votes on the IMF Executive Board. Moreover, every president of the World Bank was American, every managing director of the IMF has been an European, and every chairman of the Bank for International Settlements (BIS) has been from an AE. Out of 20 members on BIS’s board of directors, 16 come from AEs. Furthermore, it was not until after the Financial Crisis that EMEs were included in regulatory discussions by the Basel Committee on Banking Supervision (BCBS). Similarly, the G20 replaced the G8 as the main body for international economic cooperation only in 2009.

2 Walter (2015) finds evidence that EMEs still do not participate very much in the creation of financial rules like Basel III, which may be explained by mismatch between the agendas of these regulatory bodies and EME interests as well as EME’s comparative lack of resources and experiences navigating international financial regulation. Bair (2012) relates an account of the Basel III discussions about leverage ratios, portraying the debate as two-died between an opposing group (Germany, France, and Japan), and a supporting group (the U.S., U.K., Canada, Switzerland, the Netherlands, Sweden, and most Basel Committee members). Notably, the account made no mention of EMEs. These claims are corroborated by Watanagase (2013), who alleges that “most proposed regulations [by Basel III] are formulated and calibrated mainly based on advanced markets’ contexts and data, which can be significantly different from [EMEs].”

3 For example, in a speech on behalf of the BIS, White (2001) justified the sparse membership of the BCBS during Basel II on the grounds that representatives from AEs had greater expertise due to their more developed financial systems. Walter (2015) discusses evidence for why EMEs may have limited knowledge on constructing and implementing international financial regulations. See chapter 2 of Drezner (2007) for a game-theoretic model demonstrating how AEs can take advantage of their economic power and market size to influence international rule-setting.
Economists have recognized the crucial role of financial development to facilitate global intermediation. Notably, the United States acts as the world’s global banker, providing safe assets to the rest of the world by issuing debt liabilities and investing in risky foreign assets. However, for the U.S. or other AEs to bear global risk on behalf of EMEs requires bearing not only fundamental risk but also bearing systemic risk. While fundamental risk is compensated by excess returns (Gourinchas et al., 2010), with financial frictions or incomplete markets systemic risks (i.e., pecuniary externalities) need not be correctly compensated by returns. Systemic risk must be addressed through macroprudential regulation. In this paper, we show that it is unlikely that advanced economies are willing to bear a socially optimal level of systemic risk on behalf of EMEs.

Our results are of particular importance given the current focus on developing global recommendations for financial regulation (e.g., Basel III). In principle, policy coordination leads to outcomes at least as good as the uncoordinated equilibrium, but achieving successful coordination is nontrivial, and the benefits may accrue unequally to participants. Understanding the incentives for coordination and the incentives that cause Nash equilibrium to differ from the coordinated equilibrium offer important insight for the conduct and creation of new international regulations. While economists recognize that the effectiveness and implementation of macroprudential policies are complicated by micro-level spillovers through international capital markets, we argue that there are likely macro-level “spillovers” as well: changes in aggregate dynamics that result from global interactions in capital markets. While there is a rich literature studying the effects of macroprudential regulation and capital controls, less is understood about how countries should coordinate macroprudential policies given these cross-border interactions.

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4 See for example Gourinchas and Rey (2007), Gourinchas et al. (2010), Lane and Milesi-Ferretti (2007), and Maggiori (2017).

5 Macroprudential policies include bank capital requirements, counterparty concentration limits, interbank exposure limits, loan-to-value ratios, and reserve requirements, as well as limitations on capital inflows.

6 At the micro-level, economists have documented cross-border spillover effects in international lending due to domestic macroprudential policies such as capital requirements and loan-to-value ratio limits. For example, Buch and Goldberg (2016) find that “banks with higher initial capital were poised to increase lending internationally...when foreign countries tightened their capital requirements.” See also Obstfeld (2012, 2015), Shin (2012), and Rey (2015) for concerns about global financial linkages. Finally, Klein (2012) highlights that the effectiveness of capital controls depends heavily on how broadly and frequently they are enforced.

7 The literature on macroprudential policy and pecuniary externalities includes Caballero and Krishnamurthy (2001).
In light of these considerations, our paper theoretically considers how global spillovers through international capital markets can affect countries’ macroprudential policy choices. We use a two-country, two-good, stochastic macroeconomic model, based off Brunnermeier and Sannikov (2015), in which countries have limited ability to issue equity in international markets. International financial markets are imperfect, but there are no trade frictions other than, potentially, regulations imposed by each country limiting domestic leverage or capital inflows. Because of financial market imperfections, global output depends endogenously on the relative share of wealth in each country, and debt imbalances (i.e., leverage or capital inflows) create volatility that increases the fraction of the time the global economy spends with misallocated capital (one or the other country has very low levels of relative wealth). Hence, greater leverage and capital inflows lead to a better static allocation of capital, but with increased volatility that can hurt dynamic global stability.

We first consider how regulation in one country positively affects outcomes in the other (i.e., macro-level spillovers). We find that tighter macroprudential regulations in country A will (i) decrease global volatility when country A is relatively poor, (ii) improve the terms of trade for country A when country A is relatively poor, and crucially (iii) as a result, the frequency with which A is relatively poor will decrease. Hence, macroprudential regulation by A increases the frequency with which country B is relatively poor. Normatively, regulation by country A provides incentives for B to tighten its policy in order to increase the frequency of being relatively rich.

We then numerically solve for coordinated policies chosen by a global social planner and uncoordinated policies chosen in Nash equilibrium. For a broad set of parameters and whether countries are symmetric or asymmetric, we find that the Nash policies are tighter than would be chosen by a social planner. In fact, in Nash equilibrium countries maximally regulate to close credit flows completely, even though countries would choose some degree of openness (allowing limited leverage).

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8 In this setting, Brunnermeier and Sannikov (2015) show that completely shutting off international debt markets (no capital inflows) can lead to higher welfare as a result of this tradeoff.
9 Analogously, Phelan (2016) considers a closed economy model with a banking sector to show that, when equity markets are imperfect, a more stable economy is rich more frequently. As well, our result is consistent with the evidence in Boar et al. (2017) who find that countries that more frequently use macroprudential tools, other things being equal, experience stronger and less volatile GDP growth.
10 This result is consistent with the evidence in Agénor et al. (2018) who find that countries tend to set macroprudential policy more actively if their “neighbors” are more active, suggesting complementary policy responses.
These results extend to cases when countries can choose cyclical policies.\footnote{We find that when countries can coordinate simple cyclical macroprudential policies, they choose countercyclical regulation that completely limits leverage when capital is efficiently allocated (closed capital inflows) but allows limited leverage (limited capital inflows) when either country is in crisis (low relative wealth) and capital is misallocated. We then numerically solve for the Nash equilibrium countercyclical policies when symmetric countries cannot coordinate and find that the Nash policies call for tighter regulation.}

Crucially, these macro-level spillovers and strategic incentives have important implications for emerging economies.\footnote{When countries are symmetric, the welfare differences between coordinated and uncoordinated policy regimes are not very large, though both can be meaningfully different from laissez faire.} To that end, we suppose that as a result of financial development country $A$ (the AE) has a lower level of aggregate risk than country $B$ (the EME). As a result, agents in country $A$ have a greater ability to bear risk and so capital flows “upstream” to the low-risk country. In the absence of any regulation, the high-risk EME benefits from credit flows, which allow for better international risk sharing as the AE acts as a global intermediary, borrowing with safe debt to invest in risky capital (with less risk than EME investments). In Nash equilibrium, however, the AE is unwilling to act as a global intermediary, recognizing that credit flows create “excess” financial instability that mostly affects its own agents to the benefit of agents in the less-developed country.\footnote{Consistent with our prediction, Barth et al. (2013) show that post-crisis the U.S. has tightened regulations more relative to many of its peers. Furthermore, Jackson (2007) shows that even pre-crisis the U.S. had a higher cost of banking regulation and tended to enforce laws about securities much more relative to other countries.} In fact, lack of policy coordination primarily affects the tightness of policy in the AE, and these excessively tight regulations hurt EMEs to the benefit of the AE.\footnote{Starting from the competitive equilibrium with no policy regulation, coordination can lead to a Pareto improvement in welfare for both countries. However, starting from a point of uncoordinated policy coordination, virtually any coordinated policy would benefit EMEs at the expense of the AE, who would now be forced to bear systemic risk.}

While we do not take our quantitative results too seriously, we find that the welfare losses for the EME may be on the order of 1.5 to 3.5% of consumption.

To the extent that global recommendations for financial regulation reflect agreements reached as the outcome of (possibly complicated) negotiations among countries, our results suggest that AEs are likely to disproportionately benefit from those discussions. If EMEs do not agree with regulations that AEs prefer, AEs can always revert to Nash equilibrium and impose excessively tight macroprudential policies. Moreover, this threat would be credible since AEs prefer this outcome to the coordinated equilibrium. Our results show that AEs, especially the U.S., have strong
bargaining power in these negotiations.

**Related Literature**  Our paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014, 2015, 2016) and He and Krishnamurthy (2012, 2013, 2014), who apply continuous-time methods to analyze the non-linear global dynamics of economies with financial frictions, building on seminal results from Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999). Most closely related, Brunnermeier and Sannikov (2015) demonstrate that closed capital accounts improve welfare relative to open accounts because closing capital accounts improve global stability arising from incomplete markets. While the global economy is stable near the steady state with high output and growth, away from the steady state, the economy features high asset price volatility and nonlinear amplifications that can be dampened by capital controls. Similarly, Phelan (2016) illustrates that leverage limits improve macroeconomic stability by endogenously increasing the frequency with which the banking sector is well-capitalized, and this can increase welfare, and Caballero and Simsek (2017) consider how macroprudential policy can mitigate damage arising from belief heterogeneity. These results contrast with Gărleanu et al. (2015), where limits on leverage reduce agents’ incentives to diversify and result in more correlated investments, increasing financial fragility.

We extend the analysis of Brunnermeier and Sannikov (2015), who only consider completely shutting off credit flows as a policy instrument, by allowing countries to choose regulations that occasionally bind, either limiting capital inflows or limiting leverage, and by allowing countries to choose potentially different policies. By giving countries more flexibility in their policy choices, our model admits interesting policy spillovers and strategic considerations.

Economists have tended to focus on the use of capital controls by EMEs to induce favorable terms of trade (see e.g. Costinot et al., 2014; Farhi and Werning, 2014) or to mitigate the adverse

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15 In the former papers leverage limits counteract externalities which levered agents do not internalize, whereas in Gărleanu et al. (2015), they discourage investors from making socially desirable decisions.

16 Technically, we modify their model by assuming a constant growth rate and introducing leverage constraints; we solve the model analytically in closed form when leverage constraints bind; and we computationally solve the model for asymmetric volatilities. Constant growth rates simplify computation and allow us to solve the model in closed form when leverage constraints bind, but our results carry over when growth is endogenously determined by capital investment (Tobin’s Q).
consequences of fickle capital flows Caballero and Simsek (2016). Similar to Costinot et al. (2014) and Farhi and Werning (2014), in our model macroprudential policies work by changing a country’s terms of trade. However, the mechanism in our model is quite different. In Costinot et al. (2014), for example, countries “manipulate” terms of trade by differentially taxing capital inflows and outflows when growth is higher or lower than the rest of the world. In our model, limiting inflows alone are sufficient to benefit a country because the terms of trade hedge decreases systemic risk and changes the stationary distribution of an economy, dynamically benefiting the financial position of the country in the long-run.

Our model contributes to the literature on coordinated policy in international settings due to spillovers. Most similar to our analysis, Agénor et al. (2018) consider a linearized core-periphery model when financial intermediation is subject to frictions and show that policies trade off mitigating the short-run consequences of the financial accelerator (and spillovers) and the long-run effects of higher costs of capital. The gains from cooperation are on the order of 1% of consumption and asymmetric.

There is a large literature studying the effectiveness of macroprudential regulation and capital controls, as well as the potential spillover effects. Analysis of early experiences with macroprudential instruments shows that some tools can reduce banks’ asset growth within countries (Claessens et al., 2013). However, effectiveness may be weakened when risky or excessive lending moves outside of the regulatory perimeter to non-covered entities or activities (Bengui and Bianchi, 2014; Aiyar et al., 2014; Reinhardt and Sowerbutts, 2015). Kuttner and Shim (2016) find that debt-to-income ratios significantly affect housing credit growth. Bruno et al. (2016) document the role of macroprudential policies and capital controls in mitigating credit growth in 12 Asian economies. Berrospide et al. (2016) find that some regulatory changes spill over.

17 For example, Banerjee et al. (2016) explore the impact of spillovers from the macroeconomic policies of advanced economies to emerging market economies, and Ghosh and Masson (1991) find that with learning, coordinated policies outperform activist uncoordinated policies or exogenous money targets. In addition to the literature on macroprudential policy and pecuniary externalities cited earlier, Hahm et al. (2011) evaluate the impact of macroprudential policies when applied to open emerging economies as opposed to advanced economies. Korinek and Simsek (2016) apply macroprudential policy in the case of liquidity traps with overborrowing, finding that interest rate policy is inferior in dealing with excessive leverage. Farhi and Werning (2016) incorporate nominal rigidities and financial market frictions into a general theory and provides a simple formula that characterizes optimal financial market intervention.

18 Specially, a foreign country’s tightening of limits on loan-to-value ratios and local currency reserve requirements...
ited effectiveness of partial or limited capital controls see Klein (2012) and Klein and Shambaugh (2015). Empirical evidence about the effect of capital account liberalizations are mixed, e.g., Obstfeld and Taylor (2004) and Magud et al. (2011).

Outline  The remainder of the paper is organized as follows. Section 2 presents the model and the equilibrium conditions. Section 3 numerically solves for equilibrium in a “calibrated” symmetric economy and illustrates the effects of regulations, both symmetric and asymmetric. Section 4 solves for optimal coordinated and uncoordinated policies (constant/fixed leverage limits) with symmetric and asymmetric economies. Section 5 concludes.

2 The Model

This section presents a slightly modified version of the two-country model in Brunnermeier and Sannikov (2015). The global economy is populated by agents who live in two different countries, A and B. Agents use capital to produce the two intermediate consumption goods a and b, and agents in country A have productive advantages at producing good a (vice versa B and b). Capital trades in a competitive market, but agents in each country may be subject to leverage constraints owing to macroprudential policies in each country. Financial frictions limit international credit flows to risk-free debt (non-state contingent contracts).

