Should Monetary Policy Target Financial Stability?

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Abstract

Monetary policy can improve household welfare by affecting banks' leverage decisions and the rate of bank equity growth. We show this result using a model in which banks borrow using deposits and invest in productive projects, and monetary policy affects risk-premia. Because banks do not actively issue equity, aggregate outcomes depend on the aggregate level of bank equity and equilibrium is inefficient. A Fed Put is ex-ante stabilizing, decreasing volatility and the likelihood of crises, and enabling higher leverage in bad times only. The marginal welfare benefits of targeting financial stability outweigh costs from distortions caused by nominal rigidities.

Keywords: Monetary policy, Leaning against the wind, Financial stability, Macroeconomic instability, Banks, Liquidity.

JEL classification: E44, E52, E58, G01, G12, G20, G21.

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1 Introduction

Economists increasingly debate whether monetary policy should be used to stabilize the financial system. It is widely recognized that central bankers may pursue aggressive policies to stabilize the financial system during downturns—i.e., enacting a "Fed Put" to cut borrowing costs. Critics worry that a Fed Put encourages excessive risk-taking and leverage ex-ante, and so a Fed Put may backfire, increasing the probability of financial crises by causing riskier behavior in good times. Accordingly, some economists suggest that central banks should "lean against the wind" ("LAW") in good times to counteract excessive risk-taking and mitigate overheating in the the financial sector. Proponents argue that systematically raising the cost of intermediation in good times will decrease the probability of (extremely costly) financial crises.[1] There are prevailing doubts that the benefits of LAW outweigh the costs, and LAW may even make crises worse if the economy enters a crisis starting from a weaker position (Svensson, 2017).

Core to this debate is whether the financial sector creates inefficiencies with aggregate consequences that monetary policy can adequately address. Monetary policy should target financial conditions in addition to (or independent of) output gaps only if the financial sector creates externalities that affect the broader economy. In other words, to justify targeting financial stability in addition to output gaps, the financial sector must be more than a source of shocks to the rest of the economy; it must be an inefficient source of shocks.[2] This raises several fundamental questions. How do monetary policy rules affect financial stability? Can monetary policy effectively correct financial-sector externalities? Are the benefits of targeting financial stability worth the costs of deviating from standard monetary policy objectives?

To answer these questions, we use a continuous-time stochastic general equilibrium model in which financial frictions endogenously create inefficient instability and systemic risk, building on Brunnermeier and Sannikov (2014). Banks allocate resources to productive projects (banks invest in certain projects more efficiently than households can directly), but banks can issue only risk-free

[1] Proponents of this view include BIS (2014, 2016), Borio (2014); Borio et al. (2018) and Juselius et al. (2017). See also Adrian and Liang (2016); Adrian and Duarte (2016); Adrian et al. (2019).

[2] If there were no externalities, then targeting the output gap in general would also indirectly address output gaps caused by shocks to the financial sector; financial-sector shocks would just be demand or supply shocks to respond to in the usual way.

debt and not equity. As a result, banks invest more when they have more equity, and the economy's resources are allocated more efficiently when banks are well-capitalized. Limited equity issuance creates a distortion between the private and social values of bank equity, and so policies that improve financial stability can potentially increase household welfare. To this setting we add the model of monetary policy transmission from Drechsler et al. (2018) in which the nominal interest rate determines the liquidity premia on banks' investments. We investigate the impact of interest rate policy on the amplification of shocks, nonlinear dynamics, and systemic risk, which are central features of the framework in Brunnermeier and Sannikov (2014). Finally, we add nominal rigidities following the New Keynesian literature in order to analyze the welfare consequences of monetary policy rules that respond to the financial sector.

In our model, monetary policy can affect the return on banks' investments, the rate of bank equity growth, and banks' leverage decisions. Through these effects, monetary policy can change the frequency and duration of good and bad outcomes. We solve for the global dynamics of the economy and find that the impact of interest rate rules on bank profitability, and thus on bank equity growth, varies systematically with the state of the economy. How bank leverage varies over the financial cycle depends primarily on how monetary policy *changes* over the cycle, much more than on the overall level of rates.

We find the following results. First, a Fed Put increases stability *ex-ante* and does not lead to excessive leverage in good times. A Fed Put increases leverage *in bad times*, which improves aggregate outcomes and increases the expected rate at which banks recapitalize in bad times. As a result, asset-price volatility is lower nearly everywhere and the probability of crises is lower with a Fed Put. Second, leaning against the wind in good times improves stability, and thus combining LAW and Put maximizes financial stability. Third, the effects of monetary policy on leverage, volatility, and stability are state-dependent. The timing of interest rate movements matters. As a result, a large rate cut can stabilize the financial sector better than a series of shallow rate cuts that start "early."

Normatively, using monetary policy to target financial stability can improve welfare even when there is a tradeoff between financial stability objectives and standard objectives of minimizing inflation and output gaps. Deviations in monetary policy impose costs relative to the standard policy objectives because with nominal rigidities, deviations in interest rates create distortions as a result of inflation—this is the standard cost of having interest rates deviate from a target rule. We find that the marginal benefits of using monetary policy to improve financial stability (and thus aggregate stability) exceed the costs caused by distortions from nominal rigidities. Our results suggest that financial-sector considerations *alone* justify at least an additional rate cut during a financial crisis beyond what would be implied by a standard Taylor rule responding to output gaps.

Related Literature Methodologically, our paper follows the stochastic continuous-time macro literature, pioneered by Brunnermeier and Sannikov (2014, 2015, 2016) and He and Krishnamurthy (2012, 2013, 2014), who analyze the nonlinear global dynamics of economies with financial frictions, building on seminal results from Bernanke and Gertler (1989); Kiyotaki and Moore (1997); Bernanke et al. (1999). Within this literature, we combine the models of Drechsler et al. (2018) and Phelan (2016) to study how monetary policy affects global dynamics.[3] The macroeconomic framework in Phelan (2016) studies how macroprudential policies (i.e., leverage limits) can improve welfare by increasing stability. Relatedly, Caballero and Simsek (2020) study the role of macroprudential policies to mitigate the severity of demand-driven recessions.

Drechsler et al. (2018) develop a dynamic asset pricing model in which monetary policy affects the risk premium component of the cost of capital. Risk-tolerant agents take deposits from risk-averse agents to buy an asset. Lower nominal rates make liquidity cheaper and raise leverage, resulting in lower risk premia and higher asset prices and volatility.[4] Using this model of monetary policy transmission, we find that monetary policy has different effects on stability depending on the extent to which the returns on different types of investments are affected. Van der Ghote (2018) shows that in a New Keynesian version of the model in Brunnermeier and Sannikov (2014), it is

[3]For the rich asset pricing literature using this methodology, see Adrian and Boyarchenko (2012), Moreira and Savov (2014), Gârleanu and Panageas (2015), and Gârleanu et al. (2015).

[4]Drechsler et al. (2017) empirically confirm this transmission mechanism by examining how nominal rates affect the supply of retail bank deposits, an important class of liquid assets. Bernanke and Kuttner (2005) document empirically that monetary policy shocks decrease risk premia. Kekre and Lenel (2019) consider a New Keynesian model in which monetary policy shocks lower risk premia by redistributing wealth toward agents with greater propensities to invest in risky capital.

still optimal for monetary policy to mimic the natural rate. In that model, monetary policy affects price dispersion, not banks' risk-taking incentives. Van der Ghote (2019) includes money as a means of payments to examine the real effects of interest-rate corridor policies, which affect the cost of liquidity in financial markets.

Our analysis contributes to the broad literature studying the relationship between monetary policy and financial stability. Whereas Cieslak and Vissing-Jorgensen (2020) find that the Federal funds rate responds to asset prices (the stock market), we consider responses to the condition of the financial sector specifically (banks' equity levels). Several papers have cautioned against interventions in financial markets by central banks (Diamond and Rajan, 2012; Farhi and Tirole, 2012). We take maturity mismatch and correlated risks as given and then ask, in light of these features, how changes in monetary policy affect stability. Because of the general equilibrium effects in our model, a Fed Put increases leverage when rates are low but not before. The increase in leverage when rates are low is stabilizing. Moreover, a Put does not introduce a commitment problem or time inconsistency; it is ex-ante stabilizing. These results add to the analysis of Bornstein and Lorenzoni (2018), who develop a simple model where passive monetary policy causes overborrowing due to an aggregate demand externality. Our model additionally suggests that moral hazard may not be a concern when a Fed Put addresses resource misallocation.

Our paper's insight is also distinct from other research affirming the use of monetary policy to address financial stability. Stein (2012) provides a model in which monetary policy can enhance financial stability by reducing excessive short-term debt. Gertler and Karadi (2011), Curdia and Woodford (2011); Cúrdia and Woodford (2016), Christiano et al. (2015), and Caballero and Simsek (2019) study the ability of monetary policy to address aggregate demand externalities in models with financial frictions, and show that prudential monetary policy can mitigate the severity of a demand-driven recession. In our economy, monetary policy corrects a pecuniary externality that produces dynamic resource misallocation affecting both the severity and the probability of crises. Taken together, these models show monetary policy can mitigate aggregate demand *and* aggregate supply issues caused by financial frictions. Hansen (2018) characterizes optimal monetary policy via LQ approximation near an efficient (deterministic) steady state in a New Keynesian economy with a Bernanke and Gertler (1989) financial accelerator in which monetary policy works via the real interest rate. In contrast, the equilibrium in our model is inefficient and spends substantial periods of time away from the (stochastic) steady state due to nonlinearities.

2 The Baseline Model

The baseline model features financial instability but no nominal rigidities. In Section 3, we use this real model to analyze the positive consequences of monetary policy for financial stability, considering dynamic tradeoffs in the absence of standard New Keynesian tradeoffs. In Section 4.2 we add nominal rigidities in order to study the welfare consequences of these tradeoffs.

The economy is populated by households and banks, owned by households. A single factor of production ("capital") can be used to produce two intermediate goods. Banks have an advantage for producing one good (the "bank-dependent sector"). As a result, output and growth depend endogenously on capital ownership. The financial friction is costly equity issuance. Outcomes will depend on the level of equity in the banking sector. The model combines, with modifications, elements of the models in Phelan (2016) and Drechsler et al. (2018).

2.1 Technology, Environment, and Markets

Time is continuous and infinite, and there are aggregate productivity shocks that follow a Wiener process. One factor of production, capital, can be used to produce two types of intermediate goods at unit rate. The effective capital quantity y_t evolves according to equation (1),

$$\frac{dy_t}{y_t} = g_y^{ij} dt + \sigma dW_t, \tag{1}$$

where dW_t is an exogenous standard Brownian motion and g_y^{ij} depends on who manages capital and what it is used to produce. The values of g_y^{ij} for agent i and good j (Table 1) imply that banks are comparatively better at managing good-1 production (good-1 are bank-dependent investments). We define the parameter restriction on g_B more clearly later in this section.

Table 1: Expected capital productivity growth rates by agent and good produced.

g_y^{ij}	Good 1	Good 2
Households	$g-\ell$	g
Banks	g_B	g_B

Denote by Y_t the stock of effective capital at time t, which is also the flow production of goods at time t. The consumption good is produced using goods 1 and 2 according to

$$C_t = Y_{1t}^{\frac{1}{2}} Y_{2t}^{\frac{1}{2}},$$

where C_t is the quantity of the consumption good, Y_{jt} is the quantity of good j (equivalently the quantity of capital used to produce good j). Standard static optimization implies that the equilibrium prices of intermediates are given by

$$p_{1t} = \frac{1}{2}\left(\frac{Y_{2t}}{Y_{1t}}\right)^{\frac{1}{2}} p_t, \quad p_{2t} = \frac{1}{2}\left(\frac{Y_{1t}}{Y_{2t}}\right)^{\frac{1}{2}} p_t,$$

where p_t is the price of consumption. Let $\lambda_t = \frac{Y_{1t}}{Y_t}$ be the fraction of capital cultivating good 1. Then the real prices of intermediate goods ($P_{jt} = p_{jt}/p_t$) are

$$P_{1t} = \frac{1}{2}\left(\frac{1-\lambda_t}{\lambda_t}\right)^{\frac{1}{2}}, \quad P_{2t} = \frac{1}{2}\left(\frac{\lambda_t}{1-\lambda_t}\right)^{\frac{1}{2}}. \tag{2}$$

Capital is traded in a perfectly competitive market at a real price Q_t. We postulate that the real capital price (the "asset price") follows the process

$$\frac{dQ_t}{Q_t} = \mu_{Q,t}\,dt + \sigma_{Q,t}\,dW_t, \tag{3}$$

which will be determined endogenously in equilibrium. The return to owning capital includes the value of the output produced and the capital gains on the value of the capital. By Ito's Lemma, the rate of return to agent i using capital to produce good j is

$$dr_t^{ij} = \left(\frac{P_{jt}}{Q_t} + g_y^{ij} + \mu_{Q,t} + \sigma\sigma_{Q,t}\right)dt + (\sigma + \sigma_{Q,t})dW_t,$$

where g_y^{ij} is appropriately defined for agent i. The volatility of returns on investments is $\sigma + \sigma_{Q,t}$, which includes fundamental risk σ and endogenous price risk $\sigma_{Q,t}$. Denote by dr_t^{bj} and dr_t^{hj} the returns respectively to banks and households from owning capital to produce good j.

Finally, there is a market for risk-free deposits, which are in zero net-supply with endogenous real return r_t.

2.2 Households

There is a continuum of risk-neutral households denoted by $h \in [0,1]$ with initial wealths $n_{h,0}$. Lifetime utility is given by

$$V_\tau = \mathbb{E}_\tau\left[\int_\tau^\infty e^{-r(t-\tau)}(c_{h,t} + \phi_L\delta_{h,t})\,dt\right],$$

where $c_{h,t}$ is household flow consumption, r is the discount rate, $\delta_{h,t}$ are bank deposits, and $\phi_L > 0$ is the convenience yield preference parameter. Households may consume positive and negative amounts, though in equilibrium their consumption will always be positive. It follows that households require an expected real return of r on any real investment and a real return of $r_L = r - \phi_L$ on deposits due to the convenience yield. We model the convenience yield of deposits directly in the utility function for modeling ease. Deposits provide convenience yield (or liquidity value) for a variety of reasons outside of the model (see for example Diamond and Dybvig (1983), Gorton and Pennacchi (1990), or Lagos and Wright (2005)). We impose constant marginal utility for deposits to simplify the model and so that no endogenous variable affects deposit rates.

Because the real rate is constant in equilibrium, our model allows the interpretation that changes in monetary policy represent deviations from a target rate, perhaps determined by a Taylor rule.

2.3 Banks

There is a continuum of banks, denoted by $b \in [0,1]$, with initial book value ("equity") $n_{b,0}$. Banks invest in capital and issue deposits. Banks are owned by households, who choose dividend payouts, the level of deposits, the level of liquid reserves, and the portfolio weights on capital used by banks to produce goods 1 and 2. Because of un-modeled financial frictions, banks are subject to two constraints. First, equity issuance is infinitely costly (i.e., dividends must be positive). Second, the value of banks' assets minus liabilities $n_{b,t}$ cannot become negative (bankruptcy).[5]

Banks maximize the present value of dividends discounted at rate r (households' time preference) subject to its constraints. Because banks can borrow using debt at a real interest rate $r_L = r - \phi_L < r$, banks will never choose a capital structure that is completely equity. To avoid bankruptcy, banks will never choose a capital structure that is completely debt. We assume that $g_B = g - \phi_L$ so that banks have a net advantage at cultivating good 1 but not at cultivating good 2.