2.1 Setup

Technology.  Time is infinite and continuous. Capital can be used to produce either good a or good b, which are then combined into an aggregate (final) consumption good. Final consumption is given by

\[ y_t = \left( y^a_t \right)^{1/2} \left( y^b_t \right)^{1/2}, \]  

(1)

where \( y^a_t \) is the supply of good a and \( y^b_t \) is the supply of good b.
Agents in either country can produce goods $a$ and $b$ using linear production technologies, but agents in country $A$ have a superior technology for good $a$ while agents in country $B$ have a superior technology for good $b$. In particular, given $k_t$ units of capital, agents in country $A$ produce good $a$ at rate \( \bar{a}k_t \) and good $b$ at rate \( ak_t \), where \( \bar{a} > a > 0 \). Country $B$ agents face the reverse situation, with good $a$ produced at rate \( ak_t \) and good $b$ at rate \( \bar{a}k_t \).

Denote the aggregate amount of world capital available at time $t$ by $K_t$, and denote the share of world capital held by agents in country $A$ and in country $B$ by $\psi^A_t$ and $\psi^B_t$, respectively. Furthermore, denote the fraction of world capital devoted to production of goods $a$ and $b$ by $\psi^A_{ta}$, $\psi^A_{tb}$, $\psi^B_{ta}$, and $\psi^B_{tb}$, where the first superscript denotes the country. By definition,

$$\psi^A_t + \psi^A_{tb} + \psi^B_{ta} + \psi^B_{tb} = 1.$$  

The aggregate supply of good $a$ and $b$ then are given by

$$Y^a_t = (\bar{a}\psi^A_t + a\psi^B_t)K_t, \quad Y^b_t = (a\psi^A_t + \bar{a}\psi^B_t)K_t,$$

yielding the total supply of the aggregate good

$$Y_t = (Y^a_t)^{1/2}(Y^b_t)^{1/2}. \quad (2)$$

Let the final good be the numeraire. Then the prices of $a$ and $b$ can be written as

$$P^a_t = \frac{1}{2} \left( \frac{Y_t}{Y^a_t} \right), \quad P^b_t = \frac{1}{2} \left( \frac{Y_t}{Y^b_t} \right). \quad (3)$$

There is a single type of physical capital. We model productivity shocks as shocks directly to capital, which can be interpreted as shocks to “effective capital.” Capital in country $I$ evolves according to

$$\frac{dk_t}{k_t} = gdt + \sigma_I dZ^I_t, \quad (4)$$

where $dZ^I_t$ is a standard Brownian motion. The two Brownian motions, $dZ^A_t$ and $dZ^B_t$, are indepen-
dent and exogenous. This specification is tractable and admits the interpretation of global aggregate shocks with idiosyncratic (negatively correlated) country specific shocks. Thus, the shocks can capture country-specific productivity gains as well as redistributive shocks (e.g. international law suits). It follows that aggregate capital follows the law of motion

\[
\frac{dK_t}{K_t} = gd_t + \psi_t^A \sigma_t^A dZ_t^A + \psi_t^B \sigma_t^B dZ_t^B.
\] (5)

For modeling tractability we suppose that capital grows at a constant rate \( g \). This simplifies the analysis and the computations but does not significantly affect our results.\[^{19}\]

**Preferences.** All agents have log utility with intertemporal preferences described by the expected utility function

\[
E \left[ \int_0^\infty e^{-\rho t} \log(c_t) \, dt \right],
\] (6)

where \( c_t \) is the consumption of the final good at time \( t \) and \( \rho \in (0, 1) \) is the discount factor.

**Markets for Physical Capital and Risk-free Bonds.** Agents can trade physical capital in a competitive international market. We denote the equilibrium market price of capital per unit by \( q_t \). Hence, capital \( k_t \) has market value \( q_t k_t \). We postulate that \( q_t \) evolves endogenously according to

\[
\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^{qA} dZ_t^A + \sigma_t^{qB} dZ_t^B,
\] (7)

where \( \mu_t^q \), \( \sigma_t^{qA} \), and \( \sigma_t^{qB} \) will be determined in equilibrium.

Agents can also trade a risk-free bond that is in zero net supply. We denote the endogenously determined risk-free return by \( dr_t^F \). Agents can borrow or save in the risk-free asset, but they may face borrowing limits imposed by regulation in their country.

\[^{19}\]Our results regarding how regulation affects equilibrium are the same if we follow Brunnermeier and Sannikov (2015) and suppose that capital grows at an endogenous rate \( \Phi(t_t) \), where \( t_t \) is the endogenous investment rate. Intuitively, if agents invest according to Tobin’s \( Q \), then endogenous investment will produce feedback between capital prices and growth. In bad times, this feedback should make asset price crashes \textit{worse} than in our model, making excess leverage more harmful. However, solving for the Nash Equilibrium with uncoordinated policy is computationally intensive, which is why we use the simpler setup. We obtain similar results if we suppose that the capital growth rate is a function of the capital price without directly modeling investment.
Returns from Capital. The return from holding capital can be written as diffusion processes by summing capital gains \( \frac{d(q, k)}{q, k} \), which has two volatility terms, and the dividend yield from using capital to produce good \( a \) or \( b \), which has no volatility terms. We denote the return from an agent in country \( I \) buying physical capital and using it to produce good \( j \) by \( dr_I^j_t \). Given equations (4) and (7) for the laws of motion for capital and the capital price and using Ito’s Lemma, returns for country \( A \) when producing good \( a \) are given by

\[
d r^A_{t} = \left( \frac{\tilde{a} + \mu^q + g + \sigma^A \sigma^{qA}}{q} \right) dt + (\sigma^A + \sigma^{qA}) dZ^A_t + \sigma^{qB} dZ^B_t,
\]

and analogously for the other country-goods pairs.\(^{20}\) Notably, country \( A \) shocks affect the returns to capital used in country \( A \) directly (\( k_t \) changes) and indirectly through the effect on the capital price \( q_t \); however, country \( B \) shocks affect returns indirectly through the effect on the capital price (analogously for how shocks affect country \( B \) returns).

Incomplete Markets, Financial Frictions, and Macroprudential Regulation. The key financial friction in this model is the inability of countries to issue equity to each other. Agents can only trade risk-free bonds to purchase capital, and as a result, markets are incomplete because they are limited to non-contingent financial contracts. This reliance on risk-free debt generates financial fragility that would be attenuated if agents could purchase equity in the other country. In addition, agents cannot sell short investment in the production of good \( a \) and \( b \). Incomplete international markets can be motivated by home bias in equity holdings and micro-founded by agency problems and asymmetric information.\(^{21}\)

The heart of our analysis involves macroprudential regulations in each country. We model macroprudential regulation as borrowing limits requiring that leverage not exceed a country-specific

\[^{20}\text{e.g., } dr^B_{t} = \left( \frac{\tilde{a} + \mu^q + g + \sigma^B \sigma^{qB}}{q} \right) dt + (\sigma^B + \sigma^{qB}) dZ^B_t + (\sigma^B + \sigma^{qB}) dZ^B_t.\]

\[^{21}\text{Heathcote and Perri (2013) provide an explanation for the empirically observed bias toward domestic asset holdings. See Jensen and Meckling (1976), Bolton and Scharfstein (1990), and Holmström and Tirole (1997) for theories of agency problems and asymmetric information limiting state-contingent contracts. More broadly, a home equity bias can be rooted in a fundamental trade-off between the diversification benefits and participation costs of investing in international markets. Garleanu et al. (2015) show that limited market integration endogenously arises because of this trade-off, and since portfolios comprise more correlated securities, risk premia are higher relative to the frictionless benchmark.}\]

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threshold. As will become clear, in the aggregate, imposing leverage limits is equivalent to imposing limits on capital inflows.

**Consumption and Portfolio Choice** Each agent decides her consumption rate $c_t$ as well as how to allocate remaining wealth. Agents face a portfolio choice problem of how much capital to invest in the production of goods $a$ and $b$, and how much to invest in the risk-free bond. We denote the portfolio weights by $(x_t^a, x_t^b, 1 - x_t^a - x_t^b)$, where $x_t^a$ is the fraction of wealth invested in capital used to produce good $a$, $x_t^b$ is the fraction of wealth invested in capital used to produce good $b$, and $1 - x_t^a - x_t^b$ is the fraction of wealth invested in the risk-free asset. Portfolio weights $x_t^a$ and $x_t^b$ must be non-negative.

Denote $L_A^I$ and $L_B^I$ as the leverage constraints imposed on country $A$ and $B$, respectively. These constraints impose that the market value of an agent’s borrowing (debt) cannot exceed $L_I^t$ times their wealth (leverage of 0 implies no borrowing). In other words, for an agent in country $I$

$$x_t^a + x_t^b \leq L_I^t + 1.$$  

(9)

Given a consumption rate $c_t^I$ and portfolio weights $(x_t^a, x_t^b, 1 - x_t^a - x_t^b)$, the the net worth $n_t^I$ of an agent evolves according to

$$\frac{dn_t^I}{n_t^I} = x_t^a dr_t^{Ia} + x_t^b dr_t^{Ib} + (1 - x_t^a - x_t^b) dr_t^F - \frac{c_t^I}{n_t^I} dt.$$  

(10)

Thus, agents’ problems can be summarized as maximizing utility (6) subject to the budget constraint (10) together with the solvency constraint $n_t \geq 0$ and the borrowing constraint (9). Since agents have log utility, optimal consumption is to consume a fraction $\rho$ of net wealth, implying $c_t = \rho n_t$ for all agents.

**Definition 1** (Equilibrium). For any initial allocation of wealth, an equilibrium is a map from histories of shocks $\{Z^A_s, Z^B_s, s \in [0, t]\}$ to the allocation of capital $(\psi_t^{Ia}, \psi_t^{Ib}, \psi_t^{ba}, \psi_t^{bh})$ and the aggregate consumption good $(C_t^A, C_t^B)$ as well as price $q_t$ and risk-free rate $dr_t^F$ such that

1. All agents solve their optimal consumption and portfolio choice problems, subject to the
solvency constraint on their net worth and leverage constraints.

2. Capital, consumption, and debt markets clear.

2.2 Solving for Equilibrium

Since countries cannot issue equity internationally but can only trade in capital and risk-free assets, an agent’s portfolio decisions depend on her level of wealth, so equilibrium depends on the aggregate level of wealth in each country. For example, as country A’s wealth increases, its aggregate capital holdings will increase, which will increase the fraction of global capital used to produce good \(a\). Thus, capital allocations and final good production will depend on country’s relative wealth, which will vary in response to global shocks. We use stochastic continuous-time methods to solve for global equilibrium dynamics. We solve for a recursive (or Markov), rational-expectations equilibrium in which the single state variable is the relative wealth of the two countries.

Denote the aggregate net worth of agents in country \(A\) at time \(t\) by \(N_t\). Then the relative share of net wealth held by country \(A\) is defined to be \(\eta_t \equiv \frac{N_t}{q_t K_t}\). Thus, \(\eta_t\) represents the share of global wealth held by country \(A\). The aggregate portfolio choice of countries \(A\) and \(B\) can be written respectively as

\[
\left( \frac{\psi_{Aa}^t}{\eta_t}, \frac{\psi_{Ab}^t}{\eta_t}, 1 - \frac{\psi_{A}^t}{\eta_t} \right), \quad \left( \frac{\psi_{Ba}^t}{1 - \eta_t}, \frac{\psi_{Bb}^t}{1 - \eta_t}, 1 - \frac{\psi_{B}^t}{1 - \eta_t} \right),
\]

where \(\psi_{A}^t = \psi_{Aa}^t + \psi_{Ab}^t\) and \(\psi_{B}^t = \psi_{Ba}^t + \psi_{Bb}^t\). Equilibrium, therefore, consists of an endogenous law of motion for \(\eta_t\) and capital allocations and prices which are functions of the state variable \(\eta_t\). Since all agents consume a fraction \(\rho\) of their wealth, market clearing for the final consumption good implies that the equilibrium capital price satisfies

\[
q_t = \frac{(\bar{a}\psi_{Aa}^t + a\psi_{Ba}^t)^{1/2}(a\psi_{Ab}^t + \bar{a}\psi_{Bb}^t)^{1/2}}{\rho}.
\]

Asset-Pricing Equations. Since all agents have log utility, when leverage constraints do not bind, they choose portfolios such that excess returns equal their volatility of net worth (see Appendix A and Brunnermeier and Sannikov (2015) for technical details). However, when leverage
constraints bind, the excess returns can exceed the risk premium (see Appendix A). The leverage constraint will bind for agents in country \( A \) when, subject to no constraints, \( \psi_t^A / \eta_t \geq L^A + 1 \) and for country \( B \) when, subject to no constraints, \( \psi_t^B / (1 - \eta_t) \geq L^B + 1 \). Thus \( \psi_t^A / \eta_t \leq L^A + 1 \) and \( \psi_t^B / (1 - \eta_t) \leq L^B + 1 \) over the state space. A country with positive leverage invests using capital inflows. Thus, limiting leverage has the effect, in the aggregate, of restricting capital inflows.

**Characterizing Equilibrium** Using the returns equations, together with market clearing for capital and consumption, we can characterize equilibrium as a system of differential equations in the capital price \( q_t \). We first characterize equilibrium when leverage constraints do not bind, which follows immediately from Brunnermeier and Sannikov (2015).

**Proposition 1.** When leverage constraints do not bind, the equilibrium law of motion of \( \eta \) will be endogenously given as

\[
\frac{d\eta_t}{\eta_t} = \mu_t \eta_t dt + \sigma_t^A dZ_t^A + \sigma_t^B dZ_t^B,
\]

where

\[
\mu_t^\eta = \left( \frac{\psi_t^A}{\eta_t} \right)^2 \left[ (1 - \eta_t)^2 (\sigma^A + \sigma_t^{qA})^2 + (1 + \eta_t^2)(\sigma_t^{qB})^2 \right] - \left( \psi_t^B \right)^2 \left( \frac{\eta_t}{1 - \eta_t} \right) \left[ (\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2 \right] + \left( \frac{\psi_t^A}{\eta_t} \right) \left( \psi_t^B \right) \left[ (2\eta_t - 1)(\sigma^A + \sigma_t^{qA})\sigma_t^{qA} + (2\eta_t + 1)(\sigma^B + \sigma_t^{qB})\sigma_t^{qB} \right],
\]

\[
\sigma_t^{\eta A} = \frac{1 - \eta_t}{\eta_t} \psi_t^A (\sigma^A + \sigma_t^{qA}) - \psi_t^B \sigma_t^{qA},
\]

\[
\sigma_t^{\eta B} = \frac{1 - \eta_t}{\eta_t} \psi_t^A \sigma_t^{qB} - \psi_t^B (\sigma^B + \sigma_t^{qB}).
\]

Asset prices satisfy

\[
\frac{\bar{a} \left( P_t^A - P_t^B \right)}{q_t} + \sigma^A \sigma_t^{qA} - \sigma^B \sigma_t^{qB} = \frac{\psi_t^A}{\eta_t} \left( (\sigma^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2 \right) - \frac{1 - \psi_t^A}{1 - \eta_t} \left( (\sigma^B + \sigma_t^{qB})^2 + (\sigma_t^{qA})^2 \right),
\]

14
The state space is divided into 3 regions. For $\eta < \eta^a$, both countries produce good $a$ and country $B$ produces good $b$. For $\eta > \eta^b$ both countries produce good $b$ and country $A$ produces good $a$. For $\eta \in [\eta^a, \eta^b]$ countries specialize, using only their most productive technology. Goods prices satisfy

$$\frac{a}{a} \leq \frac{P^b}{P^a} \leq \frac{\bar{a}}{a},$$

where the first (second) inequality becomes equality in the left (right) region of the state space.