Following Drechsler et al. (2018) we assume deposits are subject to funding shocks, requiring banks to self-insure by holding liquid assets. Since this behavior is not frictionless, borrowing with deposits imposes a "liquidity premium" of operations that is proportional to the nominal interest rate, which we denote by $LP_t \equiv \gamma i_t$. See Appendix B for details. Our results do not rely on a particular microfoundation for how monetary policy affects liquidity premia. Other mechanisms would imply similar results. Nominal interest rates may determine liquidity premia because banks have market power in the deposit market (see Drechsler et al., 2017; Brunnermeier and Koby, 2018) or because banks use short bonds as collateral to back inside money (Lenel et al., 2019).

[5] The assumption that bank equity is "sticky" is empirically supported by Acharya et al. (2011), which shows that the capital raised by banks during the crisis was almost entirely in the form of debt and preferred stock and not in the form of common equity. Adrian and Shin (2010, 2011) provide evidence that the predetermined balance sheet variable for banks and other financial banks is equity, not assets. Our results generalize so long as banks do not issue equity too frequently. Relatedly, Gambacorta and Shin (2016) provide evidence that bank capital matters for monetary policy transmission.

2.4 Monetary Policy

We suppose the central bank sets the nominal interest rate i_t on deposits. Inflation is determined according to a standard Fisher equation

$$i_t = r_L + \pi_t,$$

where $r_L = r - \phi_L$ and households' preferences pin down the real risk-free rate. We define policy functions for i_t later. We assume inflation is locally deterministic, i.e. $\frac{dp_t}{p_t} = \pi_t \, dt$.

We interpret changes in monetary policy in this model as deviations from a desired target rate $i^* = r^* + \pi^*$, where π^* is the inflation target and r^* may be the desired real interest rate coming out of a New Keynesian model. Deviations from i^* represent changes in the policy target in response to the state of the financial sector. The state of the financial sector will also determine an equilibrium "output gap" due to misallocation of resources, a mechanism distinct from the standard New Keynesian model. See Appendix B for details on implementation.

2.5 Banks' Problem

Let $x_t = (x_{k1,t}, x_{k2,t}, x_{M,t})$ be portfolio weights (summing to one) on capital used for good 1, capital used for good 2, and reserves. We let $x_{k,t} \equiv x_{k1,t} + x_{k2,t}$ refer to banks' share of wealth invested in capital. Formally, banks solve the problem

$$\max_{\{x_t, d\zeta_t\}} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \, d\zeta_t \right],$$

subject to

$$\frac{dn_{b,t}}{n_{b,t}} = \big(r_t - (x_{k,t} - 1)\gamma i_t\big) \, dt + (x_{k1,t} \, dr_t^{b1} + x_{k2,t} \, dr_t^{b2} - x_{k,t} r_t \, dt) - \frac{d\zeta_t}{n_{b,t}} \tag{4}$$

$$n_{b,t}, x_{k1,t}, x_{k2,t}, x_{M,t} \, d\zeta_t \geq 0. \tag{5}$$

Banks earn the deposit rate, pay the liquidity premium from holding reserves, earn the risk premium on capital holdings, and pay dividends at rate $d\zeta_t$.

By homogeneity and price-taking, we can write the maximized value of a bank with equity $n_{b,t}$ as

$$\theta_t n_{b,t} \equiv \max_{\{x_t \geq 0, d\zeta_t\}} \mathbb{E}_t \left[\int_t^\infty e^{-r(s-t)} d\zeta_s \right], \tag{6}$$

where θ_t is the marginal value of equity, which summarizes how market conditions affect the value of the bank's value function per dollar of equity. The value θ_t equals 1 plus the multiplier on the equity-issuance constraint and reflects the aggregate condition of the financial sector.

We can further characterize the optimality conditions in the following way.

Proposition 1. *Consider a finite process*

$$\frac{d\theta_t}{\theta_t} = \mu_{\theta,t}\, dt + \sigma_{\theta,t}\, dW_t, \tag{7}$$

with $\sigma_{\theta,t} \leq 0$. Then $\theta_t n_t$ represents the maximal future expected payoff that a bank with book value n_t can attain, and $\{x_t, d\zeta_t\}$ is optimal if and only if (i) $\theta_t \geq 1\ \forall t$, and $d\zeta_t > 0$ only when $\theta_t = 1$, (ii) $\mu_{\theta,t} = \phi_L - \gamma i_t$, (iii) $\mathbb{E}[dr_t^{bj}] - r_t - LP_t \leq -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})$, with strict equality when $x_{jk,t} > 0$, (iv) The transversality condition $\mathbb{E}[e^{-rt}\theta_t n_t] \to 0$ holds under $\{x_t, d\zeta_t\}$.

Hence, $RP_t \equiv -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})$ represents the bank's required risk premium (or instantaneous level of risk aversion), which must at least equal the expected excess return over the liquidity premium, which is $LP_t \equiv \gamma i_t$. Cutting interest rates increases the drift of θ_t, and banks will not pay dividends when $\theta_t \geq 1$; θ_t can never be less than one because banks can always pay out the full value of equity, guaranteeing a value of at least $n_{b,t}$.

2.6 Equilibrium Asset Pricing

Households will always produce good 2 and sometimes produce good 1. Because households require a return of r on real investments, it follows that household returns satisfy

$$\frac{P_{2t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} = r, \tag{8}$$

$$\frac{P_{1t}}{Q_t} + g - \ell + \mu_{Q,t} + \sigma\sigma_{Q,t} \leq r, \tag{9}$$

where the inequality reflects that households may not always produce good 1 in equilibrium. Since we look for an equilibrium where the risk premium and liquidity premium are both non-negative, banks will never produce good 2. From the banks' investment in good 1 we have

$$\frac{P_{1t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} = r + RP_t + LP_t. \tag{10}$$

When households do not produce good 1 (banks and households specialize), we obtain a market-clearing condition for capital allocations by taking the difference between the equations (8) and (10):

$$RP_t + LP_t = \frac{P_{1t} - P_{2t}}{Q_t}. \tag{11}$$

With specialization, changes in the liquidity premium may affect the difference in returns between goods 1 and 2. In contrast, when households produce good 1, we have $\frac{P_{1t}-P_{2t}}{Q_t} = \ell$, which implies

$$RP_t + LP_t = \ell, \tag{12}$$

i.e., the sum of banks' investment premia equals the household return disadvantage. The differences between equations (11) and (12) provide a crucial insight. When banks are the marginal investors in the intermediation sector (specialization), a decrease in interest rates might, all else equal, decrease the relative return between sectors 1 and 2 ($P_1 - P_2$ decreases). When households are the marginal investor in the intermediation sector, a decrease in interest rates must increase banks' equilibrium risk-premium, which will occur through higher leverage.

2.7 Characterizing Equilibrium

A competitive equilibrium is characterized by the market price for the risky asset, together with portfolio allocations and consumption decisions such that given prices, agents optimize and markets clear. Due to equity issuance frictions, banks' decisions depend on their level of equity, and so equilibrium depends on banks' equity levels and monetary policy has scope to affect equilibrium.

We solve for the global equilibrium dynamics using the methods in Brunnermeier and Sannikov (2014). Define $N_t = \int n_{b,t} db$ as aggregate bank equity. Because capital grows geometrically and the bank problem is homogenous, the equilibrium state variable of interest is aggregate bank equity as a fraction of total value of capital:

$$\eta_t = \frac{N_t}{Q_t Y_t}.$$

Equilibrium consists of a law of motion for η_t, and asset allocations and prices as functions of η. The asset prices are $Q(\eta)$ and $\theta(\eta)$, and the flow allocations and goods prices are $\lambda(\eta)$, $\psi(\eta)$, $P_1(\eta)$, $P_2(\eta)$. We derive the evolution of η_t using Ito's Lemma and the equations for returns and budget constraints.

Lemma 1. *The equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_{\eta,t} dt + \sigma_{\eta,t} dW_t + d\Xi_t, \tag{13}$$

where $d\Xi_t$ is an impulse variable creating a regulated diffusion. Furthermore,

$$\mu_{\eta,t} = \frac{P_{1t}}{Q_t} + (\lambda_t - \psi_t)\ell - (1-\psi_t)\phi_L - \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma + \sigma_{Q,t})(\sigma_{\theta,t} + \sigma + \sigma_{Q,t}),$$
$$\sigma_{\eta,t} = \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma + \sigma_{Q,t}), \qquad d\Xi_t = \frac{d\zeta_t}{N_t},$$

where $d\zeta_t = \int d\zeta_{b,t} db$ and $\psi_t = x_{k,t}\eta_t$ is the fraction of capital held by banks.

We convert the equilibrium conditions into a system of differential equations ("ODE") in the asset prices Q and θ. Given $Q(\eta)$, $Q'(\eta)$, $\theta(\eta)$, and $\theta'(\eta)$ we can use equilibrium returns and allocations to derive $Q''(\eta)$ and $\theta''(\eta)$. We solve the ODE using appropriate boundary conditions (additional details are in Appendix A).

Proposition 2. *The equilibrium domain of the functions* $Q(\eta), \theta(\eta)$*, and* $\lambda(\eta)$*,* $\psi(\eta)$*,* $P_1(\eta)$*,* $P_2(\eta)$ *is an interval* $[0,\eta^*]$*. The function* $Q(\eta)$ *is increasing,* $\theta(\eta)$ *is decreasing, and the following boundary conditions hold: (i)* $\theta(\eta^*) = 1$*; (ii)* $Q'(\eta^*) = 0$*; (iii)* $\theta'(\eta^*) = 0$*; (iv)* $Q(0) = \underline{Q}$*; (v)* $\lim_{\eta \to 0^+} \theta(\eta) = \infty$*. Over* $[0,\eta^*]$*,* $\theta_t \geq 1$ *and* $d\zeta_t = 0$*, and* $d\zeta_t > 0$ *at* η^* *creating a regulated barrier for the process* η_t*. We refer to* η^* *as the* stochastic steady state. $\underline{Q}$ *is the price of capital in an economy with no banks.*

Hence, the system ranges between 0 and η^*, at which point banks pay dividends because the marginal attractiveness of debt outweighs the marginal attractiveness of an additional unit of equity. When interest rates do not vary too much, there exists $\bar{\eta} \in (0,\eta^*)$ such that $\psi(\eta) = \lambda(\eta)$ for $\eta > \bar{\eta}$ and $\psi(\eta) < \lambda(\eta)$ for $\eta \leq \bar{\eta}$.[6] For high levels of η, banks and households specialize in their relative investment sectors (i.e., households do not produce good 1), but below $\bar{\eta}$ households produce good 1. The evolution of η induces a stationary distribution (PDF) $f(\eta)$ with CDF $F(\eta)$; the distribution $f(\eta)$ solves a Kolmogorov-Forward equation.

We define a crisis as when banks are so constrained that they have to sell capital at fire-sale prices and households intermediate capital in the bank-dependent sector (i.e., $\eta < \bar{\eta}$). We define the stability of the economy as the fraction of time the economy is not in a crisis:

$$\text{Stability} = 1 - F(\bar{\eta}).$$

If the price function is twice-continuously differentiable, then the evolutions of the capital price and marginal bank value (equations (3) and (7)) are functions of η

$$\frac{dQ_t}{Q_t} = \mu_Q(\eta_t)dt + \sigma_Q(\eta_t)dW_t, \quad \frac{d\theta_t}{\theta_t} = \mu_\theta(\eta_t)dt + \sigma_\theta(\eta_t)dW_t,$$

where the drift and variance terms are determined by the derivatives of $Q(\eta)$ and $\theta(\eta)$. For the remainder of the paper, we suppress the dependence on η_t and on time t for notational ease.

[6] If interest rates change significantly (high slope), then there can be multiple regions in which specialization occurs and the state space is no longer cleanly separated into two regions around a single $\bar{\eta}$. In this case, we still define stability as the measure of states when specialization occurs.

3 Monetary Policy and Equilibrium Stability

The global dynamics illustrate the positive effects of monetary policy on macroeconomic stability. We solve the model numerically using the parameters from Phelan (2016) and Drechsler et al. (2018). While not a rigorous calibration, the parameter choices provide reasonable output. The two most important variables are volatility σ and the monetary policy transmission value γ. The volatility $\sigma = 2\%$ corresponds roughly to the volatility of TFP and also the typical volatility of bank assets. The value of $\gamma = 10.2\%$ implies the empirically realistic result that changing the nominal rate by 100bps changes the liquidity premium by roughly 10bps.[7]

We first solve for equilibrium with nominal rates held constant at $i = 0\%$ and $i = 4\%$. Comparing these equilibria illustrates the effects of globally higher rates. We then consider raising rates only in good times to stem the build up of systemic risk while still lowering rates in bad times once a financial crisis has actually occurred. The state-dependent consequences of monetary policy are a key component to this implementation of LAW. The time at which monetary policy switches from high rates to low rates determines the ability of LAW to arrest the build up of systemic risk and the degree of inefficiency in capital allocations.

To simplify the analysis, we first consider piecewise interest rate policies around a "strike" η^{Put} defined as follows:[8]

$$i(\eta) = \begin{cases} i^{Put} & \text{if } \eta < \eta^{Put} \quad \text{(i.e., "bad times")}, \\ i^{LAW} & \text{if } \eta \geq \eta^{Put} \quad \text{(i.e., "good times")}. \end{cases}$$

Varying the levels of i^{Put} and i^{LAW} correspond to the degree to which policy leans against the wind in good times or supports the financial sector in bad times. We set $i^{LAW} = 4\%$ and $i^{Put} = 0\%$ so that in good times rates are kept high but in bad times the central bank cuts rates. We set $\eta^{Put} = 1.75\%$ so that rate changes occur in the crisis region where households produce good 1.

The effects of monetary policy on asset prices and volatility in our model are similar to what

[7]Parameters values are $r = 4\%$, $g = 2\%$, $\sigma = 2\%$, $\ell = 1\%$, and $\phi_L = 2\%$; the micro-level parameters in Drechsler et al. (2018) are $m = 4$, and $\kappa = 0.4085$, and $\gamma = \frac{\kappa}{m}$. See those papers for detail on parameter choices.

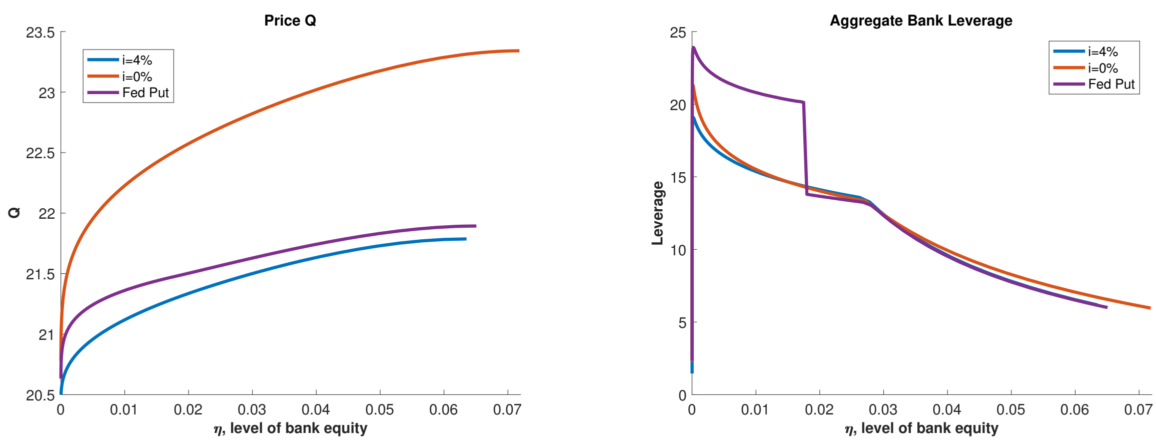
[8]Because these policies are piecewise, some equilibrium variables like leverage will exhibit jumps. If a plotted variable sharply changes, then it is a jump rather than a continuous change.