However, equilibrium is slightly modified when leverage constraints bind. Crucially, leverage constraints affect equilibrium capital prices and allocations only when they bind, but when constraints do not bind equilibrium capital prices and allocations are the same as in an economy in which leverage constraints never bind. Additionally, with constant leverage constraints over the binding region, we can analytically solve for the capital price $q(\eta)$ as well as its derivative $q'(\eta)$. When leverage constraints cease to bind, the capital price (and as a result capital allocations) are the same as in an economy in which leverage constraints never bind. Second, over the range of $\eta$ where the leverage constraint binds, $\eta_i$ follows a different law of motion until the leverage constraint no longer binds, at which point the law of motion reverts to (12).

**Proposition 2.** When country $A$’s leverage constraint binds, $\psi^A_t / \eta_t = 1 + L^A$, and:

(i) when countries specialize in production the asset price satisfies

$$q(\eta) = \frac{\bar{a}}{\rho} \sqrt{\eta(1+L^A)(1-\eta(1+L^A))}, \quad (14)$$

$$q'(\eta) = \frac{q}{2} \left[ \frac{1-2\eta(1+L^A)}{\eta(1-\eta(1+L^A))} \right], \quad (15)$$

and (ii) when countries do not specialize,

$$q(\eta) = \frac{a\sqrt{\tau}}{2\rho} (1 + \eta(1+L^A)(\tau - 1)), \quad (16)$$

$$q'(\eta) = q \left( \frac{(1+L^A)(\tau - 1)}{1+\eta(1+L^A)(\tau - 1)} \right), \quad (17)$$

where $\tau = \frac{\bar{a}}{\bar{a}}$. When leverage constraints bind, the range of $\eta$ for which countries specialize or not
can be solved in closed form using the above expressions.

Furthermore, the evolution of $\eta_t$ and $q_t$ are given by equations 23 and 24 in Appendix A.

We can similarly solve for capital prices, allocations, and equilibrium evolutions when $B$’s constraints bind.

**Welfare** The main goal of our analysis is to understand how countries choose regulation in order to maximize the welfare of its agents. We can evaluate the effects of leverage constraints on the welfare of agents in a country using Propositions 1 and 2. From Brunnermeier and Sannikov (2015), the value function for the representative agent for country $A$ is of the form

$$V^A(N_t, \eta_t) = \frac{\log N_t}{\rho} + h^A(\eta_t),$$

where $h^A(\eta_t)$ depends on the market frictions in the model. We can write $N_t = \eta_t q_t K_t$, so

$$V^A(N_t, \eta_t) = V^A(\eta_t) = \frac{\log \eta_t}{\rho} + \frac{\log K_t}{\rho} + \frac{\log q(\eta_t)}{\rho} + h^A(\eta_t).$$

Solving the Hamilton-Jacobi-Bellman equation is equivalent to solving the following second-order differential equation for $H^A(\eta_t) \equiv \log(q(\eta_t))/\rho + h^A(\eta_t)$.

$$\rho H^A = \log(\rho q(\eta_t)) + \frac{\mu_t^\eta}{\rho} - \frac{(\sigma_t^\eta A)^2 + (\sigma_t^\eta B)^2}{2\rho} + \frac{g}{\rho}$$

$$- \frac{(\psi_t^\eta A)^2 + (\psi_t^\eta B)^2}{2\rho} + \mu_t^\eta \eta_t (H^A)' + \frac{(\sigma_t^\eta A)^2 + (\sigma_t^\eta B)^2}{2} \eta_t^2 (H^A)''. \tag{18}$$

$V^B(N_t, \eta_t) = V^B(\eta_t)$ follows a symmetric equation.

Because agents have log utility, we can sharply quantify a country’s consumption-equivalent welfare gains or losses in different equilibria. Let $V_c$ be the value function in the current equilibrium with consumption $c_t$, and let $V_d$ be the value function in a different equilibrium. Then the
consumption-equivalent $\phi$ solves

$$V_d = \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log((1 + \phi)c_t) \, dt\right]$$

$$= \int_0^\infty e^{-\rho t} \log(1 + \phi) \, dt + \mathbb{E}\left[\int_0^\infty e^{-\rho t} \log(c_t) \, dt\right] = \frac{1}{\rho} \log(1 + \phi) + V_c.$$ 

Re-arranging yields $\phi = \exp((V_d - V_c) \cdot \rho) - 1$.

3 Numerical Results for a Symmetric Economy

We first illustrate the effects of leverage constraints on equilibrium stability and welfare in a symmetric economy. We solve the model numerically using parameters roughly calibrated to match advanced economies (our qualitative results are robust across a range of parameters). The most important parameters are the volatilities, which we set to $\sigma^A = \sigma^B = 3\%$, which is roughly the volatility of TFP shocks. We normalize productivity to $\bar{a} = 1$ and set $\varrho = 0.8$, implying gains from specialization in trade of 25 percent. The discount factor has negligible effects on the results (we set $\rho = 4\%$) and the growth rate only affects the level of welfare (we set $g = 2\%$).

We suppose countries choose a simple, fixed leverage constraint $L^A$ and $L^B$. We first consider how varying constraints affect the stationary distribution (stability) of the global economy, and we then consider how constraints in each country affect welfare in $A$.

3.1 Stability

In the absence of any policy, the economy features a stochastic steady state at $\eta = 0.5$ where capital is best allocated (since countries are symmetric), and the economy tends to drift toward $\eta = 0.5$ after shocks move the system away. Hence the distribution of $\eta$ is centered around when flow output is highest (i.e., the economy is typically stable).

Figure [1] plots the stationary distribution of $\eta$ varying $L^A$ for two cases. Panel (a) considers when country $B$ does not impose leverage constraints, and Panel (b) considers the other extreme when country $B$ sets leverage to zero (closed capital account). The blue distribution in Panel (a)
Figure 1: Stationary distribution of $\eta$ (stability). Tighter constraints in A shift mass toward higher $\eta$ (good for A), and tighter constraints in B shift mass toward lower $\eta$ (good for B).

thus shows the distribution in the laissez-faire economy without constraints. Notably, it is the most disperse distribution. This is precisely the notion of systemic risk: the distribution of $\eta$ in competitive equilibrium is more dispersed than any country would like. Tighter regulation can shift mass in the direction a country prefers. Of course, the flip-side is that when country A tightens, the distribution must worsen from the perspective of country B. Contrasting Panels (a) and (b) clearly illustrates how tightening by B has shifted the distribution toward low $\eta$ (bad for A).

Tighter leverage constraints increase the stability of the economy and reduce systemic risk because leverage decreases the volatility of $\eta$ and improves the terms of trade, thus increasing the drift of $\eta$. These two forces together mean that following bad shocks which push the economy away from the steady state, the economy recovers more quickly with leverage constraints. Figure 2 plots changes in the drift and volatility of $\eta_t$.

However, tighter regulation is not without cost. When constraints bind, the flow allocations are worsened and output suffers. Misallocation is reflected in a lower capital price and a widening of terms of trade ($P^a_t/P^b_t$), as plotted in Figure 3.

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22 In this way, the effect of capital controls in our model is quite complementary to the effect in Costinot et al. (2014). Because in our model taxing/limiting inflows improves the distribution of the economy, taxes on inflows need not be coupled with taxes on outflows.
Figure 2: Evolution of $\eta$. Tighter constraints in A decrease volatility and (generally) increase drift, creating a more stable economy. (Country B unconstrained)

3.2 Welfare

In this economy the dynamic benefits of a more stable economy generally outweigh the flow costs, and as a result tightening constraints in one country generally improves welfare for that country. However, there are important cross-country spillovers. Figure 4 plots welfare for country A evaluated at the stochastic steady state (i.e., $V_A(0.5)$) as a function of the leverage constraint chosen by B, for three levels of constraints in A.

There are two important patterns to notice. First, given the policy in country B, a tighter regulation in A (lower $L_A$) improves welfare in country A. This is because the stability trade-off outweighs the flow cost, as discussed earlier. Second, though, given $L_A$ in country A, welfare in country A is increasing in $L_B$. In other words, tightening in B decreases welfare in A. While tighter constraints in country A generally increase welfare for country A (though the relationship is not strictly monotonic), tighter constraints in country B decrease welfare in country A. Hence, macro-level spillovers from macroprudential policy are reflected in changes in the stationary distribution of $\eta$ and changes in welfare.

There are two complementary intuitions for this result and it is important to consider both. First, when B tightens, that shifts mass toward states when A is relatively poor. As we have seen, if B tightens and shifts mass toward states when A is poor, A can shift mass away from those states by
tightening as well. Hence, uncoordinated regulation to improve stability has a “zero-sum” element: a country cannot shift mass away from \( \eta = 1 \) without shifting it toward \( \eta = 0 \). As we will see later, this mechanism is precisely why uncoordinated (Nash) policy choices will be tighter than policies coordinated by a social planner, since each country will have an incentive to tighten in order to improve the distribution of outcomes (from its perspective).  

Second, when agents in \( B \) use leverage, they are acting as intermediaries for \( A \), borrowing with safe debt and investing in risky capital. When \( B \) tightens, in the aggregate \( B \) bears less risk when its agents would use leverage and thus \( B \) provides less intermediation for country \( A \). Crucially, when a country takes on leverage (intermediating risk), this implies bearing system risk (endogenous volatility). This is reflected in Figure 2 where the volatility of \( \eta \) decreases when country \( A \) tightens regulation—and importantly, the volatility of \( \eta \) decreases only at times when \( A \) is relatively poor, which is precisely when \( A \) does not want systemic risk. Thus, a key motivation for limiting leverage is to decrease systemic risk when the country plays the role of intermediary. But as we’ve seen, limiting leverage in order to bear less systemic risk is costly to the other country. This intuition will be crucial for understanding welfare consequences in an asymmetric economy with and an advanced and emerging country, where the advanced economy will typically play the

\[23\] Indeed, considering the placement of the curves in Figure 4 already indicates that countries are likely to compete toward \( L = 0 \) but combined welfare would be higher if they choose positive leverage limits.
role of global intermediary.

4 Coordinated and Uncoordinated Policies

In this section we let countries choose policies to maximize welfare in each country. To isolate the strategic component of leverage decisions, we first consider a symmetric economy where countries are permitted to use fixed leverage limits. Appendix D presents an extension to countercyclical constraints with similar results. In Section 4.2 we then consider an asymmetric economy in which one country has higher volatility than the other. In this global economy, the low-volatility economy (AE) will bear more risk in stochastic steady state, intermediating risk on behalf of the high-volatility EME. This case allows us to discuss the welfare consequences of policy coordination between advanced and emerging economies.

Before diving into the results, we clarify the policy game being played by countries. There are two players, countries A and B (which are the AE and EME, respectively, in the asymmetric case). Before time $t = 0$, each country chooses and commits to a fixed leverage constraint. They
maximize their expected lifetime utility subject to constraints stated in Section 2, taking as given the other country’s policy choice and the initial state of the economy. We solve the uncoordinated Nash equilibrium and the coordinated equilibrium numerically, and our results for the parameters we chose suggest that the equilibria we found are unique (i.e., there is a single equilibrium solving the game).

We compute expected utility for both the symmetric and asymmetric economies using two different welfare measures. First, we suppose that countries know the initial state of the economy, and we set it to be \( \eta = 0.5 \). For example, if \( V^A(\eta) \) is country A’s value function, then A chooses its leverage constraint \( L^A \) to maximize \( V^A(0.5) \), taking as given B’s policy \( L^B \). Second, we take an ex-ante perspective and suppose that countries do not know the initial state of the economy. Instead, countries assume that the initial state of the economy is drawn from its stationary distribution. Countries then maximize their expected lifetime utility with respect to the stationary distribution. To be explicit, with a slight abuse of notation, this expectation is

\[
E \left[ \int_0^\infty e^{-\rho t} \log(c_t) \, dt \middle| \eta \right] = E \left[ E \left[ \int_0^\infty e^{-\rho t} \log(c_t) \, dt \middle| \eta \right] \right] = E \left[ V^A(\eta) \right].
\]

4.1 Symmetric Economies

We solve for the optimal set of policies using two welfare measures described above. Since countries are symmetric, when computing the ex-ante measure, we maximize \( E[V^{A+B}(\eta)] \) where the expectation is calculated using the equilibrium stationary distribution given policy constraints. We compute the optimal coordinated and uncoordinated policy choices given this ex-ante objective function. Table 1 presents the results using the initial condition and compares to the welfare in

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24 If countries do not know the initial location of the economy, then they need to adopt a belief about it. In the long run, the probability that \( \eta \) takes any given value in \([0, 1]\) is described by the stationary distribution. Since countries have no other information, they choose the stationary distribution as their belief.

25 When setting up the model, we did not explicitly indicate in equation (6), the statement of agents’ preferences, that the expectation was only with respect to the filtration induced by the 2-dimensional Brownian motion \((dZ^A_t, dZ^B_t)\). To be fully precise, country A’s value function \( V^A \) is actually its expected lifetime utility conditional on the economy’s initial location \( \eta_0 \).

26 To calculate the Nash equilibrium, we iterate best responses for A, starting at multiple initial levels for B. Since the best responses iterate to the same level regardless of the starting value for B, we are confident that the equilibrium is unique for these parameters. Indeed, it appears that for these parameters (and a neighborhood around them), that \( L = 0 \) is a dominant strategy and so clearly a Nash equilibrium.
competitive equilibrium (no constraints), and Table 2 presents the results using expected welfare.