Drechsler et al. (2018) find in their model with two agents with heterogeneous risk aversion. In contrast, the stationary distribution in our model is bimodal and leverage behaves slightly differently; they find that higher rates lead to lower leverage everywhere, which we do not find, and we find that changes in rates are the key determinant of leverage. This difference in leverage behavior has important consequences for the effects of a Fed Put/LAW.

3.1 Equilibrium Dynamics and Monetary Policy

While interest rates and asset prices under the Put always fall between the constant levels considered, the resulting levels of bank leverage, investment returns, and volatility do not fall between the levels corresponding to the constant-rate policies.

Asset Prices Figure 1(a) plots the asset price $Q(\eta)$ under each policy rule. As expected, higher interest rates lead to lower asset prices. The asset price under the Fed Put is slightly higher than when rates are constant at 4%. A simple heuristic of "higher asset prices means more ex-ante instability" turns out to be wrong. Importantly, even though interest rates are always between 0% and 4%, and even though asset prices are between the levels when rates are at 0% and 4%, the behavior of leverage, volatility, and stability are quite different.



(a) Asset Price

(b) Leverage

Figure 1: Equilibrium prices and leverage with constant rates and Fed Put.

Leverage Figure 1(b) plots leverage levels across η. With constant rates, the level of interest rates hardly affects leverage at each η. Changes in monetary policy have much more significant effects on bank leverage than parallel shifts in rates. For low levels of equity the Put dramatically props up bank leverage. These changes occur even though interest rates in each case are the same as one of the constant-rates policies.[9] In contrast, leverage levels in good times are quite similar for all policies. This is a counterintuitive result; a typical concern about a Fed Put is that expectations for low rates in the future will lead banks to take more risk in good times. Instead, interest rate policy in good times primarily affects asset prices, not allocations.

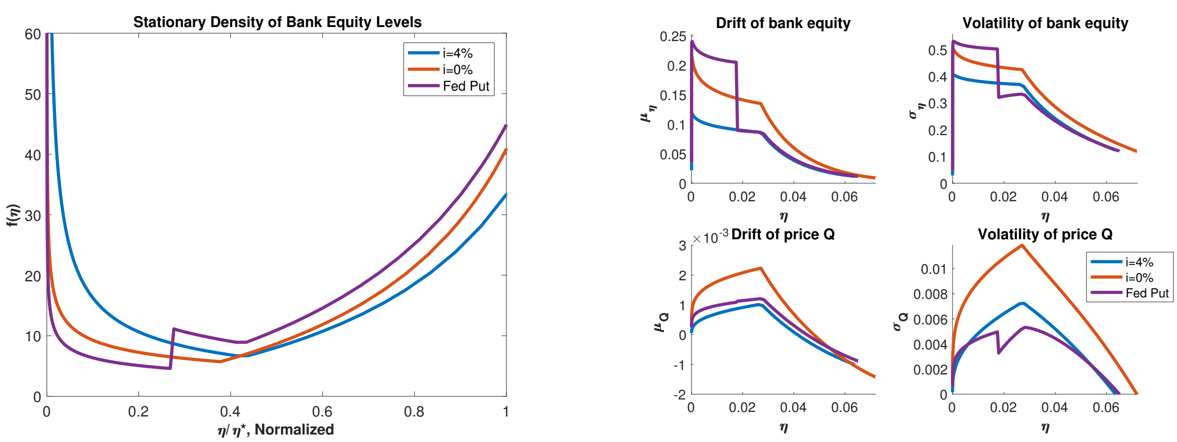
Households are the marginal pricers of capital in bad times, but banks are the marginal pricers in good times. In bad times, asset prices fall because households will only buy capital to produce good 1 at fire-sale prices (households are the marginal pricers). All else equal, low rates in bad times incentivize banks to demand more capital. Banks' higher demand for capital is met by higher supply because households are less productive with good 1, so in equilibrium banks borrow more. In contrast, during good times banks are the marginal pricers of capital. Lower rates in the future increase banks' demand for capital today but do not incentivize households to sell their capital since the allocation of capital is almost efficient, and so the price of capital rises to accommodate banks' demand.

Another way to understand the behavior of leverage in good times is to consider a simpler setting with a single Lucas tree: there is only one sector (one good), and banks have either a financing advantage and/or growth advantage when intermediating capital. The static-efficient allocation of capital would be to have banks intermediate all capital (hold the Lucas tree). For high levels of equity, the allocation would be $\psi = 1$ and leverage would be $1/\eta$ (leverage is ψ/η). Any policy that increased the demand for capital would affect prices alone, because market clearing requires that $\psi \leq 1$. In good times, leverage would be $1/\eta$ for any monetary policy. This simpler

[9] In terms of prices of intermediate goods, the price of good 1 is higher when η is low because banks hold less capital and thus the supply of good 1 is lower. For high η, P_1 is higher when interest rates are high because the liquidity premium passes through to banks' investments, thus increasing P_1 so that investment returns are higher. In general there is a kink at $\bar{\eta}$ when households begin to produce good one; below this level the price P_1 plateaus because households are willing to produce good 1 even if banks have lower equity. Under the Fed Put, there is an additional kink in the price P_1, which drops when rates drop, corresponding to the increase in bank leverage which increases the supply of good 1.

setting “rigs” the result to highlight general equilibrium mechanisms. The two-sector model does not assume away the potential for “moral hazard” (higher leverage in good times), but the line of reasoning still applies. In good times bank capital holdings could increase by reallocating toward good-1 production, but monetary policy primarily affects asset prices.

Stability Figure 2(a) plots the stationary distribution of the economy under each policy, normalizing the x-axis to account for (small, endogenous) changes in η^*. The economy is most stable ex-ante under the Fed Put. In contrast to the worry that a Fed Put would be destabilizing, we find the exact opposite. High interest rates in good times and low interest rates in bad times yield greater stability than always keeping interest rates low. The economy is not in crisis 80% of the time for the Put economy, compared to 75% and 55% for the $i = 0$ and $i = 4\%$ economies. The effects of the Put on leverage explain our findings. High bank leverage during crises allows banks to earn greater excess returns and to rebuild equity quickly. The kink in the PDF for the Put occurs at the rate cut η^{Put}, below which bank leverage and earnings increase, increasing the rate at which banks rebuild equity.



(a) Stationary distribution (normalized support)

(b) Evolutions $d\eta$ and dQ

Figure 2: Equilibrium stability and evolutions with constant rates and Fed Put.

Volatility Figure 2(b) plots equilibrium drifts and volatilities for Q and η. The right-most kinks in drifts, and volatilities occur at $\bar{\eta}$, below which households produce good 1 (roughly $\eta = 2.75\%$

in these economies), thus preventing goods prices from rising if a crisis intensifies (bank equity decreases); the lower kink occurs at the Put strike when leverage spikes. The volatilities of the asset price and bank equity are higher with low constant interest rates, consistent with the standard intuition that low rates may be destabilizing by increasing volatility. Lower constant rates increase the drift of bank equity (μ_η is higher), which is consistent with the intuition that low rates allow banks to finance themselves more cheaply and to build up equity.

While interest rates under the Put always fall between the constant levels considered, the equilibrium levels of volatility do not fall between the levels under the constant-rate policies. Surprisingly, asset price volatility σ_Q is *lowest* with the Fed Put. Asset prices are globally more stable because the economy is overall more stable. With the Put, the drift of bank equity is substantially higher for low η, even higher than would be if rates were constant at zero. The reason is that leverage for low η is much higher than leverage with constant rates. Since banks earn excess returns, greater leverage means greater expected profitability and faster equity growth. Bank equity is less volatile for high levels of η because asset prices are generally less volatile and leverage is roughly the same across policies. Higher leverage for low η means bank equity is more volatile, but since equity growth is so high, banks spend less time in the region with $\eta < \eta^{Put}$. These forces together produce a much more stable financial sector.[10]

Our results contrast with the view that a Fed Put could be destabilizing by leading banks to take excessive leverage in good times, thus leading to higher volatility. Our dynamic results highlight a limitation in the reasoning implicitly based on a quasi-static, partial equilibrium setting. The same forces that could plausibly lead banks to take more leverage in good times *also* provide incentives for banks to take leverage in moderate or not-so-good times before a crisis occurs. If a Put will occur in a crisis, banks can afford to hold assets before the crisis occurs, which means less need to rebalance their portfolios, and thus less asset price volatility. The global behavior of leverage, incentivized by a Put in a crisis, actually provides ex-ante stability.

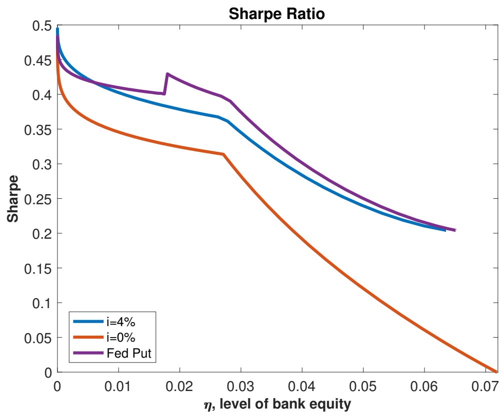
[10] A "Fed Call" with $i^{LAW} = 0\%$ and $i^{Put} = 4\%$ creates dynamics that are nearly the reverse of the Put. The Call economy is the least stable (not in a crisis only 45% of the time). Volatility is globally higher under the Call. Leverage collapses when rates increase during the Call, as does the drift of bank equity.

Unconditional LAW When we only consider policies with constant (unconditional) rates, lower constant interest rates generally improve stability. Even though lower rates produce higher asset price volatility globally, low rates allow banks to rebuild equity quickly and so capital is better allocated in general. These results with constant interest rates provide a strong argument against unconditionally leaning against the wind. Globally higher rates do not mitigate excessive risk-taking enough to offset the losses from inefficient capital allocations arising from low levels of bank equity.

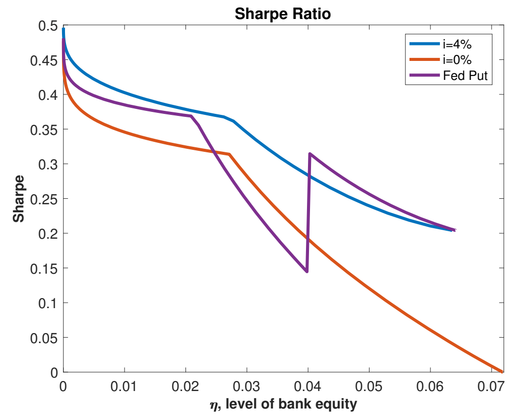
3.2 State-dependent Consequences of Monetary Policy

The stabilizing effects of a Put depend on *when* rates decrease (η^{Put}) because the effects of higher or lower interest rates depend on whether rates primarily affect allocations or prices. When households are the marginal investors in bank-dependent assets (i.e., below $\bar{\eta}$), policy changes affect allocations (banks take on more leverage when rates decrease) without detrimentally decreasing returns on banks' assets. Consider when the policy strike is at a lower level of $\eta^{Put} = 1.75\%$, inside the crisis region, and a higher level of $\eta^{Put} = 4\%$, outside of the crisis region, where only banks produce good 1. Figure 3 plots the Sharpe ratios for these two cases. The Sharpe ratio under the Put with low strike $\eta^{Put} = 1.75\%$ is almost everywhere higher than the Sharpe ratio when rates are held constant rates at 4%. In other words, the Put improves banks risk-adjusted returns both before the Put kicks in (high eta) and even after rates are cut (low eta). However, the Sharpe ratio under the Put with a high strike is substantially below what it would be with the low strike Fed Put, and can fall below the Sharpe with other policies (Figure 3(b)).

When banks are the marginal investors in bank-dependent investments (high η^{Put}), cutting rates decreases the Sharpe ratio for a portion of the state space. In this case, changing interest rates affects goods prices, which changes the returns banks get relative to households. Bank profitability does not increase, and thus lower funding rates do not provide additional incentives to borrow. However, in the region where households produce good 1, households are now the marginal investors in bank-dependent investments. While banks lose the automatic stabilization provided by rising prices (dividend yield) on bank-dependent investments, lower rates no longer pass through



(a) Low strike, $\eta^{Put} = 1.75\%$



(b) High strike, $\eta^{Put} = 4\%$

Figure 3: Sharpe ratios varying the policy strike for piecewise Put.

to prices. Banks can hold more capital without depressing their returns, hence their profitability increases, and they borrow more. These incentives are reflected in equation (12). Banks' risk premium and liquidity premium sum to ℓ, so cutting rates must in equilibrium increase banks' risk premium. Since banks face a single balance sheet decision for risk (namely, leverage) banks' instantaneous risk premium increases (all else equal) by increasing leverage.[11]

To understand the intuition for our results, it is helpful to consider two extreme benchmarks for how monetary policy could pass through into "prices and quantities" (or "returns and leverage"). In a frictionless economy, a decrease in nominal rates would decrease the nominal return on banks' investments exactly one-for-one so that the profitability of banking would not change. There would be no change in banks' leverage, banks' rate of equity growth, or volatility. In contrast, if a decrease in nominal rates did not pass through perfectly to returns, then the profitability of banking would increase, banks would increase their leverage, and banks' rate of equity growth would in-

[11] The risk premium is $-\sigma_\theta(\sigma+\sigma_Q)$. As seen in Figure 2(b), σ_Q actually decreases with the Fed Put, so the increase in the risk premium arises from a more negative σ_θ, which is given by

$$\sigma_\theta = \frac{\theta'(\eta)}{\theta(\eta)} \frac{(\psi-\eta)\sigma}{1-\frac{q'(\eta)}{q(\eta)}(\psi-\eta)}.$$

The risk premium could be higher either because banks are more risk averse or because banks are able to take more risk. The term θ'/θ captures risk aversion while $\psi-\eta$ increases with leverage. The observed increase in leverage in equilibrium means $\psi-\eta$ becomes larger, which makes σ_θ more negative, as $\theta'(\eta) < 0$.

crease. In our model, changes in monetary policy when banks are well-capitalized primarily affect bank returns, with little effect on bank leverage because banks are the marginal investors in bank-dependent investments. Decreasing banks' funding costs (cutting rates) when banks have very low levels of capital increases banks' excess returns and encourages banks to use more leverage, enabling banks to rebuild equity more quickly.