Table 1: Coordinated and uncoordinated leverage limits given symmetric initial wealth shares.

<table>
<thead>
<tr>
<th></th>
<th>(L_A)</th>
<th>(L_B)</th>
<th>(V^{A+B}(0.5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social optimum</td>
<td>16.3%</td>
<td>16.3%</td>
<td>-44.86</td>
</tr>
<tr>
<td>Nash</td>
<td>0</td>
<td>0</td>
<td>-45.035</td>
</tr>
<tr>
<td>Competitive equilibrium</td>
<td>–</td>
<td>–</td>
<td>-44.88</td>
</tr>
</tbody>
</table>

Table 2: Coordinated and uncoordinated fixed leverage limits to maximize ex-ante welfare.

<table>
<thead>
<tr>
<th></th>
<th>(L_A)</th>
<th>(L_B)</th>
<th>(\mathbb{E}[V^{A+B}(\eta)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social optimum</td>
<td>6.7%</td>
<td>6.7%</td>
<td>-45.22</td>
</tr>
<tr>
<td>Nash</td>
<td>0</td>
<td>0</td>
<td>-45.35</td>
</tr>
<tr>
<td>Competitive equilibrium</td>
<td>–</td>
<td>–</td>
<td>-47.96</td>
</tr>
</tbody>
</table>

In both cases, the Nash equilibrium is to completely close capital accounts, while the social optimum is to allow a small amount of leverage/capital inflows. Figure 5 plots welfare in \(A\) with initial condition \(\eta = 0.5\), varying \(L_A\) for a set of \(L_B\). What is clear is that for each \(L_B\), welfare in \(A\) is maximized by \(L_A = 0\). Since we explicitly consider \(L_B = 0\), by symmetry it is clear that \(L_A = L_B = 0\) is a Nash equilibrium.\(^{27}\) (Figure 5 also makes clear that, as an example, \(L_A = L_B = 0.1\) would Pareto dominate the Nash equilibrium, if countries could commit to looser regulations.) Interestingly, when welfare is measured at the initial condition \(\eta = 0.5\), the Nash equilibrium outcome is worse than the laissez faire competitive equilibrium, and the consumption-equivalent loss is 0.31% for each country. In contrast, welfare according to the ex-ante measure is better with regulation (coordinated or not) because stability improves. In this case, the consumption-equivalent loss from lack of coordination is about 0.25% for each country.

Our main result—that uncoordinated constraints are tighter than coordinated constraints—holds broadly across parameters. For all parameters we’ve considered the Nash equilibrium is

\(^{27}\)We only present three choices for \(L_B\), but the result holds for a wide range of \(L_B\) in this range of leverage limits. The plot using the ex-ante welfare measure is qualitatively similar (zero is a dominant strategy).
zero. In this case with symmetric countries, the welfare gains from coordination are not large. With asymmetric countries, the aggregate welfare gains from coordination will be similarly negligible; however, the benefits and cost to each country will be quite distinct, with potentially large welfare transfers resulting from lack of coordination.

While macroprudential regulation in our model is highly stylized, the key insight should apply across a wide range of potential environments. Although our model lacks any distinction between the effects of limits on leverage or capital inflows, the key mechanism of our model is that, when international credit markets are imperfect, a policy that stabilizes country A will lead country A to be relatively richer more frequently. This is a negative spillover to country B, which increases

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28 For very low values of $a$ the optimal coordinated policy is also to completely close capital accounts, consistent with the result in Brunnermeier and Sannikov (2015), and so the Nash and coordinated equilibria correspond in those cases. This is consistent with the interpretation that, the more significant is the productivity gain from specialization, the more important is the terms of trade hedge. Accordingly, for $a = 0.9$, the socially optimal leverage limit is slightly higher. Furthermore, the social optimum appears to be monotonic in risk ($\sigma$), with higher $\sigma$ leading to looser constraints. In this case, higher risk means that crises are more likely (larger shocks) and so looser constraints alleviate the costs of crises. For very high levels of risk ($\sigma = 10\%$), it appears that multiple Nash equilibria are possible: one with closed capital accounts (leverage is zero) and one with high leverage almost at laissez-faire.

29 While in our model leverage limits and controls on capital inflows are identical, in reality these instruments can
the incentive for $B$ to enact stabilizing policies. While a richer setup is likely to generate positive spillovers through other sets of pecuniary externalities, we expect that the negative externality present in our model would continue to be present. Thus, while the quantitative importance of our mechanism will depend on the full set of global interactions, on the margin the mechanism we highlight will lead to tighter uncoordinated policies relative to global coordination.

4.2 Advanced and Emerging Economies: Asymmetric Volatilities

We now relax the assumption that countries are symmetric and focus on different degrees of risk as the key distinction between economies. We suppose that Country $A$, the AE, has lower aggregate risk than Country $B$, the EME ($\sigma^A < \sigma^B$). With lower aggregate risk, agents in Country $A$ are willing to invest more in capital for a given level of wealth (they are exposed to less risk), which implies that at the stochastic steady state Country $A$ will borrow from Country $B$. Thus, the AE acts as an intermediary, borrowing using safe debt and investing in risky capital. We consider the values $\sigma^B = 4\%, 5\%$, and $6\%$. All our qualitative results go through for all of these parameters, though the quantitative results of course differ. When $\sigma^B$ is larger, the benefit of AE intermediation for the EM is greater.

As discussed, a key feature of the global economy is that AEs, particularly the U.S., act as intermediaries to EMEs. In other words, there are so-called “global imbalances” as capital flows “upstream” to more developed countries. A robust literature addresses how differences in financial sectors across countries can explain these flows (see Gourinchas and Jeanne (2013) for empirical evidence). Previous papers in the theoretical literature have emphasized how upstream capital flows can be driven by differing abilities to produce financial assets (Caballero et al., 2008; Fostel et al., 2017), to insure aggregate risk (Willen, 2004) or idiosyncratic risk (Mendoza et al., 2009; Angeletos and Panousi, 2011; Phelan and Toda, 2018), or for intermediaries to fund themselves have very different roles. Korinek and Sandri (2016) quantitatively find the optimal capital control and macroprudential regulation for emerging economies to mitigate contractionary exchange rate depreciations and reduce the amount and riskiness of financial liabilities. Nonetheless, the authors find it is optimal for emerging economies to employ both instruments in order to improve stability.

For example, Buch and Goldberg (2016) find that international spillovers vary across prudential instruments and are heterogeneous across banks.
While increasing aggregate risk is not identical to these mechanisms, modifying aggregate volatility delivers the differential risk-bearing mechanisms analyzed in the literature while maintaining the tractability of the model. Mechanically, the country with lower aggregate risk has greater risk-bearing capacity because it can invest in assets with less risk than agents in the high-risk economy can. When country A borrows to invest in capital, its risky investments will have lower volatility than if country B made the investment. While in reality the U.S. borrows with debt and invests in risky foreign projects (earning favorable returns), the model with asymmetric volatilities delivers a similar result with A acting as an intermediary better able to manage risk arising from (for example) greater financial depth.

Welfare Results  First, since countries are not symmetric, a social planner may choose asymmetric Pareto weights. Second, it is no longer the case that the stochastic steady state is \( \eta = 0.5 \) and so evaluating welfare with a symmetric initial condition need not be the obvious thing to do. We have solved for coordinated and uncoordinated policies using \( \eta = 0.5 \), and with initial conditions determined by the stationary distribution in the laissez faire equilibrium (i.e., the mean, median, or mode of this distribution). The coordinated values naturally vary with the initial condition chosen, whether the ex-ante measure is used, and the social planner’s Pareto weights. In any case, the result is the same: Nash policies are zero leverage (closed capital accounts), which are tighter than the coordinated level.

Figure 6 plots welfare pairs \((E[V^A], E[V^B])\) for each country when \( \sigma^B = 5\% \) in different regulation regimes (ex-ante welfare measure). The yellow dot is the welfare pair in Competitive equilibrium (no constraints), the black asterisk is the welfare pair in Nash equilibrium (zero leverage), and the blue line is the Pareto frontier (in the upper left corner the Pareto weight \( \alpha \) on country A is near zero, and \( \alpha \) increases toward 1 moving clockwise along the curve). The figures for \( \sigma^B = 4\% \) and \( \sigma^B = 6\% \) are similar and presented in Appendix C.

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31 As an example of a model with heterogeneous risk aversion, see Gärleanu and Panageas (2015), which considers the implications of heterogeneity for asset pricing.

32 Indeed, it appears that \( L = 0 \) is still generally a dominant strategy even in this asymmetric economy.

33 Similarly, the corresponding figures using \( V(0.5) \) as the welfare measure look similar, with the main quantitative...
Figure 6: $E[V^B]$ against $E[V^A]$ as the Pareto weight $\alpha$ on country $A$’s expected welfare varies from 0 to 1 with parameters $\sigma^A = 3\%$ and $\sigma^B = 5\%$. The values from the competitive and Nash equilibria are also plotted.

These results show the stark contrast in welfare relative to the competitive equilibrium when countries choose policies according to Nash: country $A$ benefits substantially while country $B$ is hurt. With weights approximately in the interval $[.35, .4]$, the policy chosen by a planner will Pareto dominate the allocation in competitive equilibrium. The Nash equilibrium is strictly inside the Pareto frontier, reflecting the potential gains from coordination. When the planner puts non-trivial weight on country $B$, then it is optimal (globally) for country $A$ to intermediate risk (i.e., to bear financial instability) for the sake of country $B$. But without coordination country $A$ is not willing to bear the financial risk that benefits the rest of the world.

In general, we observe that the social optimum involves looser leverage constraints in the low-volatility AE, and essentially zero leverage (completely closed accounts) in the high-volatility EME. This is sensible since the risk-bearing role of the AE is more important than it was in the symmetric economy. Since Country $A$ borrows at the stochastic steady state, thus behaving as a
difference being that the competitive equilibrium is much closer to the pareto frontier. The exact shapes of those figures depend on the initial condition chosen, and in particular on what side of the stationary distribution it is.
Figure 7: Optimal Leverage Varying the Pareto Weight on Country A, $\sigma^B = 5\%$.

global intermediary, the optimal leverage policy maintains some leverage for Country A to absorb global risk. Indeed, the higher is aggregate risk in Country B, the higher is the unconstrained leverage limit for Country A, reflecting a greater role for risk-bearing by country A.

Figure 7 plots the optimal leverage chosen by a global social planner as we vary the weight put on Country A. When all weight is put on the EME, the Social planner calls for very high leverage by the AE and zero leverage for the EME. This is sensible: the social planner desires to have the AE bear all risk (fundamental and systemic) and to provide intermediation to the EME. As more weight is put on the AE, the social planner imposes tighter leverage in the AE, thus protecting the AE from systemic risk. If the social planner cares only about the AE, then the EME will be called on to provide very negligible intermediation.\footnote{Results for Country B are smoothed to remove numerical noise.}

Tables 4–6 in Appendix C present the results when we maximize the sum of utilities, $\mathbb{E}[V^A + V^B]$, thus applying equal Pareto weights to each country. The critical result that emerges in the asymmetric analysis is that the high-volatility EME is most hurt by lack of policy coordination. In contrast, in Nash equilibrium the low-volatility AE benefits compared to coordination (with equal Pareto weights) and especially compared to laissez faire. In every case, the high-volatility country suffers in the Nash equilibrium relative to the optimum. For each $\sigma^B$, the coordinated optimal $L^B$ is essentially zero, while $L^A$ is positive. Hence, country A benefits from lack of coordination (choos-
ing tighter policy than the coordinated optimum) but country $B$ is hurt from lack of coordination (choosing essentially the same policy in each case). Furthermore, while in the symmetric economy countries strictly benefited from regulation (with or without coordination) under the ex-ante measure, country $B$ can be worse off with regulation than compared to laissez faire without any regulations.

In terms of consumption-equivalents, the consequences of regulation can be significant. Table 3 presents the consumption-equivalent gains and losses for each country when welfare is calculated using $\eta = 0.5$ as an initial condition. The damage done to the developing country is evident. Even with just a one percentage point asymmetry in fundamental volatility, country $B$ already suffers welfare losses of 1.59% in consumption equivalents. As the asymmetry grows, the losses grow nonlinearly and reach nearly 3.5% when $\sigma^B = 6\%$. While we do not take these welfare losses too seriously, they do highlight the severe impact these macro-level spillovers can have on advanced and emerging economies, even in the simple setting we consider.

Table 3: Consumption-equivalent changes using $V(0.5)$

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma^B = 4%$</th>
<th>$\sigma^B = 5%$</th>
<th>$\sigma^B = 6%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi^A$</td>
<td>0.30%</td>
<td>0.90%</td>
<td>1.86%</td>
</tr>
<tr>
<td>$\phi^B$</td>
<td>-1.59%</td>
<td>-2.20%</td>
<td>-3.48%</td>
</tr>
</tbody>
</table>

4.3 International Negotiations and Bargaining Power

Global recommendations for financial regulation reflect agreements reached as the outcome of (possibly complicated) negotiations among countries. Because countries can always revert to Nash policies and because domestic macroprudential policies can be chosen unilaterally, our results show that AEs (especially the U.S.) have strong bargaining power when negotiating international agreements. Therefore, even though policy coordination nominally should be no worse than non-cooperation, this asymmetry in bargaining power means that the benefits of coordination may

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35We present the consumption-equivalent changes for country $B$ under the ex-ante measure in Appendix C. The losses in that case are much larger because the deterioration in the stationary distribution (for country $B$) is more significant for calculating ex-ante welfare. Following earlier discussion, the consumption-equivalent gains depend on the welfare measure used, including the particular $\eta$ when the initial condition is considered.
accrue disproportionately to AEs. Crucially, the incentives for coordination differ between AEs and EMEs: AEs prefer the Nash outcome while EMEs are substantively hurt by it. This asymmetry is not present when countries are symmetric precisely because neither country makes *systematic* use of the other’s greater risk-bearing capacity. Instead, symmetric countries make *temporary* use of each other’s risk-bearing capacity resulting from temporary relative wealth fluctuations. Hence, in the symmetric case, no country has the incentive to use Nash equilibrium (mitigating systemic risk) into a bargaining tool. With forces driving upstream capital flows, the EME acquires an additional reason for its preference that the other country adopt systematically looser leverage constraints, leading the EME to demand intermediation by the AE relatively more than the AE demands intermediation by the EME.