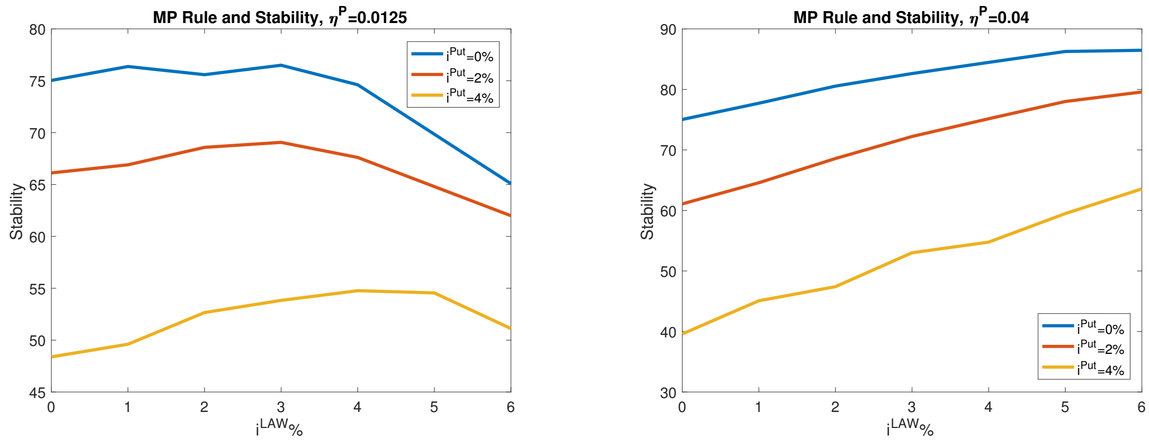
In Appendix C we examine the state-dependent effects of monetary policy by considering the marginal impacts of extending a Fed Put (changing η^{Put}), and by comparing a Put to constant-rates policy with the same average level of rates and vary η^{Put}. These two exercises give us complementary measures of the state-dependent effects of monetary policy on financial stability.

3.3 State-Contingent Easing and Leaning Against the Wind

Constant low interest rates lead to a more stable distribution but higher volatility, but a state-dependent policy can achieve both stability objectives: high interest rates (at times) to generate low volatility, and low interest rates (at other times) to generate a favorable stationary distribution. Leaning against the wind must be state-contingent. The global behavior of leverage depends on the state-dependent behavior of interest rates much more than on the overall level. We now consider timing considerations regarding when to lean against the wind and when to ease.

First, cutting rates in bad times is stabilizing, as is raising rates in good times. We find that LAW generally improves stability when the central bank raises rates outside of crises. We solve the model with $i^{LAW} \in [0, 6\%]$ and $i^{Put} \in [0, 4\%]$. Figure 4 plots the stability measures for $\eta^{Put} = 1.25\%$ and 4%. Two results emerge clearly. A more aggressive Fed Put (lower i^{Put}) is always more stabilizing regardless of the level of i^{LAW}, and the position of the policy strike matters. When η^{Put} is outside the crisis region, then a higher i^{LAW} is more stabilizing—LAW in good times is an effective policy. However, if η^{Put} is low so that rates are high during crises, a higher i^{LAW} leads to a less stable economy. When η^{Put} is sufficiently low (1.25% in this case), LAW can be counterproductive because rates are too high in crisis times.

Since higher i^{LAW} implies higher interest rates *on average*, it is instructive to compare the LAW/Put policy to a constant-rate policy with the interest rate set equal to the average rate under



(a) $\eta^{Put} = 1.25\%$ (b) $\eta^{Put} = 4\%$

Figure 4: Stability with Leaning against the Wind together with a Fed Put, varying (i^{LAW}, i^{Put}) and varying η^{Put}.

the LAW/Put policy. We fix $i^{Put} = 0\%$ and vary i^{LAW}, and we then compute the average interest rate $\bar{i}(i^{LAW})$ under the piecewise policy. We then compare stability under the piecewise policy to stability under the constant-rate policy with the same level of average rates. While in an absolute sense LAW may or may not improve stability (see $\eta^{Put} = 1.25\%$), LAW always leads to stability gains compared to the constant-rate policy, and those gains are larger for higher i^{LAW}.

The evidence of this section suggests that LAW in good times can be an effective policy toward improving financial stability. Increasing interest rates has a detrimental effect on stability when higher rates increase bank funding costs during crises, precisely when increasing bank equity is most valuable, and precisely when changing funding costs has minimal effect on banks' investment returns. A carefully targeted policy of leaning against the wind in good times, and only in good times, can improve financial stability.[12]

Second, we find that central banks should cut rates early to avoid entering a crisis. We find that central banks should "keep their powder dry" by waiting to cut but then cutting quickly. We analyze the issue of timing in detail in Appendix D and provide the summary of our results here. If stability is the only objective, the optimal policy is to cut just before a crisis occurs and then

[12]While endogenous instability, represented either by σ^Q or σ^η, is highest for moderate values of η, excessive risk taking in terms of the effect on stability is highest for high η. This is evident from the behavior of σ^Q in Figure 2. A policy that raises interest rates for middle values of η does not improve stability and generally harms stability.

to cut almost as much as possible. Because our model features non-linear dynamics, *changes* in rates matter more for some variables than the overall level of rates. If the central bank has the flexibility to respond aggressively to financial distress, then it pays to keep the powder dry and then cut fast. These equilibrium dynamics occur when the policy is known ex-ante. If the central bank is constrained so that it cannot promise to respond aggressively to financial distress, then it is better to cut early rather than to delay.

In summary, monetary policy affects financial stability differently at different points in the cycle. Policy decisions in good (or bad) times affect the ex-ante behavior of the financial sector in *both* good and bad times as well. The consequences of a Fed Put, both ex-ante and ex-post, depend on how aggressively monetary authorities lean against the wind in good times, and also on when authorities switch from leaning against to supporting the financial system. The efficiency of the Put depends on agents expecting a Put in the future, and those expectations affect the global dynamics of the system. Understanding the effects of monetary policy on financial stability requires considering how the state-dependent policies of LAW and Fed Put together affect the global dynamics of the financial sector.

Macroprudential Tools A key considerations is the extent to which macroprudential policy measures can be used instead of monetary policy. Appendix E considers two types of macroprudential policies, leverage constraints and equity injections (or tail risk insurance). Monetary policy affects stability in quite different ways from leverage constraints. Equity injections can act somewhat as a substitute for active monetary policy, and with sufficiently aggressive equity injection, monetary policy is unnecessary to provide stabilization.

4 Monetary Policy and Welfare

We now consider whether or not the welfare consequences justify using monetary policy to target financial stability. We first solve for welfare in the baseline real model. We then incorporate reduced-form utility losses from inflation, reflecting New Keynesian mechanisms not explicitly included in our model. This formulation reflects a standard result that welfare losses in a New

Keynesian model with Calvo price setting can be represented by flow quadratic utility losses (see Section 4.4 of Galí, 2015). Our welfare calculations give us a model-specific measure of the costs and benefits of using monetary policy for financial stability.

4.1 Welfare in the Baseline Model

Welfare in our model is easy to characterize. Because capital grows geometrically we can write household welfare as

$$V_t = V(\eta)Y_t,$$

where $V(\eta)$ implicitly includes how the evolution of η affects capital growth. Because households are risk neutral and their investments earn expected return r and $r-\phi_L$ for deposits, expected discounted utility is equal to wealth

$$V(\eta) = (1+(\theta-1)\eta)Q, \tag{14}$$

i.e., the capital price and the present value of banks are together sufficient summaries of the expected discounted value of consumption and liquidity. Given the dynamics of the model, $V(\eta)$ is an increasing function, meaning that expected discounted household utility is higher when the financial sector is well-capitalized. In the baseline model, the costs of using monetary policy for financial stability are potential decreases in the capital price and the present value of banks.[13]

Given the liquidity costs from positive nominal interest rates, in the absence of financial frictions the Friedman Rule would be optimal in our model. Proposition 3 states this explicitly for the case when banks can freely issue equity.

Proposition 3. *When banks can freely issue equity (no financial frictions), the optimal nominal interest rate is $i^* = 0$ so that the Friedman Rule holds.*

The model with financial frictions features pecuniary externalities so that banks generally take excessive risk. A positive liquidity premium can improve welfare by decreasing banks' risk-taking.

[13] While the asset pricing implications of our model are similar to those in Drechsler et al. (2018), their model with heterogeneous agents does not include the same tight relationship between asset prices and welfare.

Proposition 4. *With costly equity issuance, the optimal interest rate at the stochastic steady-state η^* is positive; a positive liquidity premium local to η^* corrects the pecuniary externality from financial frictions.*

Positive liquidity premia can improve stability. Nonetheless, it appears that even with equity issuance constraints the Friedman Rule is still nearly optimal and Proposition 4 is very much a local result. We have not been able numerically to find policies with positive interest rates that increase welfare.

Accordingly, instead of focusing on the absolute level of interest rates in the baseline model, we first focus on comparing a Fed Put policy with the constant interest rate policy holding rates at the average level $\bar{i}$. This exercise takes some baseline level of rate $\bar{i}$ as given and considers increasing rates in good times to lean against the wind, and cutting rates in bad times to support the financial sector. Our results give the welfare consequences of such a policy deviation.

In practice central banks are likely to adopt policies that respond substantially more gradually than a piecewise Put, so for welfare results we consider linear Fed Put policies. To analyze the effect of cutting early and slow compared to cutting early and late, we consider state-dependent policies with two thresholds: above η^{LAW} rates are held constant at i^{LAW}; below η^{Put}, rates are held constant at i^{Put}; between η^{LAW} and η^{Put}, rates change linearly between i^{LAW} and i^{Put}. Formally, we let rates take the following form:

$$i(\eta) = \begin{cases} i^{Put} & \text{if } \eta \leq \eta^{Put}, \\ i^{Put} + \left(\frac{i^{LAW} - i^{Put}}{\eta^{LAW} - \eta^{Put}}\right)(\eta - \eta^{Put}) & \text{if } \eta \in (\eta^{Put}, \eta^{LAW}), \\ i^{LAW} & \text{if } \eta \geq \eta^{LAW}. \end{cases} \tag{15}$$

When the initial condition is known, the effect of the Fed Put on welfare depends (unsurprisingly) on the initial condition. When the economy is presently at low η, the Fed Put increases welfare compared to a policy of constant rates, since the "bailout component" of the policy is most salient, in expected present value terms, thus increasing welfare. When the economy is presently at a high η, the costs of LAW/Put policies (primarily in lowering asset prices for high η), outweigh the benefits from stability. The baseline model with welfare evaluated at a high initial state provides results arguably consistent with the cost-benefit analysis in Svensson (2017). Appendix C.5 provides details of the analysis.

Accordingly, since the welfare consequences of Put policies depend on the initial condition, implying that the "optimal" policy depends on the current state, we primarily consider a "timeless" perspective in which the initial condition is not known. We consider an agnostic view on the initial condition and take expectations according to the stationary distribution. This "timeless" perspective calculates welfare by letting the stationary distribution $f(\eta)$ determine the ex-ante distribution of initial conditions and then computes $\mathbb{E}[V(\eta)]$ using the stationary distribution occurring in equilibrium. This exercise adopts a prior on the capitalization of the financial sector and computes the expected welfare. The welfare results using this timeless perspective on the initial condition show that Fed Put always improves welfare relative to constant rates.

Figure 5 shows the percentage gain in welfare comparing Fed Puts to policies of constant rates. We calculate welfare according to this ex-ante measure, comparing Fed Puts for various levels of η^{Put} and η^{LAW} to the corresponding policies of constant rates at the average level. In each case the Fed Put increases welfare relative to constant rates, and the effects are non-monotonic in the strike η^{LAW}. The higher strike policy has a greater effect on stability, thus increasing the fraction of time in states with higher welfare, but it is precisely in those high states that the Fed Put hurts welfare relative to constant rates.

In the baseline model, whether the stability-benefits of monetary policy outweigh the costs depends on the perspective one takes. Adopting the stationary density as a prior will tend to give crisis regions substantial weight when considering losses. In reality, if the economy is in a crisis then there likely would not be much uncertainty about the initial η. Similarly, when the economy is in "good times," policymakers will be more concerned that the economy is teetering toward a crisis than actually being in one. A more sensible prior for this case may place substantially greater weight on "good times," and this could lead to welfare losses from imposing LAW/Put policies. One credible conclusion is that even without a cost for deviating from $\bar{i}$ (nominal rigidities), the benefits of LAW/Put policies may not always outweigh the costs.

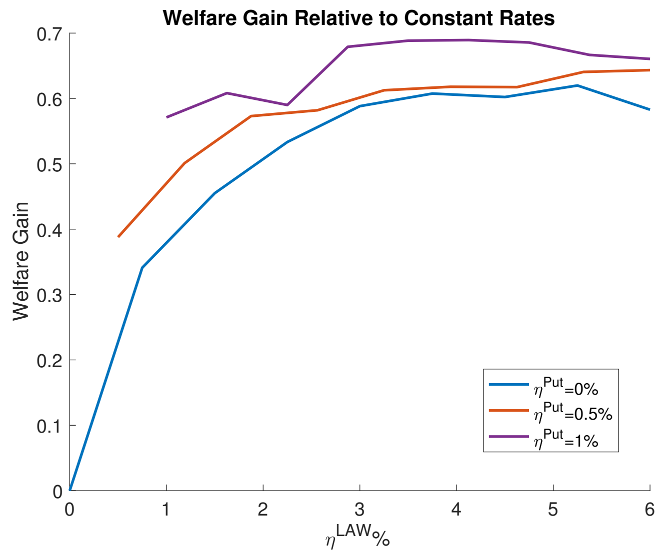


Figure 5: Welfare gains from Fed Put using ex-ante measure, compared to constant interest rates.

4.2 Welfare With Losses From Nominal Rigidities

We now introduce a utility cost of having inflation deviate from target, which reflects output losses that would result form distortions when firms face nominal rigidities (e.g., à la Calvo): households suffer quadratic flow utility loss from inflation. This formulation, centered around zero inflation, is a standard way to represent welfare losses from output gaps in a New Keynesian model (see Section 4.4 of Galí, 2015; see Nuño and Thomas, 2016 for a formalization in continuous time). The central bank now has a reason to avoid interest rates far below target because low interest rates exacerbate distortions arising from nominal rigidities.

Household utility is now

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} (c_{h,t} + \phi_L \delta_{h,t} - \varepsilon_\pi \pi_t^2 Y_t)\, dt \right], \tag{16}$$

where ε_π is the measure of inflation costs and we scale by aggregate capital to maintain homogeneity in welfare. The value of ε_π is a function of structural parameters governing firms' demand elasticity, the Calvo price-setting frequency, and the curvature of the production function. The structural parameters in Galí (2015) imply a value of $\varepsilon_\pi = 216$.

We let $L(\eta)$ denote the discounted expected inflation loss, which solves the following HJB equation:

$$rL(\eta) = \varepsilon_\pi \pi(\eta)^2 + L'(\eta)\eta\mu^\eta + \frac{1}{2}L''(\eta)(\eta\sigma^\eta)^2. \tag{17}$$

Household welfare is now given by $V(\eta) - L(\eta)$. The inflation loss does not have any consequences for prices or quantities in equilibrium but only affects household welfare. The higher are inflation costs, the less likely is welfare to increase when rates deviate from i^*. Given our chosen parameters, $i^* = 2\%$ and we use $\varepsilon_\pi = 216$ as noted.