To formalize this logic, we adapt the game-theoretic model of regulatory coordination in Drezner (2007). In this model, there are two countries who can either adhere to their own preferred regulatory standards or switch to the standards preferred by the other country. Coordination on the same regulatory standards yields benefits to both countries but with some cost of adjustment. When countries can have asymmetric market sizes, Drezner (2007) shows that the likelihood of coordination at a given country’s preferred standards is increasing in their market size because the benefits of coordination will accrue primarily to the country with smaller market size. The intuition is that coordination should reduce barriers to efficient economic interactions, but because of the asymmetry in market size, the magnitude of benefits from coordination will be relatively larger for the smaller country. Since there is a size asymmetry, the larger country can also apply coercion, such as economic sanctions, to further increase the likelihood of coordination.

Adapting this game to our model is straightforward by considering an asymmetry in financial development. The AE’s preferred financial regulatory standards are $L^A = 0$ and the EME does not use leverage constraints, while the EME’s preferred standards are $L^B = 0$ and the AE does not use leverage constraints. If both countries choose their preferred macroprudential policy, we obtain the Nash outcome $(L^A, L^B) = (0, 0)$ from the static game we modeled earlier. If both countries switch to the others’ preferred standards, then they both adopt open capital accounts, which is the laissez-faire equilibrium. The Nash outcome is coercive in the sense that the EME is disproportionately
harmed while the AE actually prefers it relative to the other three outcomes. Fixing the AE’s market size, the greater the asymmetry in financial development, the worse the Nash outcome is for the EME, increasing the likelihood that the two countries coordinate on the AE’s preferred regulatory standards.

While our model is highly stylized, these results should still hold in some form across a wide range of potential environments. We have abstracted away from channels for positive macro-level spillovers, and richer set-ups would likely incorporate them. However, the key insight of our model is that a stabilizing policy in one country means that country becomes relatively richer more frequently, and we expect this mechanism, on the margin, to remain in models with positive externalities. A more realistic Nash outcome would likely involve tighter leverage constraints relative to the coordinated equilibrium, but constraints would perhaps be positive rather than zero. Since the EME would still want the AE to provide financial intermediation, the Nash outcome will remain a coercive option for the AE.

Our results also corroborate and supplement commentary about the ways in which AEs both control and benefit the most from recent international financial regulations. Indeed, there is some concern that AEs design global macroprudential policies that are not well-suited, at a micro level, for EMEs.\textsuperscript{36} When creating international regulatory standards in response to the financial crises of the 1990s, developed countries either constructed new institutions like the Financial Stability Forum rather than use established organizations like the IMF or vested new powers in organizations with predominantly AE membership.\textsuperscript{37} The standards written by these bodies nevertheless became de facto international standards for EMEs since AEs linked them to participation in international institutions and to market mechanisms.\textsuperscript{38}

\textsuperscript{36}In the popular press, Taylor (2010) writing in the Financial Times emphasizes that Basel III regulations are too complex for emerging markets to implement well (e.g. risk management offices are just not sophisticated enough).

\textsuperscript{37}Despite AEs having a strong presence on the IMF Executive Board, it is precedent that decisions are not made unless consensus is achieved, so they would have to consider the concerns of EMEs when writing new international financial regulations. Through the creation of new institutions and the use of existing ones with membership dominated by their own representatives—whether this was the desired intention or not—AEs could write regulations based primarily on their own preferences. For more details, see chapter 5 in Drezner (2007).

\textsuperscript{38}Access to resources from organizations like the IMF began to require compliance with financial regulatory standards written by AE-dominated organizations like the Financial Stability Forum, so even though developing countries did not participate in the creation of those standards, they nevertheless had to follow them to acquire needed resources. Beck et al. (2018) finds evidence that EMEs, when seeking advice from the IMF and World Bank, have been advised
Notably, EMEs were included in Basel discussions only after the Financial Crisis, at which point both Basel I and Basel II had already become standards for regulation of internationally active banks. Walter (2015) observes this narrow membership led to a focus during Basel III on regulations most relevant to banks residing in developed countries. Shimizu (2013) corroborates this claim, asserting that Basel standards center around issues relevant to financial institutions in the U.S. and U.K. but not to Asian countries, where commercial banks supply the bulk of funds. Some of the regulations, like the more stringent capital adequacy regulation, may impair economic growth in Asian economies without substantially improving financial stability because of the differences in the structure of their financial systems. The implementation of Basel III’s leverage ratio may similarly penalize EMEs because it may render traditional financial products like trade finance more costly than holding off-balance-sheet items like derivatives.

While Basel III focused on many rules-based regulations, EMEs tend to favor more discretionary regulations because their financial systems are different from both each others’ and the financial systems of AEs. Discretion is needed to adapt the regulations to the particulars of each or encouraged to adopt parts of Basel II and III, despite these standards not being official requirements. Markets also reinforced the need to comply with these regulations. Failure to demonstrate compliance would generate negative signals in financial markets. To facilitate expansion by both EME banks into foreign markets and AE banks into EME markets, many EMEs adopted Basel II and Basell III because they are de facto international regulatory standards, as Beck et al. (2018) notes. See chapter 5 in Drezner (2007) for more exposition on international financial regulations in the 1990s.

It has been argued that some of the support for the international regulations proposed in the 1990s stemmed from a desire to limit the growing presence of Japanese banks. Whether these claims hold water or not, Shimizu (2013) provides evidence that the introduction of these regulatory standards sharply contracted the supply and weakened the demand for credit in Japan, contributing to the country’s “lost decade.” Substantial portions of Shimizu (2013) are devoted to criticisms of Basel’s capital adequacy regulation as ineffective for addressing systemic risk and as a major cause of Japan’s credit crunch during the 1990s.

Most banks in Asian EMEs focus on supplying traditional financial products and especially trade finance. For example, Watanagase (2013) reports that at the time of the Financial Crisis, only 0.1 percent of total assets held by the Thai banking sector were invested in products related to collateralized debt obligations. On the margin, the comparatively higher cost of traditional products may also incentivize financial institutions to invest more of their portfolio in sophisticated and riskier products.

Shimizu (2013) pushes Basel III to allow flexibility in the national implementation of its standards because rules targeting the causes of financial crises in Western AEs like the U.S. may not be effective in Asian economies and could constrict the flow of funds needed for growth. Other central banks, notably the Reserve Bank of India, have also seen success using dynamic sector risk-weight adjustment, where regulators use their discretion to identify the systemic risk of different asset classes and re-weight their supervision accordingly (see Acharya (2013)). Watanagase (2013) advocates discretionary risk-based supervision as an essential component of financial regulations because exceedingly rules-based regulations may not adapt to new technologies and changing sources of systemic risk. Knowledge of the linkages between the financial sector and the real economy is also limited, so regulatory rules may not be complete in their scope, making discretion a necessary dimension for policymakers.
country’s financial system. Basel III and unilateral regulatory reforms in response to the financial crisis (e.g., ring-fencing in the UK and the Volcker rule in the US) may also reduce the volume of cross-border transactions with EME banks due to higher costs, adversely impacting EMEs.\footnote{Alford (2013) worries that, because of additional supervision, systemically important financial institutions (SIFI) may reduce their investment in emerging markets. Beck (2018) discusses some of the channels through which fund flows may be reduced.}

Finally, Moody’s (2013) concluded that Basel III does little to improve GDP in developing countries (i.e., the negative effects of reduced banking activity exceed the potential gains from reduced probability of crisis).\footnote{The two primary reasons for this finding are that developing countries have smaller banking systems and most EMEs have higher average levels of bank capitalizations than Basel III’s minimum requirement. The first reason leads to smaller losses in a financial crisis, and the second mitigates the actual impact Basel III has on bank activity, resulting in limited benefits from more stringent financial regulations.}

Several dimensions of our model can be further extended to provide greater insight as to how global macroprudential regulation should be conducted. Our analysis ignores any important heterogeneity within countries. Indeed, Buch and Goldberg (2016) find that heterogeneity among banks lead to different responses to cross-border spillovers. Thus, a homogenous constraint may not be the best choice, and further research should be conducted to determine how agent heterogeneity influences the optimal constraint.\footnote{To capture some of the additional mechanisms that could be present in a richer model, we extended our analysis to suppose that the capital growth rate is an increasing function of the capital price, $g(q)$. Specifically, we suppose that if $q$ falls below a threshold $\bar{q}$, adverse selection in capital production leads the growth rate to drop. Our results (not presented) are qualitatively identical in this environment and generally quantitatively more significant in the sense that the optimal leverage is higher and thus even greater than the Nash level.}

Second, including richer heterogeneity across countries (besides asymmetric volatilities), and particularly across financial sectors, would likely provide additional forces for strategic interactions between countries.

5 Conclusion

Emerging market economies depend on “center” countries in the global financial system to provide intermediation and risk-bearing, but to do so requires that AEs bear systemic risk. We have shown that policy coordination may not necessarily improve outcomes for all countries unless financially developed countries agree to bear nontrivial levels of systemic risk. When international credit markets are imperfect, more stable countries are more likely to be relatively wealthy compared to less
stable countries. Macroprudential regulation, when effective, increases economic stability, creating a macro-level spillover through international capital flows. As a result, tight macroprudential policy in one country provides strategic incentives for tight policy in the other. With asymmetries in financial development, developing economies suffer from these strategic incentives while advanced economies benefit because intermediating fundamental risk requires bearing systemic risk, and advanced economies may not receive sufficient returns to compensate.

References


——— (2010): “Adjusting to capital account liberalization,”.

BAIR, S. (2012): Bulls by the Horn: Fighting to Save Main Street from Wall Street and Wall Street from Itself, Free Press.


Appendices for Online Publication

A Proofs and Equations

A.1 Additional Equations

In equilibrium, when leverage constraints do not bind, returns on production must satisfy

\[
\mathbb{E}[dr_t^{Aa}] - dr_t^F = \frac{\psi_t^A}{\eta_t} (\sigma_t^A + \sigma_t^{qa})^2 + \frac{\psi_t^A}{\eta_t} (\sigma_t^{QB})^2, \\
\mathbb{E}[dr_t^{Ab}] - dr_t^F \leq \frac{\psi_t^A}{\eta_t} (\sigma_t^A + \sigma_t^{qa})^2 + \frac{\psi_t^A}{\eta_t} (\sigma_t^{QB})^2, \\
\mathbb{E}[dr_t^{Ba}] - dr_t^F \leq \frac{\psi_t^B}{1-\eta_t} (\sigma_t^{QA})^2 + \frac{\psi_t^B}{1-\eta_t} (\sigma_t^B + \sigma_t^{QB})^2, \\
\mathbb{E}[dr_t^{Bb}] - dr_t^F = \frac{\psi_t^B}{1-\eta_t} (\sigma_t^{QA})^2 + \frac{\psi_t^B}{1-\eta_t} (\sigma_t^B + \sigma_t^{QB})^2,
\]

where the term on the right hand side can be interpreted as the risk premium that agents must earn in order to invest in the production of good $a$ or $b$.\footnote{See Brunnermeier and Sannikov (2015) for technical details.} Notice that the excess return on the
production of the disadvantaged good is less than or equal to the risk premium, which reflects that in equilibrium there exist ranges of $\eta_t$ where country $A$ produces only good $a$ and country $B$ produces only good $b$.

Equations from Proposition 2 When country $A$’s leverage constraint binds, $\psi^A_t/\eta_t = 1 + L^A$, and $\eta_t$ follows

$$
\frac{d\eta_t}{\eta_t} = \left[ \frac{\psi^A_t}{\eta_t} \left( \frac{\bar{a}(P_t^a - P_t^b)}{q_t} \right) + \left( \frac{aP_t^b}{q_t} \right) - \rho + \left( 1 + \frac{L^A}{1 - \eta_t} \right) \left( (\sigma_t^{qA})^2 + (\sigma_t^{qB})^2 \right) \right] \, dt 
+ (\psi^A_t)^2((\sigma^A)^2 + (\sigma^B)^2) - \left( \frac{\psi^A_t}{\eta_t} \right) \left[ (\sigma^A + \sigma_t^{qA})\sigma_t^{qA} + (\sigma^B + \sigma_t^{qB})\sigma_t^{qB} \right] \, dt
+ \left[ \left( \frac{\psi^A_t}{\eta_t} - \psi^A_t \right) \left( \sigma^A \sigma_t^{qA} - \sigma^B \sigma_t^{qB} \right) + \psi^A_t(1 + L^A)(\sigma^B \sigma_t^{qB} - (\sigma^A + \sigma_t^{qA})) \right] \, dt
+ \left[ 2\psi^A_t(\sigma^A \sigma_t^{qA} - \sigma^B(\sigma^B + \sigma_t^{qB})) - \psi^A_t \frac{L^A}{1 - \eta_t} ((\sigma_t^{qA})^2 + (\sigma_t^{qB})^2) \right] \, dt
+ \left[ (1 - \eta_t)(1 + L^A)\sigma^A + L^A \sigma_t^{qA} \right] \, dZ_t^A + \left[ L^A \sigma_t^{qB} - (1 - \eta_t(1 + L^A))\sigma^B \right] \, dZ_t^B,
$$

(23)

and $q_t$ evolves according to

$$
\frac{dq_t}{q_t} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t \mu^A_t \eta_t dt + \frac{q'(\eta_t)}{q(\eta_t)} \eta_t ((1 - \eta_t)(1 + L^A)\sigma^A + L^A \sigma_t^{qA}) dZ_t^A
+ \frac{q'(\eta_t)}{q(\eta_t)} \eta_t (L^A \sigma_t^{qB} - (1 - \eta_t(1 + L^A))\sigma^B) dZ_t^B
+ \frac{1}{2} \frac{q''(\eta_t)}{q(\eta_t)} \eta_t^2 (((1 - \eta_t)(1 + L^A)\sigma^A + L^A \sigma_t^{qA})^2 + (L^A \sigma_t^{qB} - (1 - \eta_t(1 + L^A))\sigma^B)^2) dt,
$$

(24)

where the volatilities $\sigma_t^{qA}$ and $\sigma_t^{qB}$ can be explicitly written as

$$
\sigma_t^{qA} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t (1 - \eta_t)(1 + L^A)\sigma^A \frac{1 - q'(\eta_t)}{q(\eta_t)} \eta_t L^A, \quad \sigma_t^{qB} = -\frac{q'(\eta_t)}{q(\eta_t)} \eta_t (1 - \eta_t(1 + L^A))\sigma^B \frac{1 - q'(\eta_t)}{q(\eta_t)} \eta_t L^A.
$$

(25)
A.2 Proof of Proposition 2: Law of Motion for \( \eta_t \).