Before turning to state-dependent policy in our model, we first consider two benchmarks for interest rates. First, we calculate the optimal interest rate i^{FB} in an economy without financial frictions but with inflation costs. The Friedman Rule is no longer optimal, so the optimal interest rate i^{FB} balances the liquidity cost from positive interest rates with the standard New Keynesian costs of deviating from i^*.[14] For these parameters, $i^{FB} = 1.47\%$. As a second benchmark, we restrict monetary policy to a constant nominal rate $\hat{\imath}$ and maximize expected welfare under the timeless initial condition. For these parameters, welfare is maximized near $\hat{\imath} = 1.62\%$.

We now solve for "optimal" monetary policy functions restricted to piecewise linear rules as in equation (15), choosing η^{LAW}, η^{Put}, i^{LAW}, and i^{Put} to maximize expected welfare using the timeless perspective. For this class of functions, the best policy appears to be to set policy roughly as follows: $\eta^{Put} = 1\%$, $\eta^{LAW} = 3.5\%$, $i^{Put} = 1.4\%$, and $i^{LAW} = 1.7\%$.[15] Compared to the first-best benchmark, the optimal policy leans against the wind by about 20bps in good times, and accommodates by about 10bps in bad times (comparing to the constant-rates benchmark the numbers are 10bps and 20bps). It is optimal to begin cutting rates before a crisis, but not too early. Thus, the central bank should keeps its powder dry for a while.

Table 2 displays the results, also varying ε_π higher and lower to 144 and 324. The optimal policy strikes are roughly the same ($\eta^{Put} = 1\%$ and $\eta^{LAW} = 3.5\%$) except that with the lower inflation costs welfare is slightly higher for $\eta^{LAW} = 3.25\%$ so that it is optimal to hold powder dry

[14] We solve for i^{FB} using equation (21) in the Appendix to solve for the optimal allocation λ^* as a function of i and then maximize asset prices subject to discounted inflation costs, hence $Q^*(\lambda^*) - \varepsilon_\pi \frac{i-i^*}{r}$.

[15] The objective function is flat in this neighborhood, and calculating expected welfare requires solving an integral (which induces noise), so small perturbations do not have meaningful changes in welfare.

for a little longer before cutting. The optimal constant interest rate $\hat{i}$ is higher than the first-best i^{FB} (the gap is decreasing in the inflation cost). When considering state-dependent policies, it is always optimal to pursue a Fed Put and to lean against the wind. The extent of the rate drop is decreasing in the inflation cost, and for the baseline inflation cost is on the order of a single rate change (25bps).

Table 2: Optimal Interest Rate Policies with Inflation Costs

ε_π	144	216	324
i^{FB}	1.19%	1.47%	1.65%
$\hat{i}$	1.43%	1.62%	1.75%
i^{LAW}	1.47%	1.70%	1.78%
i^{Put}	1.13%	1.40%	1.66%
Rate Drop	34bps	30bps	12bps

For ε_π even lower than 144, the optimal policy has a lower Put strike η^{Put} and also cuts to a much lower i^{Put}. Comparing the extent of a rate drop is somewhat harder to do in this case since two dimensions are moving: how deep to cut, and over what range. Simply subtracting i^{Put} from i^{LAW} doesn't tell the full story of how much rates are responding since policy takes longer to reach i^{Put}. If we consider $\varepsilon_\pi = 108$ (half the benchmark number) and fix $\eta^{Put} = 1\%$ and $\eta^{LAW} = 3.25\%$ (the optimal values for $\varepsilon_\pi = 144$), then the optimal rate cut is roughly 62bps, indicating that two and a half rate cuts could be justified to support the financial sector. Lower values of ε_π may indeed be justified depending on the microfoundation for nominal rigidities (Nuño and Thomas (2016) consider a continuous-time New Keynesian model with Rotemberg pricing and argue for a much lower calibration of inflation costs).

We find evidence in favor of leaning against the wind in good times and pursuing a Fed Put in crises. Nominal rigidities motivate keeping interest rates near a baseline target, but financial instability motivates a degree of state-dependent policy in response to conditions in the financial sector. Even with large inflation costs, a rate cut to support the financial sector can be justified.[16]

[16]Our stability and welfare results hold broadly in an environment with risk averse agents. We considered another economy with two distinct risk-averse agents, where one agent type is a "banker" and provides intermediation to the other "household" type. Our results are broadly the same in this different environment. We prefer our baseline model for several reasons. First, with two risk-averse agents, shifting mass in the density toward the banker does not

Remark on Moral Hazard We have considered moral hazard in the sense that in good times banks may take more leverage expecting rate cuts in bad times. Section 3.1 shows that leverage does not increase by much in good times because banks are the marginal pricers of capital. The prospect of lower rates in bad times primarily raises asset prices in good times, reflecting bankers' higher valuation for capital. Our welfare analysis confirms that stability gains translate into welfare gains.

However, this form of moral hazard is "honest," in the sense that there are no agency problems for banks beyond the equity issuance constraint, while policymakers are also concerned by "dishonest" moral hazard. We have not modeled any reason why banks may take actions that households would not find desirable at the individual level. Moral hazard occurs only because expectations of lower interest rates tomorrow might incentivize more leverage today, all else equal.

In reality, the bankers who manage banks can be "dishonest" and misbehave. For example, it is common to assume that bankers can "steal" capital and divert it for their own consumption (Di Tella (2019) discusses a variety of reasons why "stealing" is a reasonable approximation of actual misbehavior by financial intermediaries). Di Tella (2019) studies this form of dishonest moral hazard within a continuous-time macro-finance model and finds that the potential to steal produces an inefficient competitive equilibrium because bankers do not internalize the impact of their individual decisions on asset prices. Bankers face a greater incentive to steal when prices are higher because they can earn more per unit of capital. In our setting, this externality would be worsened by accommodative monetary policy. If we place a similar mechanism into our model, then moral hazard may become a larger concern and mitigate the capacity of monetary policy to address financial stability. A tractable approach would be the incentive constraint used in Gertler and Karadi (2011): a Fed put generally raises θ, which would relax the incentive constraint and permit more leverage.

necessarily benefit the household since it means less consumption for the household. Our model does not have this problem and so the behavior of the stationary density is easier to interpret for welfare. Lowering the probability of financial crisis implies higher consumption, which is better for households. Second, welfare analysis is straightforward when households own banks (as in our benchmark model), but with two agents requires Pareto weights or some narrow form of welfare analysis. Depending on the reason for financial intermediation, additional complications can emerge. Third, with two risk-averse agents, when bankers accumulate enough wealth they often have an incentive to lend to households or take zero leverage. In contrast, banks are always leveraged in our baseline model.

Alternative forms of dishonest moral hazard would also temper our results. Banks function as monitors for depositors (see Diamond (1984)), but screening risky investments is costly. When financial conditions are safer, it is a reasonable conjecture that effort exhausted during screening may decrease. This moral hazard may cause banks to systematically choose poorer quality projects, whose returns are either more volatile or lower in expectation. We could also motivate a value-at-risk (VaR) constraint as the outcome of a moral hazard problem between depositors and the bank (see Adrian and Shin (2011)). Accommodative monetary policy may loosen banks' VaR constraints and produce even more leverage in good times, which could undo the stabilizing effects of monetary policy.

Nevertheless, careful considerations of additional moral hazard problems temper our results on the margin but are unlikely to reverse them. Leverage does not always systematically increase when a central bank commits to an interest rate rule which responds to financial conditions, and if it does, then the increase does not always harm welfare. Within our model, monetary policy improves or worsens outcomes depending on whether households or banks are the marginal pricers of capital. More broadly, we hypothesize that honest moral hazard should not be a concern as long as monetary policy only lowers interest rates when capital is used inefficiently by the marginal pricer. When capital is efficiently used, lower rates tomorrow will cause the marginal pricers of capital to simply raise their valuations without changing their leverage substantially. If policymakers and economists believe that lower rates in bad times may still lead to excessive risk taking in good times, then they should examine forms of dishonest moral hazard.

5 Conclusion

We provide a macroeconomic model with a financial sector in which monetary policy endogenously determines the stability of the economy and therefore determines the probability and severity of crises. By affecting risk and liquidity premia, monetary policy can improve the stability of the financial sector and increase household welfare. Policies that combine leaning against the wind in good times with accommodative rates during financial distress can substantially improve stability. The consequences of monetary policy for financial stability are state-dependent, and so the stability benefits of monetary policy depend critically on the timing, with the greatest potential benefits coming when rate cuts occur during financial crises. Nominal rigidities introduce a tradeoff between using monetary policy for standard output-inflation stabilization and for financial stability. We find evidence that the marginal benefits of using monetary policy to target financial stability outweigh the potential welfare costs from distortions caused by nominal rigidities.

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Appendices

A Proofs and Additional Equations

A.1 Proofs

Proof of Proposition 1. Homogeneity and price-taking imply that banks' value function takes the form $U_t = \theta_t n_{b,t}$, where θ_t is the marginal value of banks' equity. The HJB can be written as

$$r\theta_t n_{b,t} = \max_{x_{k1,t}, x_{k2,t}, x_{M,t}, d\zeta_t} d\zeta_t + \mathbb{E}[d(\theta_t n_{b,t})], \tag{18}$$

subject to the constraints (4) and (5).

By Ito's product rule,

$$\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} = (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t})\, dt + (\sigma_{\theta,t} + \sigma_{nb,t})\, dW_t.$$

Suppressing the controls and dropping the differential dt, equation (18) simplifies to

$$r\theta_t n_{b,t} = \max d\zeta_t + \theta_t n_{b,t} \left(\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t}\right).$$

Using the dynamic budget constraint (4),

$$
\begin{aligned}
r\theta_t n_{b,t} &= \max d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\
&\quad + \theta_t n_{b,t}\left(r_t - \gamma(x_{k,t}-1)i_t + x_{k1,t}(\mathbb{E}[dr_t^{b1}] - r_t\,dt) + x_{k2,t}(\mathbb{E}[dr_t^{b2}] - r_t\,dt) - \frac{d\zeta_t}{n_{b,t}}\right) \\
&= \max\,(1-\theta_t)d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\
&\quad + \theta_t n_{b,t}\left(r_t - \gamma(x_{k,t}-1)i_t + x_{k1,t}(\mathbb{E}[dr_t^{b1}] - r_t\,dt) + x_{k2,t}(\mathbb{E}[dr_t^{b2}] - r_t\,dt)\right).
\end{aligned}
$$

In real terms, bank returns on capital holdings satisfy:

$$
dr_t^{bj} = \left(\frac{P_{jt}}{Q_t} + g - \phi_L + \mu_{Q,t} + \sigma\sigma_{Q,t}\right)dt + (\sigma + \sigma_{Q,t})\,dW_t, \tag{19}
$$

where $j = 1,2$ and $P_{jt} = p_{jt}/p_t$ is the real price of intermediate good j. Using these returns, we may write $\sigma_{nb,t} = x_{k,t}(\sigma + \sigma_{Q,t})$. Our remaining controls are $x_{k1,t}, x_{k2,t}, d\zeta_t$. The linearity in $d\zeta_t$ implies that banks use consumption to create a reflecting barrier whenever $\theta_t \leq 1$. Taking FOCs w.r.t. portfolio shares, we obtain, with a slight abuse of notation, the asset-pricing condition

$$
\mathbb{E}[dr_t^{bj}] - r_t \leq -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t, \tag{20}
$$

where $j = 1,2$. Intuitively, this expression says that in equilibrium the excess (real) returns on capital holdings equal the sum of a risk premium and a fixed multiple of the liquidity premium. □

Proof of Proposition 2. We follow the same steps as Brunnermeier and Sannikov (2014) and Phelan (2016). □

Proof of Proposition 3. When banks can freely issue equity, then equilibrium is stationary with constant asset price Q^*, constant land allocation λ^*, and no endogenous risk or instability, implying banks do not have a required risk premium. From equations (8) and (10) prices satisfy

$$
Q^* = \frac{P_2^*}{r-g}, \quad \frac{P_1^* - P_2^*}{Q^*} = LP
$$

where P_1^* and P_2^* are calculated using equation (2) at λ^*. Rearranging, it follows that

$$\lambda^* = \frac{1}{\frac{LP}{r-g}+2}. \tag{21}$$

It therefore follows immediately that Q^* is maximized for $LP = 0$, which yields $\lambda^* = \frac{1}{2}$ with the static-efficient allocation of capital. $\square$

Proof of Proposition 4. The welfare function $V(\eta)$ solves the following differential equation

$$rV(\eta) = z(\eta) + V'(\eta)\eta\mu^\eta + V(\eta)g_Y(\eta) + \frac{1}{2}V''(\eta)(\eta\sigma^\eta)^2 + V'(\eta)\eta\sigma^\eta\sigma, \tag{22}$$

where $z(\eta_t) = \lambda_t^{1/2}(1-\lambda_t)^{1/2} + \phi_L(\psi_t - \eta_t)Q_t$ is output plus convenience yield utility and $g_Y(\eta_t) = g(1-\psi_t) + \psi_t(g-\phi_L) - (\lambda_t - \psi_t)\ell$ is aggregate productivity growth. Taking the derivative of the right-hand side of equation (22) with respect to ψ_t yields

$$\mathscr{L}(\psi_t) \equiv \frac{\partial z(\psi,\lambda)}{\partial\psi} + V'(\eta)\eta\frac{\partial\mu^\eta}{\partial\psi} + V\frac{\partial g_Y}{\partial\psi} + \left(V''(\eta)\eta\sigma^\eta(\psi,\lambda,\eta) + V'\eta\sigma\right)\frac{\partial\sigma^\eta}{\partial\psi}. \tag{23}$$

It suffices to show that if interest rates are zero then $\mathscr{L}(\psi_t) < 0$ near η^*. Near η^*, $\psi(\eta) = \lambda(\eta)$, so that

$$\frac{\partial z(\psi,\lambda)}{\partial\psi} = P_1 - P_2 + \phi_L q_t,$$

and $\frac{\partial g_Y}{\partial\psi} = -\phi_L$. By smooth-pasting, $V'(\eta^*) = 0$. Plugging in terms, we have

$$\mathscr{L}(\psi_t) = P_1 - P_2 + V''(\eta^*)\eta\sigma^\eta(\psi,\lambda,\eta)\frac{\partial\sigma^\eta}{\partial\psi} - \phi_L(V(\eta^*) - Q(\eta^*)).$$

Rearranging equation (22) yields $V(\eta^*) = \frac{z(\eta^*)}{r-g_Y} + \frac{1}{2(r-g_Y)}V''(\eta^*)(\sigma^\eta(\psi,\eta))^2$. The first term is the present discounted value if the system did not move from η^*. Since welfare is strictly less than $\frac{z(\eta^*)}{r-g_Y}$, $V''(\eta^*) < 0$. $V(\eta) \geq Q(\eta)$ because $\theta_t \geq 1$. When $\lambda_t = \psi_t$, $P_1 - P_2 = -\sigma_t^\theta(\sigma + \sigma^Q)Q_t$ when liquidity premia are zero. Furthermore $\frac{\partial\sigma^\eta}{\partial\psi} > 0$. In competitive equilibrium $\theta'(\eta^*) = 0$, so that $\sigma_t^\theta = 0$, which implies $P_1 = P_2$ when the liquidity premium. Hence $\mathscr{L}(\psi_t) < 0$. Since the marginal social value of ψ_t is negative at η^* if the liquidity premium is zero, welfare in competitive equilibrium would improve if leverage near η^* decreased on the margin, which occurs with a positive liquidity premium. $\square$

A.2 Equilibrium System of Differential Equations

We have $\mu_{\theta,t} = \phi_L - \gamma i_t$: $\mu_{\theta,t}$ is always less than or equal to the equivalent drift in the economy with zero liquidity shocks. Since θ_t decreases with η, a smaller drift fo θ_t implies that θ_t, in expectation, moves toward larger values and bad times at a slower rate, reflecting the partial equilibrium intuition that higher interest rates should disincentivize excessive risk-taking. By Ito's lemma, we also have that

$$\mu_{\theta,t} = \frac{\theta'}{\theta}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{\theta''}{\theta}\eta_t^2\sigma_{\eta,t}^2.$$

Setting these two equations equal to each other yields a second-order ODE in η_t for θ. Similarly, using equation (9) and Ito's lemma, we have the following equations

$$\begin{aligned}
\mu_{Q,t} &= r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t}, \\
\mu_{Q,t} &= \frac{Q'}{Q}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{Q''}{Q}\eta_t^2\sigma_{\eta,t}^2.
\end{aligned}$$

Hence, we obtain a coupled system of second-order ODEs:

$$\begin{aligned}
\theta'' &= \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(\phi_L - \gamma i_t - \frac{\theta'}{\theta}\eta_t\mu_{\eta,t}\right), \\
Q'' &= \frac{2Q}{(\eta_t\sigma_{\eta,t})^2}\left(r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t} - \frac{Q'}{Q}\eta_t\mu_{\eta,t}\right).
\end{aligned}$$

We solve for $\sigma_{Q,t}, \sigma_{\theta,t}$ in closed form using Ito's lemma, and these terms remain the same as in Phelan (2016).

A.3 Evolution of η

It remains to derive the evolution of η_t specified in Proposition. The net worth of banks is scale invariant, and banks only use capital in the production of good 1. Define $d\Xi_t \equiv d\zeta_t / n_{b,t}$. Assuming full self-insurance, the law of motion becomes

$$\frac{dn_{b,t}}{n_{b,t}} = \big(r_t - \gamma(x_{k,t} - 1)i_t\big)\, dt + x_{k,t}(dr_t^{b1} - r_t\, dt) - d\Xi_t.$$

Substituting in $r_t = r - \phi_L$ and (10) yields

$$\frac{dn_{b,t}}{n_{b,t}} = \big(r - \phi_L - \gamma(x_{k,t} - 1)i_t + x_{k,t}\left(-\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t\right)\big)\, dt - d\Xi_t + x_{k,t}(\sigma + \sigma_{Q,t})\, dW_t$$

Define ψ_t to be the share of capital held by banks. Then $x_{k,t} = \psi_t / \eta_t$, and we have

$$\frac{dn_{b,t}}{n_{b,t}} = \left(r - \phi_L + \gamma i_t - \frac{\psi_t}{\eta_t}\sigma_{\theta,t}(\sigma + \sigma_{Q,t})\right) dt - d\Xi_t + \frac{\psi_t}{\eta_t}(\sigma + \sigma_{Q,t})\, dW_t.$$

By Ito's product rule,

$$\frac{d(Q_t Y_t)}{Q_t Y_t} = (\mu_{Q,t} + \mu_{Y,t} + \sigma\sigma_{Q,t})\, dt + (\sigma + \sigma_{Q,t})\, dW_t.$$

We may write $\mu_{Y,t}$ and $\mu_{Q,t}$ as:

$$\begin{aligned}
\mu_{Y,t} &= \psi_t(g - \phi_L) + (\lambda_t - \psi_t)(g - \ell) + (1 - \lambda_t)g = g - \psi_t\phi_L - (\lambda_t - \psi_t)\ell \\
\mu_{Q,t} &= r - \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t - \frac{P_{1t}}{Q_t} - g - \sigma\sigma_{Q,t} \\
&= r - \phi_L - \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t - \frac{P_{1t}}{Q_t} - (g - \phi_L) - \sigma\sigma_{Q,t}
\end{aligned}$$

Plugging this in and applying Ito's quotient rule implies

$$\begin{aligned}
\frac{d(1/(Q_t Y_t))}{1/(Q_t Y_t)} = &\left((\sigma + \sigma_{Q,t})^2 - (r - \phi_L) + \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) - \gamma i_t + \frac{P_{1t}}{Q_t} + g - \phi_L + \sigma\sigma_{Q,t}\right) dt \\
&- \big(g - \psi_t\phi_L - (\lambda_t - \psi_t)\ell + \sigma\sigma_{Q,t}\big)\, dt - (\sigma + \sigma_{Q,t})\, dW_t.
\end{aligned}$$

By Ito's product rule,

$$
\begin{aligned}
\frac{d\eta_t}{\eta_t} &= \left(\gamma i_t - \frac{\psi_t}{\eta_t}\sigma_{\theta,t}(\sigma+\sigma_{Q,t})\right) dt - d\Xi_t + \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})\, dW_t \\
&\quad + \left((\sigma+\sigma_{Q,t})^2 + \sigma_{\theta,t}(\sigma+\sigma_{Q,t}) - \gamma i_t + \frac{P_{1t}}{Q_t} + g - \phi_L + \sigma\sigma_{Q,t}\right) dt \\
&\quad - \left(g - \psi_t\phi_L - (\lambda_t - \psi_t)\ell + \sigma\sigma_{Q,t}\right) dt - (\sigma+\sigma_{Q,t}), dW_t - \frac{\psi_t}{\eta_t}(\sigma+\sigma_{Q,t})^2\, dt \\
&= \left(\frac{P_{1t}}{Q_t} + (\lambda_t - \psi_t)\ell - (1-\psi_t)\phi_L - \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma+\sigma_{Q,t})(\sigma_{\theta,t}+\sigma+\sigma_{Q,t})\right) dt \\
&\quad - d\Xi_t + \left(\frac{\psi_t}{\eta_t} - 1\right)(\sigma+\sigma_{Q,t})\, dW_t
\end{aligned}
$$

A.4 Numerical Algorithm

Assuming that the government implements (28), we can use the exact same algorithm with only mild adjustments to asset pricing conditions and equilibrium expressions. For clarity, we derive these expressions here.

The Bellman equation is given by

$$
\begin{aligned}
r\theta_t n_{b,t} = \max_{x_{k,t}, d\zeta_t} & (1-\theta_t)\, d\zeta_t + \theta_t n_{b,t}(\mu_{\theta,t} + \sigma_{\theta,t}\sigma_{nb,t}) \\
& + \theta_t n_{b,t}\left(r_t + \gamma i_t + x_{k,t}(\mathbb{E}[dr_t^{b1}] - r_t - \gamma i_t)\right).
\end{aligned}
$$

Plugging in our first-order conditions and using the bang-bang control in $d\zeta_t$ imply

$$
r\theta_t n_{b,t} = \theta_t n_{b,t}\mu_{\theta,t} + \theta_t n_{b,t}\,(r - \phi_L + \gamma i_t) \Rightarrow \mu_{\theta,t} = \phi_L - \gamma i_t.
$$

We have that $\mu_{\theta,t}$ is always less than or equal to the equivalent drift in the economy with zero liquidity shocks. Since θ_t decreases with η, a smaller drift fo θ_t implies that θ_t, in expectation, moves toward larger values and bad times at a slower rate, reflecting the partial equilibrium intuition that higher interest rates should disincentivize excessive risk-taking. By Ito's lemma, we also have that

$$\mu_{\theta,t} = \frac{\theta'}{\theta}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{\theta''}{\theta}\eta_t^2\sigma_{\eta,t}^2.$$

Setting these two equations equal to each other yields a second-order ODE in η_t for θ.

We can similarly derive a second-order ODE for q. Using the second equation in (10), we may write

$$\mu_{Q,t} = r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t}.$$

By Ito's lemma, we also have

$$\mu_{Q,t} = \frac{Q'}{Q}\eta_t\mu_{\eta,t} + \frac{1}{2}\frac{Q''}{Q}\eta_t^2\sigma_{\eta,t}^2.$$

In this way, we obtain a coupled system of second-order ODEs:

$$\theta'' = \frac{2\theta}{(\eta_t\sigma_{\eta,t})^2}\left(\phi_L - \gamma i_t - \frac{\theta'}{\theta}\eta_t\mu_{\eta,t}\right), \tag{24}$$

$$Q'' = \frac{2Q}{(\eta_t\sigma_{\eta,t})^2}\left(r - \frac{P_{2t}}{Q_t} - g - \sigma\sigma_{Q,t} - \frac{Q'}{Q}\eta_t\mu_{\eta,t}\right). \tag{25}$$

By Ito's lemma, we can solve for $\sigma_{Q,t}, \sigma_{\theta,t}$ in closed form, and they remain the same as in Phelan (2016).

Specialization Households do not produce good 1, so $\psi = \lambda$. Taking the difference between the two equations in (10), we obtain a market-clearing condition for capital allocations:

$$P_{1t} - P_{2t} = -\sigma_{\theta,t}(\sigma + \sigma_{Q,t})Q_t + \gamma i_t Q_t \tag{26}$$

Non-Specialization Households produce good 1, so we must have

$$\frac{P_{1t}}{Q_t} + g - \ell + \mu_{Q,t} + \sigma\sigma_{Q,t} - r = \frac{P_{2t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} - r \Rightarrow P_{1t} - P_{2t} = lQ_t.$$

However, since $\psi < \lambda$, we need an additional condition to pin down ψ. In this case, we use the fact that households' asset pricing condition for good 1 is satisfied, hence

$$\frac{P_{1t}}{Q_t} + g + \mu_{Q,t} + \sigma\sigma_{Q,t} - \frac{P_{1t}}{Q_t} - (g - \ell) - \mu_{Q,t} - \sigma\sigma_{Q,t} = r - \sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t - r$$

$$\Rightarrow \ell = -\sigma_{\theta,t}(\sigma + \sigma_{Q,t}) + \gamma i_t \tag{27}$$

In other words, the household efficiency loss from investing in the "intermediation sector" equals the sum of the banks' risk premium and the liquidity premium.

Plugging in, we obtain

$$\ell = -\frac{\theta'}{\theta}\frac{Q(\psi - \eta)\sigma}{Q - Q'(\psi - \eta)}\left(\frac{\sigma Q}{Q - Q'(\psi - \eta)}\right) + \gamma i_t.$$

Let $x = \psi - \eta$ and $y = \ell - \gamma i_t$. Then

$$\begin{aligned} y\left(Q - Q'x\right)^2 &= -\frac{\theta'}{\theta}\sigma^2 Q^2 x \\ Q^2 - 2QQ'x + (Q'x)^2 &= -\frac{\theta'}{\theta}\frac{\sigma^2}{y}Q^2 x \\ x^2(Q')^2 + x\left(\frac{\theta'}{\theta}\frac{\sigma^2}{y}Q^2 - 2QQ'\right) + Q^2 &= 0. \end{aligned}$$

By the quadratic formula, we may solve for x and use that $\psi = x + \eta$.

B A Model of Monetary Policy Transmission

B.1 Monetary Policy: Nominal Rates, Inflation, and Liquidity Premia

Following Drechsler et al. (2018), monetary policy determines the opportunity cost of holding liquid assets—namely, central bank reserves—rather than capital (i.e., determines the liquidity premium). To capture the money multiplier of reserves, each dollar of reserves yields $m > 1$ effective liquid assets. The central bank can create and withdraw reserves by exchanging government bonds through open market operations (see Drechsler et al. (2018) for implementation details).

Let M_t be the total dollar value of reserves in the economy, and let s_t be the value of a dollar in consumption units. Letting reserves be the numeraire, s_t becomes the inverse price level, and the real value of liquid assets held by banks scaled by aggregate wealth is

$$S_t = \frac{s_t(m-1)M_t}{Q_t Y_t}.$$

The remaining liquidity is given by the value of government bonds held by the central bank, $s_t M_t$.

Inflation and the Nominal Rate We assume inflation is locally deterministic, i.e.

$$-\frac{ds_t}{s_t} = \frac{dp_t}{p_t} = \pi_t \, dt.$$

Since the rate on deposits pins down the risk-free interest rate, the nominal interest rate is

$$i_t = r - \phi_L + \pi_t.$$

Liquidity Premia The liquidity premium on reserves is the opportunity cost of holding liquid assets. Because reserves pay no interest, their return is equal to their capital gain, so the liquidity premium that deposits earn is

$$r_t - \frac{ds_t}{s_t} = r_t + \pi_t = i_t,$$

which is precisely the nominal interest rate.

The government earns seigniorage from the liquid assets held by banks. To close the model, we assume the seigniorage is distributed to households, so the fraction of bank equity is unaffected by seigniorage, and the government maintains zero net worth.

Policy Implementation While we relegate the entirety of the details of policy implementation to Drechsler et al. (2018), we include the formalization of policy because they allow us to derive an expression satisfied by the real value of liquidity.

Proposition 5. *To implement the nominal interest rate rule i_t, the nominal supply of reserves M_t must grow according to*

$$\frac{dM_t}{M_t} = (i_t - r_t)\,dt + \frac{dS_t}{S_t} + \frac{dQ_t}{Q_t} + \left(\frac{dS_t}{S_t}\right)\left(\frac{dQ_t}{Q_t}\right) + \frac{dY_t}{Y_t} + \left(\frac{dS_t}{S_t} + \frac{dQ_t}{Q_t}\right)\left(\frac{dY_t}{Y_t}\right), \quad (28)$$

and the real value of liquidity as a share of wealth satisfies

$$S_t = \eta_t \kappa (x_{k,t} - 1). \quad (29)$$

B.2 Banks' Problem

Following Drechsler et al. (2018) we assume deposits are subject to funding shocks, which are modeled as a Poisson process J_t with constant intensity χ. J_t is an aggregate shock, and when J_t realizes, banks must immediately redeem a fraction $\frac{\kappa}{1+\kappa}$ of their deposits, where $\kappa > 0$. Only a fraction $1-\rho \in (0,1)$ of capital's value can be recovered quickly enough to absorb a funding shock. Banks may self-insure by holding liquid assets, which can be liquidated without causing fire sales when a funding shock realizes.

Let $x_t = (x_{k1,t}, x_{k2,t}, x_{M,t})$ be portfolio weights (summing to one) on capital used for good 1, capital used for good 2, and reserves. We use the shorthand $x_{k,t} \equiv x_{k1,t} + x_{k2,t}$ to refer to banks' share of wealth invested in capital.

Formally, banks solve the problem

$$\max_{\{x_t, d\zeta_t\}} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)}\, d\zeta_t \right],$$

subject to

$$\begin{aligned} \frac{dn_{b,t}}{n_{b,t}} = {} & \left(r_t - x_{M,t} i_t + \frac{S_t}{m} i_t \right) dt + (x_{k1,t}\, dr_t^{b1} + x_{k2,t}\, dr_t^{b2} - x_{k,t} r_t\, dt) - \frac{d\zeta_t}{n_{b,t}} \\ & - \frac{\rho}{1-\rho} \max\left\{ \frac{\kappa}{1+\kappa}(x_{k,t} + m x_{M,t} - 1) - m x_{M,t}, 0 \right\} dJ_t, \end{aligned} \quad (30)$$

$$n_{b,t}, x_{k1,t}, x_{k2,t}, x_{M,t}\, d\zeta_t \geq 0. \quad (31)$$

Banks earn the deposit rate, pay the liquidity premium on government bonds, receive seigniorage payments from the government, earn the risk premium on capital holdings, and pay dividends at rate $d\zeta_t$. (We can also pay some seigniorage to banks in proportion to their wealth, which will affect the evolution of their net worth without meaningfully changing equilibrium dynamics.) The second line of (30) reflects the exposure of banks to funding shocks given their portfolio.