To write out the law of motion for \( \eta_t \), we first find how \( N_t \) and \( 1/(q_t K_t) \) evolve, and then we use Ito’s Product Rule to find \( d\eta_t/\eta_t \). The net worth of country A evolves according to

\[
\frac{dN_t}{N_t} = x^A_A d\tau^A_t + x^A_B d\tau^B_t + (1 - x^A_A - x^A_B) \nu_t - \rho dt.
\]

Since we know the laws of motion for \( q_t \) and \( K_t \), Ito’s Product Rule implies that

\[
\frac{d(q_t K_t)}{q_t K_t} = (\mu^q_t + \mu^K_t + \psi^A_t \sigma^A_t q^A_t + \psi^B_t \sigma^B_t q^B_t) dt + (\psi^A_t \sigma^A_t + \sigma^A_t q^A_t) dz^A_t + (\psi^B_t \sigma^B_t + \sigma^B_t q^B_t) dz^B_t.
\]

Ito’s Quotient Rule then implies that

\[
\frac{d(1/q_t K_t)}{1/q_t K_t} = \left( \frac{d(q_t K_t)}{q_t K_t} \right)^2 - \frac{d(q_t K_t)}{q_t K_t} \]

\[
= \left[ (\psi^A_t \sigma^A_t + \sigma^A_t q^A_t)^2 + (\psi^B_t \sigma^B_t + \sigma^B_t q^B_t)^2 - \mu^q_t - \mu^K_t - \psi^A_t \sigma^A_t \sigma^A_t q^A_t \right. \\
- \psi^B_t \sigma^B_t \sigma^B_t q^B_t ] dt - (\psi^A_t \sigma^A_t + \sigma^A_t q^A_t) dz^A_t - (\psi^B_t \sigma^B_t + \sigma^B_t q^B_t) dz^B_t.
\]

Applying Ito’s product rule then yields the general form for the evolution of \( \eta_t \):

\[
\frac{d\eta_t}{\eta_t} = \frac{dN_t}{N_t} + \frac{d(1/q_t K_t)}{1/q_t K_t} + \text{Cov} \left[ \frac{d(1/q_t K_t)}{1/q_t K_t}, \frac{dN_t}{N_t} \right].
\]

We now consider when \( \psi^A_t/\eta_t \geq L^A + 1 \). First, the leverage constraint can only bind when \( \eta_t \in [0, 0.5] \), \( \psi^{Ab}_t = 0 \), which allows us to write country A’s portfolio choice as \( (1 + L^A, 0, -L^A) \), where \( \psi^A_t = \psi^{Aa}_t = \eta_t (1 + L^A) \). Substitution of rates of return yields after cancelling and rearranging
\[
\frac{d\eta_t}{\eta_t} = \{ (1 + L^A) \left( \frac{dF_t}{q_t} \right) + L^A (\mu^A_t + \mu^K_t) + (1 + L^A)(1 - \eta_t)(\sigma^A \sigma^q_t) - \rho - L^A dF_t \\
+ (\eta_t(1 + L^A) \sigma^A + \sigma^q_t)^2 + ((1 - \eta_t(1 + L^A)) \sigma^B + \sigma^q_t)^2 - (1 - \eta_t(1 + L^A)) \sigma^B \sigma^q_t \\
- (1 + L^A)(\sigma^A + \sigma^q_t)(\eta_t(1 + L^A) \sigma^A + \sigma^q_t) - (1 + L^A) \sigma_t^q ((1 - \eta_t(1 + L^A)) \sigma^B + \sigma^q_t) \} dt \\
+ \left((1 - \eta_t)(1 + L^A) \sigma^A + L^A \sigma^q_t \right) dZ^A_t + (L^A \sigma_t^q - (1 - \eta_t(1 + L^A)) \sigma^B \sigma^q_t) \right) dZ^B_t
\]

Because the leverage constraint binds,
\[
\mathbb{E}[dF^A_t] - dF_t > \frac{\psi_t^A}{\eta_t} (\sigma^A + \sigma^q_t)^2 + \frac{\psi_t^A}{\eta_t} (\sigma^q_t)^2,
\]
so we cannot use the same procedure outlined in the appendix of Brunnermeier and Sannikov (2015). Instead, we note that country B must produce good b, implying that
\[
dF_t^F \leq \mathbb{E}[dF^B_b] - \frac{\psi_t^B}{1 - \eta_t} (\sigma^q_t)^2 - \frac{\psi_t^B}{1 - \eta_t} (\sigma^B + \sigma^q_t)^2.
\]
However, the inequality cannot be strict because that would imply that country B would prefer to produce more of good b, so B would not be optimizing its portfolio choice. In addition, given an \(\eta_t\) where country A wants to take on more leverage but cannot due to the constraint, country B must reduce its optimal investment in risk-free bonds without leverage constraints and either invest that extra capital in producing good a or lower the risk-free rate so that excess returns from producing
good $b$ rise. Therefore, the above equation holds with equality and substituting it in yields

$$
\frac{d\eta_t}{\eta_t} = \left[ (1 + L^A) \left( \frac{\tilde{a}P^a_t}{q_t^A} \right) - L^A \left( \frac{\tilde{a}P^b_t}{q_t^B} \right) + (1 + L^A)(1 - \eta_t)(\sigma^A_\eta^q + \rho) \right] dt \\
- \left[ L^A(\sigma^B_\eta^q - L^A \left( \frac{1 - \eta_t(1 + L^A)}{1 - \eta_t} \right) (\sigma^B + \sigma^q\eta^B)^2 - (1 - \eta_t(1 + L^A))\sigma^B_\eta^q \right] dt \\
+ \left[ (\eta_t(1 + L^A)\sigma^A + \sigma^q_\eta^A)^2 + ((1 - \eta_t(1 + L^A))\sigma^B + \sigma^q_\eta^B)^2 - (1 - \eta_t(1 + L^A))\sigma^B_\eta^q \right] dt \\
- (1 + L^A)(\sigma^A + \sigma^q_\eta^A)(\eta_t(1 + L^A)\sigma^A + \sigma^q_\eta^A) dt \\
- (1 + L^A)\sigma^q_\eta^B ((1 - \eta_t(1 + L^A))\sigma^B + \sigma^q_\eta^B) dt \\
+ ((1 - \eta_t)(1 + L^A)\sigma^A + L^A\sigma^q_\eta^A) dZ_t^A + (L^A_\eta^q - (1 - \eta_t(1 + L^A))\sigma^B) dZ_t^B,
$$

Similarly, when $\psi_t^B / (1 - \eta_t) \geq L^B + 1$, the leverage constraint will only bind for $\eta_t \in [0.5, 1]$, which allows us to write country $B$’s portfolio choice as $(0, 1 + L^B, -L^B)$, where $\psi_t^B = \psi_t^{Bb} = (1 - \eta_t)(1 + L^B)$. This partly pins down country $A$’s portfolio choice, requiring that $\psi_t^A = \eta_t(1 + L^B) - L^B$ and thus $\psi_t^A / \eta_t = 1 + L^B - L^B / \eta_t$. Since the portfolio shares must sum to one, we have that country $A$ loans out $1 - \psi_t^A / \eta_t = L^B (1/ \eta_t - 1)$ Therefore, after substituting in returns and cancelling terms,

$$
\frac{d\eta_t}{\eta_t} = \left\{ \psi_t^{Aa} \left( \frac{\tilde{a}P^a_t}{q_t^A} + \eta_t^q + \eta_t^K + \sigma^A_\eta^q \right) + \frac{\psi_t^{Ab}}{\eta_t} \left( \frac{\tilde{a}P^b_t}{q_t^B} + \eta_t^q + \eta_t^K + \sigma^A_\eta^q \right) - \rho \right\} dt \\
+ L^B \left( \frac{1}{\eta_t} - 1 \right) dr_t^F + (\psi_t^A_\sigma^A + \sigma^q_\eta^A)^2 + (\psi_t^B_\sigma^B + \sigma^q_\eta^B)^2 - \mu_t^q - \mu_t^K - \psi_t^A_\sigma^A_\eta^q - \psi_t^B_\sigma^B_\eta^q \\
- \left[ (1 + L^B) - \frac{L^B}{\eta_t} \right] (\sigma^A + \sigma^q_\eta^A)(\psi_t^A_\sigma^A + \sigma^q_\eta^A) - \left[ (1 + L^B) - \frac{L^B}{\eta_t} \right] \sigma^q_\eta^B(\psi_t^B_\sigma^B + \sigma^q_\eta^B) dt \\
+ \left[ (1 + L^B) - \frac{L^B}{\eta_t} \right] (\sigma^A + \sigma^q_\eta^A) - (\psi_t^A_\sigma^A + \sigma^q_\eta^A) dZ_t^A + \left[ (1 + L^B) - \frac{L^B}{\eta_t} \right] \sigma^q_\eta^B - (\psi_t^B_\sigma^B + \sigma^q_\eta^B) dZ_t^B
$$

For this range of $\eta_t$, it is the case that country $A$’s returns pin down the risk-free rate, so after substituting in

$$
dr_t^F = \mathbb{E}[dr_t^{Aa}] - \frac{\psi_t^A}{\eta_t} (\sigma^A + \sigma^q_\eta^A)^2 - \frac{\psi_t^A}{\eta_t} (\sigma^q_\eta^B)^2,
$$
the law of motion for $\eta_t$ becomes

$$
\frac{d\eta_t}{\eta_t} = \left(1 - \frac{\psi_t^{Ab}}{\eta_t}\right) \left(\frac{\partial P_t^a}{q_t}\right) + \frac{\psi_t^{Ab}}{\eta_t} \left(\frac{\partial P_t^b}{q_t}\right) - \rho + (1 - \psi_t^A)\sigma_t^{qA} \\
- \left(1 - \frac{\psi_t^{Ab}}{\eta_t}\right) + \left[(\sigma_t^A + \sigma_t^{qA})^2 + (\sigma_t^{qB})^2\right] + (\psi_t^A \sigma_t^A + \sigma_t^{qA})^2 + (\psi_t^B \sigma_t^B + \sigma_t^{qB})^2 - \psi_t^B \sigma_t^{B}\sigma_t^{qB} \\
- \left[(1 + L^B) - \frac{L^B}{\eta_t}\right] (\sigma_t^A + \sigma_t^{qA})(\psi_t^A \sigma_t^A + \sigma_t^{qA}) - \left[(1 + L^B) - \frac{L^B}{\eta_t}\right] \sigma_t^{qB}(\psi_t^B \sigma_t^B + \sigma_t^{qB}) dt \\
+ \left[(1 + L^B) - \frac{L^B}{\eta_t}\right] (\sigma_t^A + \sigma_t^{qA}) - (\psi_t^A \sigma_t^A + \sigma_t^{qA}) dZ_t^A \\
+ \left[(1 + L^B) - \frac{L^B}{\eta_t}\right] \sigma_t^{qB} - (\psi_t^B \sigma_t^B + \sigma_t^{qB}) dZ_t^B
$$

A.3 Proof of Proposition 2: Law of Motion for $q_t$.

Assume $q_t$ is a twice-continuously differentiable function of $\eta_t$. Then Ito’s Lemma implies that

$$
dq_t = q'(\eta_t) d\eta_t + \frac{1}{2} q''(\eta_t) (d\eta_t)^2
$$

Thus, if we divide both sides by $q_t$ and expand terms, $q_t$ evolves according to

$$
\frac{dq_t}{q_t} = \left[\frac{q'(\eta_t)}{q(\eta_t)} \eta_t \mu_t^\eta + \frac{1}{2} \frac{q''(\eta_t)}{q(\eta_t)} \eta_t ((\sigma_t^{\eta A})^2 + (\sigma_t^{\eta B})^2)\right] dt + \frac{q'(\eta_t)}{q(\eta_t)} \eta_t \sigma_t^{\eta A} dZ_t^A + \frac{q''(\eta_t)}{q(\eta_t)} \eta_t \sigma_t^{\eta B} dZ_t^B,
$$

where $\sigma_t^{\eta A}$ is the volatility from $dZ_t^A$ and $\sigma_t^{\eta B}$ is the volatility from $dZ_t^B$. Substituting in the drift and volatilities from Proposition 1 yields the laws of motions for $q_t$ in Proposition 2. Additionally, when the leverage constraint for country $A$ binds, we have that

$$
\sigma_t^{qA} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t \sigma_t^{\eta A} \Rightarrow \sigma_t^{qA} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t ((1 - \eta_t)(1 + L^A)\sigma_t^A + L^A \sigma_t^{qA}) \\
\sigma_t^{qB} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t \sigma_t^{\eta B} \Rightarrow \sigma_t^{qB} = \frac{q'(\eta_t)}{q(\eta_t)} \eta_t (L^B \sigma_t^{qB} - (1 - \eta_t(1 + L^B))\sigma_t^B)
$$

45
Rearranging yields the result. Similarly, when the leverage constraint for country $B$ binds,

$$\sigma^{qA} = \frac{q'(\eta)}{q(\eta)} \eta \left[ \left( \frac{\psi^A}{\eta} \right) \left( \sigma^A + \sigma^{qA} \right) - (\psi^A \sigma^A + \sigma^{qA}) \right]$$

$$= \frac{q'(\eta)}{q(\eta)} \eta \left[ \left( \frac{\psi^A}{\eta} - \psi^A \right) \sigma^A \right] + \frac{q'(\eta)}{q(\eta)} \eta \left( \frac{\psi^A}{\eta} - 1 \right) \sigma^{qA}$$

$$\sigma^{qB} = \frac{q'(\eta)}{q(\eta)} \eta \left[ \frac{\psi^A}{\eta} \sigma^{qB} - (\psi^B \sigma^B + \sigma^{qB}) \right]$$

$$= -\frac{q'(\eta)}{q(\eta)} \eta \psi^B \sigma^B + \frac{q'(\eta)}{q(\eta)} \eta \left( \frac{\psi^A}{\eta} - 1 \right) \sigma^{qB}$$

Rearranging and substitution yields a similar closed-form expression for $\sigma^{qA}$ and $\sigma^{qB}$ when constraints bind for country $B$.

### A.4 Proof of Proposition 2: Expressions for $q$ and $q'$

**Lemma 1.** Let $\tau = \bar{a}/\bar{a}$. Suppose that $\sigma^A$ and $\sigma^B$ are sufficiently close such that $\psi^{Ab} = 0$ whenever country $A$ is levered\(^{46}\). Under our other assumptions on the model, if leverage constraints for country $A$ bind, then equilibrium capital shares are characterized by

$$\psi^{Aa}(\eta) = \eta (1 + L_A)$$

$$\psi^{Ab}(\eta) = 0$$

$$\psi^{Ba}(\eta) = \begin{cases} 
\frac{1 - (1 + \tau)(1 + L_A)\eta}{2} & \text{if } \eta \leq \frac{1}{(1 + L_A)(1 + \tau)}, \\
0 & \text{otherwise.}
\end{cases}$$

$$\psi^{Bb}(\eta) = 1 - \psi^{Aa}(\eta) - \psi^{Ba}(\eta).$$

**Proof.** Suppose leverage constraints bind for country $A$. Then by definition of leverage constraints

\(^{46}\)While we never encountered a case where a large asymmetry between $\sigma^A$ and $\sigma^B$ led country $A$ to produce good $b$ even when it is levered, our results do assume this outcome.
and our hypothesis on $\psi^{Ab}$,

$$\frac{\psi^{Aa}}{\eta} = 1 + L_A \Rightarrow \psi^{Aa}(\eta) = \eta(1 + L_A).$$

The expression for $\psi^{Bb}(\eta)$ follows from market-clearing. To finish this section of the proof, the excess returns condition for country $B$ requires that $\psi^{Ba} > 0$ if and only if

$$\mathbb{E}[d\tau^{Bb}] = \mathbb{E}[d\tau^{Ba}],$$

which holds only when

$$\frac{p^{a}}{p^{b}} = \frac{\bar{a}}{a} = \tau.$$

Since the consumption good is the numeraire, we have that

$$\frac{Y^{b}}{Y^{a}} = \tau \Rightarrow \bar{a} \tau \psi^{Aa} + \bar{a} \psi^{Ba} = \bar{a} \psi^{Bb}.$$

Divide both sides by $\bar{a}$ and apply market-clearing for capital and the condition of a binding leverage constraint to attain

$$\tau \psi^{Aa} + \psi^{Ba} = 1 - \psi^{Ba} - \psi^{Aa} \Rightarrow \psi^{Ba}(\eta) = \frac{1 - (\tau + 1)\psi^{Aa}}{2} = \frac{1 - (\tau + 1)(1 + L_A)\eta}{2}.$$

This is positive when $\eta \leq (1 + L_A)^{-1}(1 + \tau)^{-1}$, and since we require capital shares to be nonnegative, $\psi^{Ba} = 0$ when this condition is not satisfied

We now proceed to prove the final statement of the proposition. Since agent preferences are identical and log utility, aggregate flow consumption will be proportional to aggregate wealth. By market-clearing for flow consumption,

$$\rho qK = Y = (Y^{a})^{1/2}(Y^{b})^{1/2} = \left(\bar{a} \psi^{Aa} K + a \psi^{Ba} K\right)^{1/2} \left(\bar{a} \psi^{Bb} K + a \psi^{Ab} K\right)^{1/2}.$$

Let $Y^{a} = \bar{a} \psi^{Aa} + a \psi^{Ba}$, $Y^{b} = \bar{a} \psi^{Bb} + a \psi^{Ab}$, and $Y = (Y^{a})^{1/2}(Y^{b})^{1/2}$. Divide both sides by $K$, and
we attain
\[ \rho q = \gamma. \]

To acquire \( q' \), note that \( \gamma^a \) and \( \gamma^b \) are functions of the capital shares \( \psi^A, \psi^B, \psi^{A'}, \) and \( \psi^{A''} \). Since we have closed-form expressions for capital shares as functions of \( \eta \), we can directly differentiate \( \gamma^a \) and \( \gamma^b \) with respect to \( \eta \). Thus, differentiating both sides with respect to \( \eta \) yields

\[
q' (\eta) = \frac{1}{\rho} \left[ \frac{1}{2} (\gamma^a)^{-1/2} (\gamma^a)' \left( \gamma^b \right)^{1/2} + \frac{1}{2} (\gamma^b)^{-1/2} (\gamma^b)' \left( \gamma^a \right)^{1/2} \right] \\
= \frac{(\gamma^a)^{1/2} (\gamma^b)^{1/2}}{2\rho} \left[ \frac{(\gamma^a)'}{\gamma^a} + \frac{(\gamma^b)'}{\gamma^b} \right] \\
= \frac{\gamma}{2\rho} \left[ \frac{(\gamma^a)'}{\gamma^a} + \frac{(\gamma^b)'}{\gamma^b} \right]
\]

Using Lemma 1, we can write \( \gamma^a, \gamma^b \), and their derivatives in terms of \( \eta, L_A \), and fundamental parameters. Simplification yields the final statement in Proposition 2.

**B Computational Algorithm**

Here, we provide a description of the algorithm used to compute dynamics and welfare.

**Unconstrained Case.** Using market-clearing for consumption to express the capital shares as functions of the capital price \( q \), the asset-pricing relationship in Proposition 2 from Brunnermeier-Sannikov (2015) form is an implicit differential equation with initial conditions \((q(0), q'(0))\).

1. We apply a small perturbation to \( \psi^A \) and \( \eta \) and apply market-clearing to retrieve \( q(0) \). We then estimate \( q'(0) \) using the equilibrium asset-pricing relationship.

2. Use Matlab’s `decic` function to compute consistent initial conditions while fixing \( \psi^A(0) = 0 \).

3. Using Matlab’s `ode15i`, calculate \((q, qp)\) from \( \eta \approx 0 \) to the level of \( \eta \) at which \( \psi^A = 0 \) and the capital price \( q \) is maximized. With these values, we can also compute \( \mu^\eta, \sigma^{\eta A}, \) and \( \sigma^{\eta B} \).
4. Repeat steps 1-3 but solving from $\eta \approx 1$ to the $\eta$ at which $q$ is maximized.

5. Estimate $H^A(0)$ and $H^A(1)$ numerically.\footnote{To test robustness of these numerical estimates, we solved the value function ODE using incorrect boundary conditions. We found that the behavior of the value function in the interior of the state space is not sensitive to the boundary conditions chosen, so our chosen welfare measures, $V(0.5)$ and $\mathbb{E}[V]$, should not be affected much by numerical errors in our estimates of boundary condition.}

6. Interpolate the capital price, capital shares, and drift and volatilities of $d\eta_t / \eta_t$ with \textit{interp1} to construct the second-order differential equation for $H(\eta)$. Solve the ODE using the boundary value problem solver \textit{bvp4c}.

7. Use the Kolmogorov Forward Equations to compute the stationary density. Numerically integrate to retrieve the CDF and $\mathbb{E}[V(\eta)]$.

**Binding Leverage Constraint.** Since we can explicitly write capital shares as functions of $\eta$ using the fact that the leverage constraint binds, we can acquire $q$ and $q'$ in closed form by market-clearing.

1. Determine for what range of $\eta$ the leverage constraint binds for countries $A$ and then $B$ using our computations from the unconstrained case. Using our closed-form expressions for $q$ and $q'$, explicitly compute capital shares and dynamics over those $\eta$ insert these values into the relevant matrix from the unconstrained case.

2. Use steps 5, 6, and 7 from the unconstrained case to compute welfare and the stationary density.

**Piecewise Leverage Constraint.** For clarity, assume $L^A_1 = L^B_1 = L_1$ and $L^A_2 = L^B_2 = L_2$.

1. Compute the constrained equilibrium twice, once using $L_1$ and again using $L_2$.

2. Consider the matrices from using $L_1$. Cut the matrices for dynamics, $\eta$, $q$, and $q'$ into two groups: values whose corresponding $\eta \leq \eta^A_*$ or $\eta \geq \eta^B_*$.

3. Consider the matrices from using $L_2$. Slice the matrices for those values whose $\eta \in (\eta^A_*, \eta^B_*)$. 
4. Join these matrices produced from these three sections. Proceed using steps 5, 6, and 7 from the unconstrained case.

**Solving for the Coordinated and Uncoordinated Equilibrium.** We assume here that $L_2 = 0$. We first consider the coordinated algorithm and then the uncoordinated one.

1. Create a function that takes as arguments $L_A^1$, $\eta_A^*$, $L_B^1$, and $\eta_B^*$ and gives as output the expectation of the value function. Multiply by $-1$ to turn the problem into a minimization problem.

2. Apply `fminsearch` to the function, minimizing over the vector $[L_I^1, \eta_I^*]$, as given symmetric $\sigma_A$ and $\sigma_B$, the coordinated equilibrium will have symmetric policies.

3. Set $L_B^1$ and $\eta_B^*$ to the coordinated values. Holding them fixed, now minimize the expectation of the value function with respect to $L_A^1$ and $\eta_A^*$ to determine country A’s best response. Change $L_B^1$ and $\eta_B^*$ to country A’s best response. Repeat until Nash equilibrium is found (i.e., policy responses converge to a fixed point).

4. If the coordinated equilibrium yields lower utility, repeat step 2 using a different initial value, as there may exist several local minima.

C Additional Figures and Tables

C.1 Pareto Frontiers and Welfare Changes for Asymmetric Economies

In the main text, we only reported the Pareto frontier for the case of $\sigma^A = 3\%$ and $\sigma^B = 5\%$. For robustness, we also computed the Pareto frontier in our two other cases with asymmetric volatility. Figure 8 shows these Pareto frontiers as well as the expected value functions from the competitive and Nash equilibria. Like the reported case, both equilibria are clearly within the frontier, confirming further that gains from coordination exist across a wide scope of parameters. The relative locations of the expected value functions for both equilibria vary between the plots,
and the movement appears to be nonlinear, but we have not conducted a thorough investigation. More likely than not, this phenomenon occurs due to the nonlinearities present in the model.

Tables 4–6 present the results when we maximize the sum of utilities, $E[V_A + V_B]$, thus applying equal Pareto weights to each country.

**Table 4: $\sigma^A = 3\%$, $\sigma^B = 4\%$**

<table>
<thead>
<tr>
<th>Case</th>
<th>$L^A$</th>
<th>$L^B$</th>
<th>$E[V_A]$</th>
<th>$E[V_B]$</th>
<th>$E[V_A + V_B]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>3.98%</td>
<td>0.02%</td>
<td>-23.105</td>
<td>-22.618</td>
<td>-45.723</td>
</tr>
<tr>
<td>Nash Equilibrium</td>
<td>0</td>
<td>0</td>
<td>-22.74</td>
<td>-23.08</td>
<td>-45.82</td>
</tr>
</tbody>
</table>

**Table 5: $\sigma^A = 3\%$, $\sigma^B = 5\%$**

<table>
<thead>
<tr>
<th>Case</th>
<th>$L^A$</th>
<th>$L^B$</th>
<th>$E[V_A]$</th>
<th>$E[V_B]$</th>
<th>$E[V_A + V_B]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>9.74%</td>
<td>.01%</td>
<td>-23.720</td>
<td>-22.568</td>
<td>-46.288</td>
</tr>
<tr>
<td>Nash Equilibrium</td>
<td>0</td>
<td>0</td>
<td>-22.83</td>
<td>-23.76</td>
<td>-46.59</td>
</tr>
<tr>
<td>Competitive Equilibrium</td>
<td>–</td>
<td>–</td>
<td>-29.05</td>
<td>-20.27</td>
<td>-49.32</td>
</tr>
</tbody>
</table>

In Table 7, we report the consumption-equivalent welfare losses of country $B$ in Nash equilibrium relative to competitive equilibrium, using the expected value function of country $B$ computed in Tables 4–6.
Table 6: $\sigma^A = 3\%$, $\sigma^B = 6\%$

<table>
<thead>
<tr>
<th>Case</th>
<th>$L^A$</th>
<th>$L^B$</th>
<th>$E[V^A]$</th>
<th>$E[V^B]$</th>
<th>$E[V^A + V^B]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>13.42%</td>
<td>.01%</td>
<td>-24.041</td>
<td>-23.065</td>
<td>-47.106</td>
</tr>
<tr>
<td>Nash Equilibrium</td>
<td>0</td>
<td>0</td>
<td>-22.826</td>
<td>-24.848</td>
<td>-47.674</td>
</tr>
</tbody>
</table>

Table 7: Consumption-equivalent losses for EME using ex-ante welfare measure

<table>
<thead>
<tr>
<th>Case</th>
<th>$\sigma^B = 4%$</th>
<th>$\sigma^B = 5%$</th>
<th>$\sigma^B = 6%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>5.84%</td>
<td>13.0%</td>
<td>23.9%</td>
</tr>
</tbody>
</table>

Welfare losses under the ex-ante measure are substantially larger. This is because the deterioration in the stationary distribution (from the perspective of country B) has greater consequences.

C.2 Symmetric Constraints in the Symmetric Economy

Here we show positive results for a symmetric economy with symmetric leverage constraints $L^A = L^B = L$.

Figure 9 displays the allocations of world capital between country A and B. Panel (a) plots capital held by country A and used for good a ($\psi^{Aa}$). When leverage constraints bind $\psi^{Aa}$ varies linearly by $L + 1$, but $\psi^{Aa}$ equals the unconstrained curve when constraints do not bind (i.e., country A’s optimal choice of $\psi^{Aa}$ can now be achieved even with the leverage constraint). The kinks on the right side of the state space correspond to when B’s leverage constraints bind. Panel (b) plots country A’s leverage over the state space. When the constraint binds, the leverage ratio is constant, as evidenced by the flat dashed lines for $\eta < 0.5$. When country A holds a majority of world wealth, its leverage ratio dips below 1, indicating that some share of its portfolio is now being lent out (capital outflows), and eventually the ratio returns to 1, reflecting that at $\eta = 1$ or $\eta = 0$, one country holds all capital. Results for B are symmetric.

Panel (c) illustrates more clearly how leverage constraints affect prices. When leverage constraints are tighter, A cannot hold as much capital and will produce less of good a. Since country A has a comparative advantage in producing a, the supply of a decreases much faster, resulting in a
sharper increase in the price ratio. Furthermore, the "terms of trade hedge" discussed in Brunnermeier and Sannikov (2015) is nullified much quicker, as country $B$ will shift into the production of good $a$ much faster due to country $A$’s reduced production, and this requires that $\frac{aP_t^a}{P_t^b} = \bar{a}/\bar{a}$.