Given the following conditions, it is optimal for banks to fully self-insure in equilibrium.

Lemma 2. *Suppose χ and ρ are such that, for all i_t,*

$$\chi \frac{\rho}{1-\rho} \frac{\kappa}{1+\kappa} \geq LP_t, \tag{32}$$

where $LP_t \equiv \frac{\kappa}{m} i_t$ is the liquidity premium at interest rate i_t. Then banks fully self-insure, and their liquidity demand is given by

$$mx_{M,t} = \max\left\{\kappa(x_{k,t}-1), 0\right\}.$$

Hence, nominal interest rates i_t determine the liquidity premium LP_t in equilibrium, where $\gamma = \frac{\kappa}{m}$ in the main text. By Ito's product rule,

$$\begin{aligned}\frac{d(\theta_t n_{b,t})}{\theta_t n_{b,t}} &= (\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t})\, dt + (\sigma_{\theta,t} + \sigma_{nb,t})\, dW_t \\ &\quad - \frac{\rho}{1-\rho} \max\left\{\frac{\kappa}{1+\kappa}(x_{k,t} + mx_{M,t} - 1) - mx_{M,t}, 0\right\} dJ_t.\end{aligned}$$

Suppressing the controls and dropping the differential dt, equation (18) simplifies to

$$r\theta_t n_{b,t} = \max d\zeta_t + \theta_t n_{b,t}\left(\mu_{\theta,t} + \mu_{nb,t} + \sigma_{\theta,t}\sigma_{nb,t}\right) - \chi \frac{\rho}{1-\rho} \max\left\{\frac{\kappa}{1+\kappa}(x_{k,t} + mx_{M,t} - 1) - mx_{M,t}, 0\right\}$$

Under the assumption of full self-insurance, we may ignore the Poisson term. Because bank deposits provide liquidity services, they will always be levered, so we may directly substitute $mx_{M,t} = \kappa(x_{k,t} - 1)$ into the Bellman equation.

Proof Lemma 2. Taking the first derivative w.r.t. $mx_{M,t}$ and multiplying through by κ, we have that an optimal choice of $mx_{M,t}$ satisfies

$$0 = -\frac{\kappa}{m} i_t - \chi \kappa \frac{\rho}{1-\rho} \max\left\{\frac{\kappa}{1+\kappa} - 1, 0\right\} \geq -\frac{\kappa}{m} i_t + \chi \frac{\rho}{1-\rho} \frac{\kappa}{1+\kappa}$$

Using our hypothesis, we have

$$0 \geq -\frac{\kappa}{m} i_t + \frac{\kappa}{m} i_t = 0,$$

so the FOC is always satisfied. Since i_t in equilibrium depends on η_t, a bounded variable, there always exist χ, ρ sufficiently large to ensure that (32) always holds.

Finally, to ensure that the max function returns zero, we need

$$0 = \frac{\kappa}{1+\kappa}(x_{k,t} - 1) - \frac{1}{1+\kappa} mx_{M,t},$$

which implies the desired demand function. □

C State-dependent Consequences of Monetary Policy

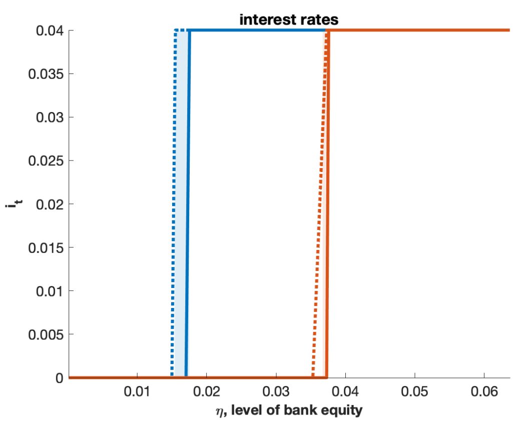
We now consider the state-dependent effects of monetary policy in two ways. First, we consider the marginal impacts of extending a Fed Put (changing η^{Put}). Second, we compare a Put to constant-rates policy with the save average level of rates and vary η^{Put}. These two exercises give us complementary measures of the state-dependent effects of monetary policy on financial stability.

C.1 Marginal Impacts of Monetary Policy

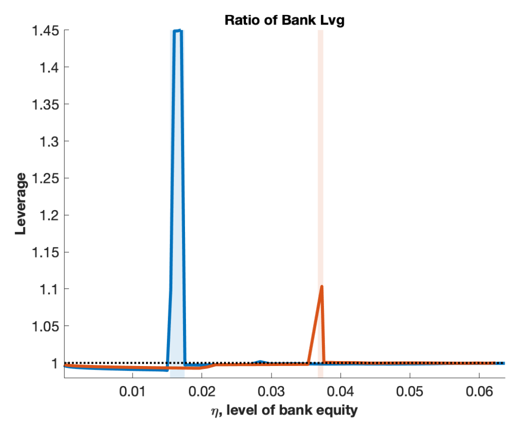
We now examine the marginal impacts of changing fed policy depending on the state of the economy η. We consider piecewise rules with $i^{Put} = 0\%$ and $i^{LAW} = 4\%$: policy holds rates at zero for $\eta < \eta_0^{Put}$ but set rates to 4% for high levels of η. We then set η^{Put} to a higher level $\eta_1^{Put} > \eta_0^{Put}$, to extend the range over which rates are held at zero by the equivalent of 1% of time the economy is in that range according to the stationary distribution (the exact number 1% is not critical, we get similar results with 5% though the implications are obviously less localized). Denoting outcomes for variable X under each policy by X_0 and X_1, we then compute the ratio $\frac{X_1(\eta)}{X_0(\eta)}$ to demonstrate how the equilibrium outcome changes when policy holds rate low for longer.[17]

Figure 6(a) plots the interest rate rules we consider. The shaded portion corresponds to the range over which the central bank extends zero rates. Thus, the blue exercise extends low rates from $\eta_0^{Put} = 1.54\%$ to $\eta_1^{Put} = 1.75\%$, and the red exercise extends low rates from $\eta_0^{Put} = 3.67\%$ to $\eta_1^{Put} = 3.75\%$. These levels of η^{Put} contrast results when policies primarily change when households are, or are not, the marginal investors in bank-dependent assets.

Figure 6(b) plots the ratio of leverage in each case. Leverage spikes during the extension period, with a smaller spike when extension occurs at higher η^{Put}. Hence, the marginal impact on leverage is greatest when η^{Put} is low. This is consistent with the observation that when policy occurs when banks are the marginal investor in bank-dependent investments, policy changes primarily affect prices instead of allocations.



(a) Interest rate rules



(b) Change in leverage

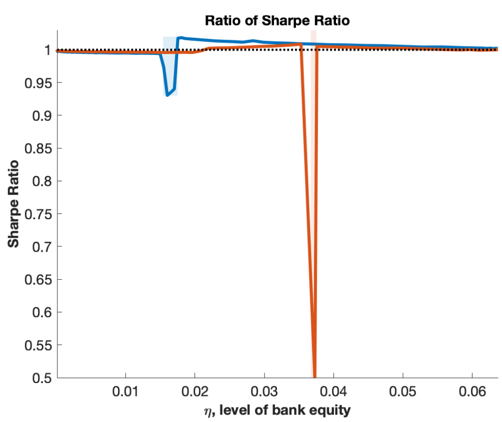
Figure 6: Extending low interest rates and marginal impacts on changes in leverage.

Figure 7(a) plots the changes in Sharpe ratios. For the high-η^{Put} policy, the marginal impact of

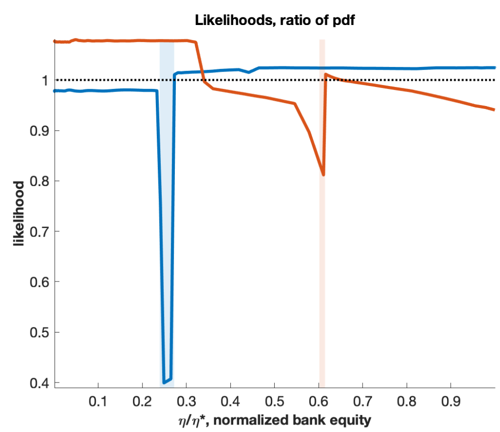
[17]This policy can also be viewed as a variation of "Forward Guidance," from the perspective of a low-η economy: it is as if the central bank announced that it will hold rates to zero until η reaches a $\eta_1^{Put} > \eta_0^{Put}$, which would imply a longer range of zero rates than previous.

extending low rates dramatically decreases the Sharpe ratio over the policy extension range, with almost no increase globally. In contrast, the marginal impact of the low-η^{Put} policy is to increase the Sharpe ratio for $\eta > \eta_1^{Put}$ with a small decrease in the Sharpe over the policy extension range.

Figure 7(b) plots the changes the stationary distribution. We normalize the support by dividing by η^* to compare the stationary distributions because the range of the economy changes substantially between those policies. The marginal impact of the low-η policy is strictly to stabilize the economy, but the marginal impact of the high-η policy can increase the likelihood of crises. In this case, the likelihood of being in high η regions actually decreases when the high-η policy is extended, which further supports the interpretation that the marginal impact of the high-η policy is destabilizing. Indeed the marginal impact on stability of extending rates up to $\eta^{Put} = 3.75\%$ is bad for stability. Extending rates up to $\eta^{Put} = 1.75\%$ has a marginal impact of decreasing the probability of crises by 1.51%, while extending rates up to $\eta^{Put} = 3.75\%$ has a marginal impact of *increasing* the probability of crises by 1.48%.[18]



(a) Change in Sharpe ratio



(b) Change in PDF of η, normalized

Figure 7: Marginal impacts of extending low interest rates on returns and stability.

[18]The policies have similar, though quantitatively different, marginal impacts on price volatility and the evolution of η. Price volatility falls for $\eta > \eta_1^{Put}$ but rises once the economy enters the policy extension range below η_1^{Put}. Thus, mechanically, price volatility is lower for a much larger range when the policy occurs at a lower η. Similarly, the marginal impacts on equity drift and volatility are much larger for the low-η policy, which is why the effect on stability is much greater for that policy.

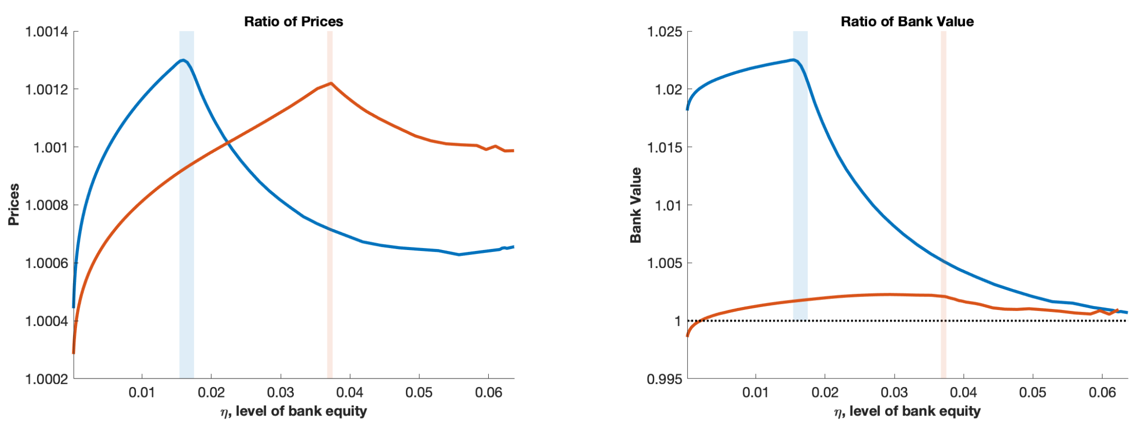
C.2 Marginal Impacts on Welfare

Table 3 calculates the marginal impact on ex-ante welfare of extending low rates. As one would expect given the positive results, the marginal impact is state-dependent and non-monotonic.

Table 3: Marginal impacts of extending low interest rates on welfare using ex-ante measure

η^{Put}	1.75%	2.75%	3.75%
Gain in $\mathbb{E}[V(\eta)]$	0.440%	0.339%	0.361%

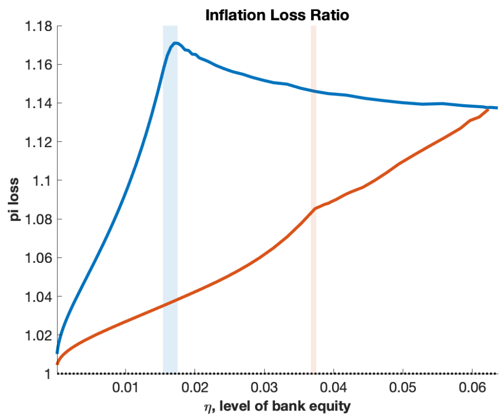
Figure 8 plots the marginal impacts on the asset price and bank value. The marginal impact of increasing prices varies over the state space similarly to the effect of "forward guidance" in Drechsler et al. (2018). Additionally, for low η, the marginal impact on the asset price is greatest when the central bank policy occurs for low η^{Put}. However, the marginal impact on bank value is always greater when the central bank policy occurs for low η^{Put}. When the central bank cuts rates at a high η^{Put}, the marginal impact of extending the rate cut has almost no effect on bank value, and even decreases bank value in the lowest-capitalized states. In essence, the effectiveness of monetary policy to support the economy (including bank values) wears off in this case, and extending the range of easing means that monetary policy becomes ineffective more quickly.



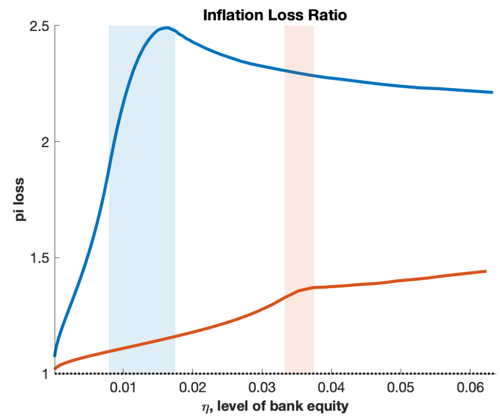
(a) Change in Q (b) Change in bank value

Figure 8: Marginal impacts of extending low interest rates on asset price and bank value.

Figure 9 plots the marginal impacts on inflation loss. Since it plots a ratio, the figure applies for all $\varepsilon_\pi > 0$. However, the inflation loss is the one variable that has meaningful qualitative changes when the policy is extended by more than 1%. Panel (b) plots the marginal impact on the inflation cost when we extend low rates by 5%.