Figure 10 plots the equilibrium drift and volatility terms for the state variable $\eta$. For $\eta \in (0, 0.5)$, $\mu^\eta > 0$ while for $\eta \in (0.5, 1)$, $\mu^\eta < 0.5$ with $\mu^\eta = 0$ at $\eta = 0.5$. As a result, $\eta = .5$ is an attracting basin. Symmetric leverage constraints do not change this property, but $\mu^\eta \cdot \eta$ peaks much sooner because the price of good $a$ relative to good $b$ increases up much faster as the economy moves away from the stochastic steady state. Consequently, dividend yields increase, causing investment in good $a$ to deliver a higher return for smaller deviations from $\eta = 0.5$ when leverage constraints are imposed, so $\eta$ returns to 0.5 at a much faster rate. On the other hand, leverage extends the range of $\eta$ for which the terms of trade hedge is effective. Comparing Panel (b) with the ratio of goods prices makes it clear that $\mu^\eta \cdot \eta$ peaks when country $B$ starts production of good $a$. The drift keeps going up precisely because the price ratio increased, but once $P_t^a/P_t^b = \bar{a}/\bar{a}$, the terms of trade hedge is nullified. Once this occurs, country $A$’s decreasing share of world capital has a dominant influence on $\mu^\eta \cdot \eta$ and pushes it down. This follows from the fact that country $A$’s net worth evolves according to

$$\frac{dN_t^A}{N_t} = \psi_t^{Aa}dr_t^{Aa} + \psi_t^{Ab}dr_t^{Ab} + (1 - \psi_t^A)dr_t^F - \rho dt. \tag{26}$$

Since $\psi_t^{Ab} = 0$ on $\eta \in [0, 0.5]$ and $\mathbb{E}[dr_t^{Aa}] > dr_t^F$, an increasingly smaller $\psi^{Aa}$, all else equal,
leads to a slower drift of $dN_t$ and thus of $d\eta_t$ as well. As a result, $\mu^n \cdot \eta$ eventually becomes larger in the unconstrained case than in the constrained cases when $\eta \in [0, 0.15]$.

Panel (b) plots the volatility of $d\eta_t$, which is obtained by computing the standard deviation of $d\eta_t/\eta_t$ and multiplying through by $\eta_t$. Explicitly, $\sigma^n = \sqrt{(\sigma^{nA})^2 + (\sigma^{nB})^2}$. Consistent with the results in Phelan (2016), the volatility of $d\eta_t$ is lower when leverage constraints are imposed. One can interpret this as evidence of lower systemic volatility because it suggests smaller changes in $\eta$ when the economy is hit with exogenous shocks. Equation (23) explains why this occurs. All else equal, as $L^A \rightarrow 0$, the impact of $\sigma^{qA}$ decreases, so $\sigma^{nA} \rightarrow (1 - \eta_t)\sigma^A$. Similarly, when $L^A$ approaches zero, it removes the effect of $\sigma^{qB}$ on $\sigma^{nB}$, and this downward pressure outweighs the increase in the size of $\sigma^B$’s coefficient.

D Extension: Countercyclical Constraints

For our main results, we considered the rather blunt policy of a single leverage constraint. Discussions of macroprudential policy, however, tend to focus on the usefulness of countercyclical policies. To reduce systemic risk, it may be advantageous to limit the expansion of credit, and in bad times, the severity of crises could be reduced by bolstering asset prices and encouraging
leverage. In light of these considerations, this section extends the model to allow countries more flexibility when choosing macroprudential policies.

To that end, we first considered symmetric, coordinated piecewise policies described by \((L_1, L_2, \eta_*)\). For \(\eta < \eta_*\) country A limits leverage to \(L_1\), and for \(\eta \in (\eta_*, .5)\) country A limits leverage to \(L_2\), and symmetrically for country B. A completely state-contingent policy is unrealistic to consider in practice, given issues of time-consistency and implementability (see [Klein, 2012]). (Additionally, solving a completely flexible state-contingent policy is too computationally difficult.) The simple, piecewise rule we consider is flexible enough to allow countries to choose pro-cyclical or countercyclical policies (or completely shutting off credit flows), and could realistically be approximated in practice.

Maximizing welfare in this way, countries optimally chose countercyclical policy. In particular, countries choose \(L_2 = 0\), completely limiting leverage (or capital inflows) when the economy is near the stochastic steady state, which is when capital is allocated more efficiently, but choose \(L_1 > 0\) when the economy is away from steady state and capital is misallocated. Furthermore, the leverage limit binds (leverage equals \(L_1\)) because there continue to be welfare gains from limiting the pecuniary externality.

Given these results, we solve for the optimal coordinated and uncoordinated policies restricted to piecewise countercyclical constraints. We suppose countries can adopt a piecewise leverage constraint described by the pair \((L^I_1, \eta^I_*)\), where \(I = A, B\). For the purpose of exposition, first consider \(I = A\). For \(\eta \leq \eta^A_*\), country A imposes \(L^A_1\) as its leverage constraint, and for \(\eta > \eta^A_*\), country A closes its capital inflows. Country B’s policy behaves similarly, except that when \(\eta < \eta^B_*\), country B closes its capital inflows, and when \(\eta \geq \eta^B_*\), B adopts leverage constraint \(L^B_1\). In an economy with symmetric capital volatilities, countries adopt \(L^I_1 > 0\) and \(\eta^I_* < .5\), which improves welfare in both countries relative to the unconstrained competitive equilibrium, indicating that countercyclical macroprudential policy can be effective. As before, we solve for the optimal policies (i) with the initial condition \(\eta = .5\) and (ii) with the ex-ante perspective using the equilibrium stationary distribution to calculate expected welfare.\(^{48}\) Table 8 presents the results using the initial condition \(\eta = .5\) and (ii) with the ex-ante perspective using the equilibrium stationary distribution to calculate expected welfare.\(^{48}\) We have not been able to rule out the possibility of multiple equilibria given the computational difficulty of choosing best responses in two variables.
tion and compares to the welfare in competitive equilibrium (no constraints). Table 9 presents the results using expected welfare.

Table 8: Optimal coordinated and uncoordinated policies given symmetric initial wealth shares.

<table>
<thead>
<tr>
<th></th>
<th>(L^A_s)</th>
<th>(L^B_s)</th>
<th>(\eta^A_s)</th>
<th>(\eta^B_s)</th>
<th>(V^{A+B}(0.5))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>9.83%</td>
<td>9.83%</td>
<td>.4714</td>
<td>.5286</td>
<td>-44.7138</td>
</tr>
<tr>
<td>Nash</td>
<td>11.97%</td>
<td>11.97%</td>
<td>.3999</td>
<td>.6001</td>
<td>-44.7992</td>
</tr>
<tr>
<td>Competitive</td>
<td>–</td>
<td>–</td>
<td>0.5</td>
<td>0.5</td>
<td>-44.8796</td>
</tr>
</tbody>
</table>

Table 9: Optimal coordinated and uncoordinated policies to maximize ex-ante welfare.

<table>
<thead>
<tr>
<th></th>
<th>(L^A_s)</th>
<th>(L^B_s)</th>
<th>(\eta^A_s)</th>
<th>(\eta^B_s)</th>
<th>(\mathbb{E}[V^{A+B}(\eta)])</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>23.93%</td>
<td>23.93%</td>
<td>.4439</td>
<td>.5561</td>
<td>-44.8050</td>
</tr>
<tr>
<td>Nash</td>
<td>10.03%</td>
<td>10.03%</td>
<td>.4075</td>
<td>.5925</td>
<td>-44.9538</td>
</tr>
<tr>
<td>Competitive</td>
<td>–</td>
<td>–</td>
<td>0.5</td>
<td>0.5</td>
<td>-47.9165</td>
</tr>
</tbody>
</table>

The uncoordinated policies are tighter in at least one of two ways. First, the Nash leverage limit \(L^I_1\) may be tighter than in the coordinated case. Second, credit flows may be closed off for a larger range of \(\eta\). From the ex-ante view, Nash policy is tighter in both senses: credit flows are closed off for a larger range of \(\eta\), and when flows are allowed, the leverage constraint is tighter. However, the initial condition \(\eta = 0.5\), credit flows are closed off for a larger range, but leverage limits are looser when credit flows are allowed. However, there is an important and drastic difference in \(\eta^I_1\), determining when credit flows are allowed, while there is a comparatively small difference in \(L^I_1\). In fact, using an appropriate measure of leverage, Nash policy is still on balance tighter. One can compute the stationary distribution corresponding to the socially optimal and Nash policies and calculate the expected leverage. Using this metric, we find that the social optimum features higher leverage overall, and hence policy is looser. Trivially, the expected leverage in the ex-ante condition is much tighter in the Nash case (since both measures are tighter).

Notably, the welfare gains from policy—coordinated or otherwise—are larger when calculated in an ex-ante way. This is because leverage constraints improve welfare by increasing global
economic stability so that the stationary distribution is more concentrated around \( \eta = 0.5 \). As a result, using the stationary distribution to calculate welfare adds an additional benefit to the welfare calculation since the distribution used for the expectation is less disperse (the welfare function \( V(\eta) \) is concave).

The result that Nash policies are tighter than coordinated policies holds across a range of parameters. The key parameters determining the welfare costs from instability arising from incomplete markets are the productivity loss \( a \) and volatility \( \sigma \). We find that across a variety of parameters, coordinated policies are looser than Nash policies in some sense. In particular, for several parameters, such as \( a = 0.6, 0.7 \), coordinated policies are looser in both leverage constraint and \( \eta^P \). For others, such as \( a = 0.5 \), while the leverage constraint is somewhat looser in Nash, the magnitude of the Nash \( \eta^P \) is comparatively much tighter than the coordinated choice. When volatility is high, the economy is less stable (i.e., the stationary distribution is more spread out) for any level of leverage limits, but based on our results with \( \sigma = 5\% \), coordinated policy is still looser.

The following tables present the results for calculating coordinated and uncoordinated countercyclical policies when varying \( a \) and \( \sigma \).

**Table 10: Optimal coordinated and uncoordinated policies: \( a = 0.5 \).**

<table>
<thead>
<tr>
<th>L^A</th>
<th>L^B</th>
<th>η^A</th>
<th>η^B</th>
<th>V^{A+B}(0.5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>.1527</td>
<td>.1527</td>
<td>.4354</td>
<td>.5646</td>
</tr>
<tr>
<td>Nash</td>
<td>.2629</td>
<td>.2629</td>
<td>.3353</td>
<td>.6647</td>
</tr>
<tr>
<td>Competitive</td>
<td>N/A</td>
<td>N/A</td>
<td>0.5</td>
<td>0.5</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>L^A</th>
<th>L^B</th>
<th>η^A</th>
<th>η^B</th>
<th>E[V^{A+B}(\eta)]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>.3592</td>
<td>.3592</td>
<td>.3997</td>
<td>.6003</td>
</tr>
<tr>
<td>Nash</td>
<td>.8426</td>
<td>.8426</td>
<td>.2737</td>
<td>.7263</td>
</tr>
<tr>
<td>Competitive</td>
<td>N/A</td>
<td>N/A</td>
<td>0.5</td>
<td>0.5</td>
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</tbody>
</table>
Table 11: Optimal coordinated and uncoordinated policies: $\alpha = 0.6$.

<table>
<thead>
<tr>
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<th>$\eta^B_s$</th>
<th>$V^{A+B}(0.5)$</th>
</tr>
</thead>
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<tr>
<td>Social Optimum</td>
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<td>.3200</td>
<td>.4122</td>
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<td>Nash</td>
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<td>.2325</td>
<td>.3500</td>
<td>.6500</td>
<td>-44.7878</td>
</tr>
<tr>
<td>Competitive</td>
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<td>N/A</td>
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<td>0.5</td>
<td>-44.8796</td>
</tr>
</tbody>
</table>

Table 12: Optimal coordinated and uncoordinated policies: $\alpha = 0.7$.

<table>
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<tr>
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<th>$\eta^A_s$</th>
<th>$\eta^B_s$</th>
<th>$\mathbb{E}[V^{A+B}(\eta)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
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<td>.5006</td>
<td>.5994</td>
<td>-44.7910</td>
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<tr>
<td>Nash</td>
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<td>.3057</td>
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<tr>
<td>Competitive</td>
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<td>N/A</td>
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<td>0.5</td>
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Table 13: Optimal coordinated and uncoordinated policies: $\alpha = 0.9$.

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<th>$\eta^A_s$</th>
<th>$\eta^B_s$</th>
<th>$\mathbb{E}[V^{A+B}(\eta)]$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.7801</td>
<td>.4679</td>
<td>.5321</td>
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<tr>
<td>Nash</td>
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<td>.1811</td>
<td>.4083</td>
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</tr>
<tr>
<td>Competitive</td>
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<td>N/A</td>
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<td>0.5</td>
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<th>$\eta^A_s$</th>
<th>$\eta^B_s$</th>
<th>$\mathbb{E}[V^{A+B}(\eta)]$</th>
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<tbody>
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<tr>
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<tr>
<td>Competitive</td>
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<td>N/A</td>
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<td>0.5</td>
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</tbody>
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Table 14: Optimal coordinated and uncoordinated policies: $\sigma = 2.5\%$.

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<th>$\eta^B_s$</th>
<th>$V^{A+B}(0.5)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
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<td>.1990</td>
<td>.4378</td>
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<td>Nash</td>
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<tr>
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<td>N/A</td>
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<td>-44.7118</td>
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<table>
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<th>$\eta^A_s$</th>
<th>$\eta^B_s$</th>
<th>$E[V^{A+B}(\eta)]$</th>
</tr>
</thead>
<tbody>
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<td>.4724</td>
<td>.4389</td>
<td>.5611</td>
<td>-44.2422</td>
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<tr>
<td>Nash</td>
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<td>-47.6353</td>
</tr>
</tbody>
</table>

Table 15: Optimal coordinated and uncoordinated policies: $\sigma = 5\%$.

<table>
<thead>
<tr>
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<th>$L^A_s$</th>
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<th>$\eta^B_s$</th>
<th>$V^{A+B}(0.5)$</th>
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</thead>
<tbody>
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<td>Social Optimum</td>
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<td>N/A</td>
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<td>N/A</td>
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<td>.5</td>
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</table>

<table>
<thead>
<tr>
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<th>$L^B_s$</th>
<th>$\eta^A_s$</th>
<th>$\eta^B_s$</th>
<th>$E[V^{A+B}(\eta)]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social Optimum</td>
<td>.2922</td>
<td>.2922</td>
<td>.3696</td>
<td>.6304</td>
<td>-46.3458</td>
</tr>
<tr>
<td>Nash</td>
<td>.2180</td>
<td>.2180</td>
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</tbody>
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