(a) Extend by 1%



(b) Extend by 5%

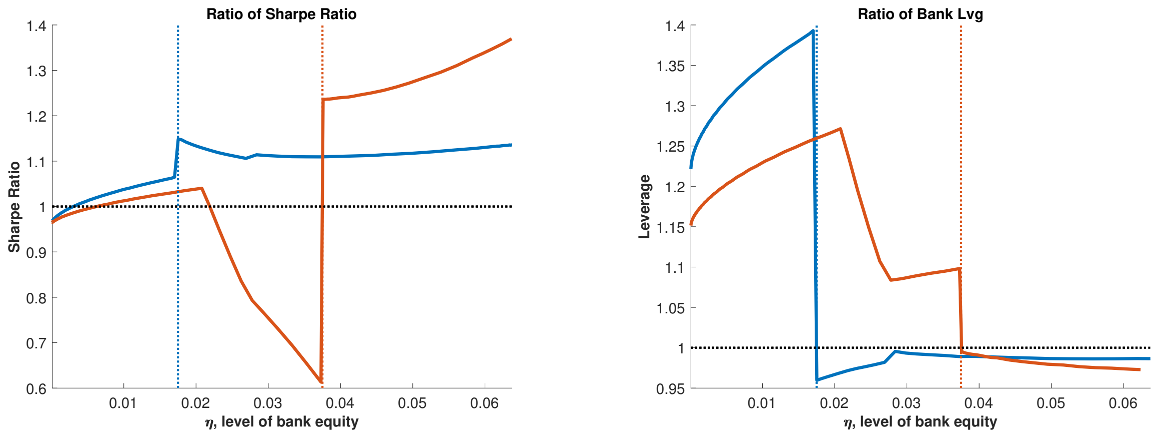
Figure 9: Marginal impact on inflation costs of extending low rates.

C.3 State-dependent Comparison of Fed Put and Constant Rates

We now consider in more detail the consequences of Fed Puts compared to to policies of constant interest rates. We first consider piecewise Puts, which have the starkest results, and then we consider linear Puts.

C.3.1 Piecewise Puts

We set $i^{Put} = 0\%$ and $i^{LAW} = 4\%$, and let $\eta^{Put} = 1.75\%$ and 3.75%, which correspond to average rates of 3.53% and 2.83% respectively. Compared to constant interest rates at the average level, the Put increases leverage below η^{Put} but not otherwise; improves stability (decreases the fraction of time banks have low equity and increases the fraction of time with high equity); decreases price volatility; and increases Sharpe ratios, though non-monotonically depending on when the Fed Put kicks in. While a Fed Put is generally stabilizing, the results on Sharpe ratios and leverage depend on the location of η^{Put}.



(a) Sharpe ratio

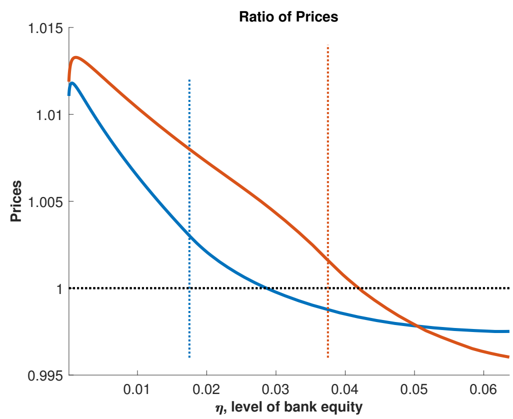
(b) Leverage

Figure 10: Effects of Fed Put on Sharpe Ratios and Leverage compared to constant interest rates.

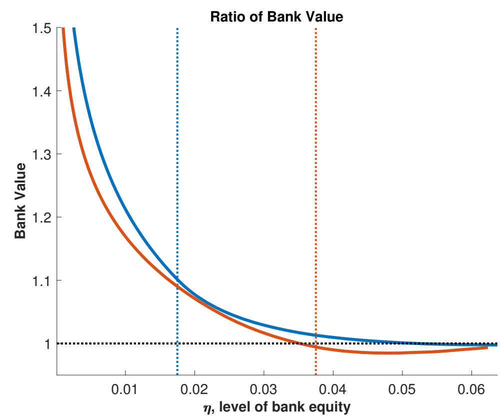
Figure 10(a) plots the ratio of Sharpe ratios is each case; the vertical dotted line marks the policy strike. Compared to constant rates, the Fed Put with a low strike leads to higher bank Sharpe ratios almost everywhere. In contrast, the high-strike Put decreases the Sharpe between $\bar{\eta}$ and η^{Put}, the range over which banks are the marginal investors, so rate cuts primarily pass through to returns.

Figure 10(b) plots the ratio of leverage in each case. As discussed, the Put has the effect of propping up leverage when rates are cut: compared to the constant-rate policy, leverage is generally higher during for $\eta < \eta^{Put}$. However there are two differences to note. First, leverage can be lower for $\eta > \eta^{Put}$ (the effect is small for high η). In particular, when η^{Put} is very low (the blue at 1.75%), leverage just above η^{Put} is noticeably lower than would occur under constant rates. In other words, there is no "moral hazard": banks do not take on high leverage in anticipation of being "bailed out" later by a rate cut. Second, the Put with high strike props up leverage by much less, especially over the range where banks are the marginal investor.

Figure 11 plots the changes in the asset price and bank value. With the Fed Put, prices are higher in bad times and lower in good times, but the highest effectiveness to raise bad-time prices occurs for low strike puts. Banks nearly always like the Fed Put.



(a) Asset price Q

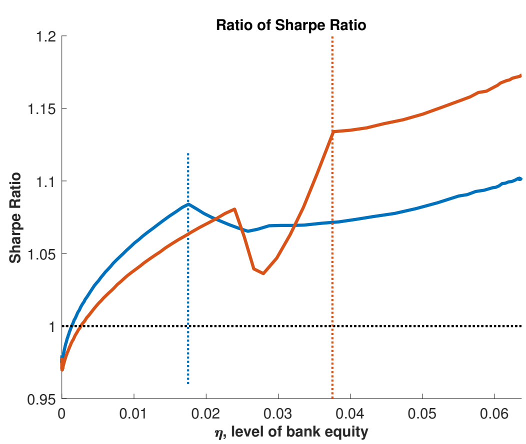


(b) Bank Value

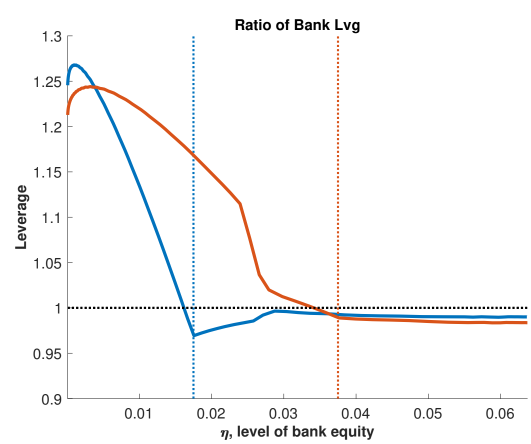
Figure 11: Effect of Fed Put on asset price and bank value.

C.3.2 Linear Puts

Figure 12 plots the ratio of Sharpe ratios and the ratio of leverage in each case. Compared to the piecewise case, the change in the Sharpe ratio is not nearly as dramatic—the high-strike Put increases the Sharpe ratio everywhere, but not always by as much, and not monotonically in η. Under the linear Put, leverage does not begin to increase at all (rather than mildly) until $\bar{\eta}$.



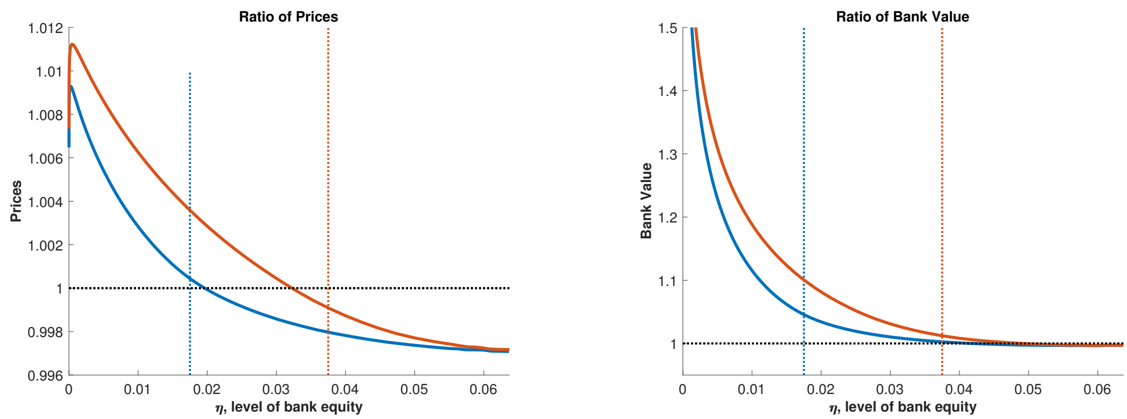
(a) Sharpe ratio



(b) Leverage

Figure 12: Effects of Fed Put on Sharpe Ratios and Leverage compared to constant interest rates.

Figure 13 plots the changes in the asset price and bank value. In this case, the Put always increases bank value. In the model, banks are owned by households, so there is no conflict of interest regarding how households would value the Fed Put relative to a policy of constant rates. However, household welfare is not everywhere higher with the Fed Put (because the asset price falls), and so if households and banks had competing interests, banks would lobby for a Fed Put.



(a) Asset price Q (b) Bank Value

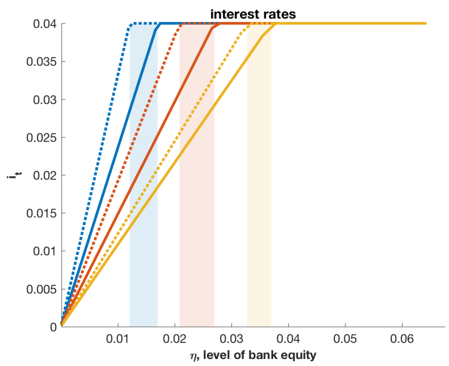
Figure 13: Effect of Fed Put on asset price and bank value.

We now consider Puts that decline linearly to zero at $\eta = 0$, starting from $i^{LAW} = 4\%$, and as before we vary the strike.

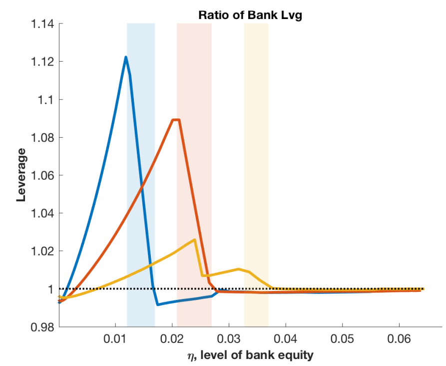
These results reinforce the state-dependent effects of monetary policy. The marginal gains from monetary policy are greatest when policy occurs during crises and not before. Targeted monetary policy can have significant effects on stability.

C.4 Marginal Impacts of Extending Linear Fed Puts

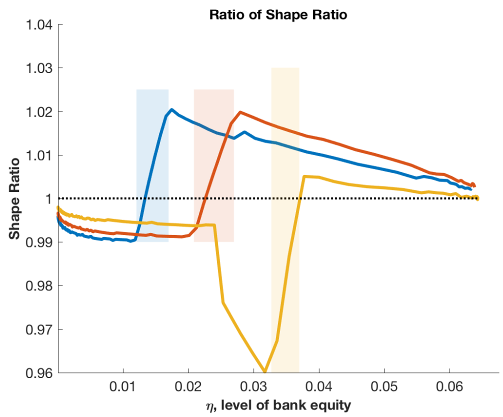
Figure 14 plots the marginal impact of extending the Fed Put by 5%. Most importantly, the marginal impact on the Sharpe ratio can be very negative if the Fed Put occurs at high η when banks are the marginal investors in bank-dependent investments.



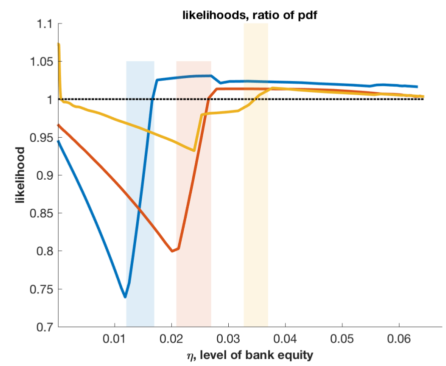
(a) Interest rate rules



(b) Leverage



(c) Sharpe ratio



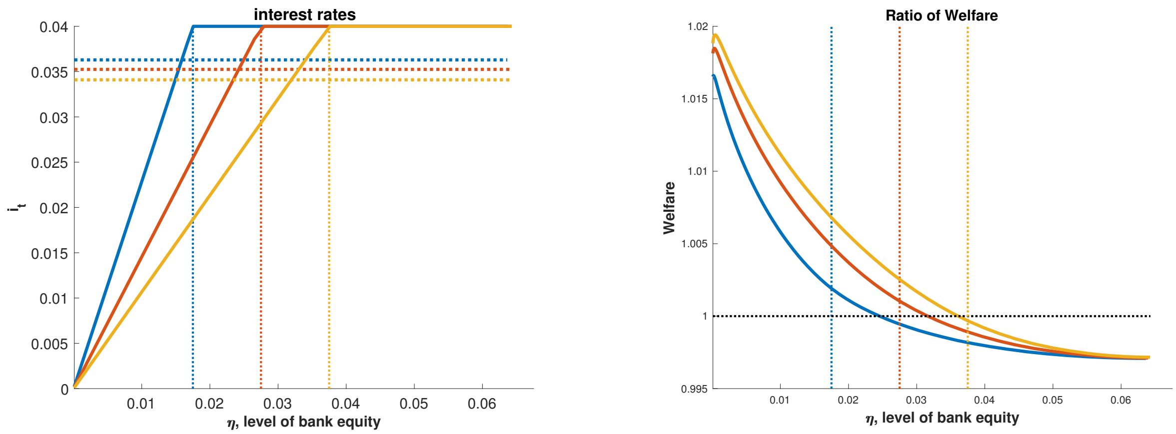
(d) Distribution (PDF) of η

Figure 14: Marginal impact of extending Fed Put.

C.5 Welfare with Known Initial Condition in Baseline Model

In practice central banks are likely to adopt policies that respond substantially more gradually than a piecewise Put, so for welfare results we consider linear Fed Put policies in which $i_t = 4\%$ for $\eta > \eta^{LAW}$, but below η^{LAW} rates vary linearly to $i^{Put} = 0\%$ at $\eta^{Put} = 0$, thus giving $i(\eta) = \min\left\{4\%, \frac{4\%}{\eta^{Put}}\eta\right\}$. These policies respond gradually to conditions in the financial sector, though they can still represent aggressive interest rate responses. Denote welfare under the Put by $V(\eta)$ and denote welfare when rates are held constant at $\bar{i}$ by $\bar{V}(\eta)$.

Figure 15(a) plots the Put policies we consider and the average rates implied by those policies (dotted). Figure 15(b) plots $\frac{V(\eta)}{\bar{V}(\eta)}$ at every η, which is the ratio of welfare under the Put compared



(a) Interest rates

(b) Welfare ratio

Figure 15: Effect of Fed Put on welfare.

to welfare under constant rates $\bar{i}$. The effect of the Fed Put on welfare depends (unsurprisingly) on the initial condition: relative to constant rates the Fed Put can either increase or lower welfare depending on the current state of the financial sector. In particular, for low levels of η, the Fed Put increases welfare, which is not surprising given the nature of the policy (and the behavior of the price Q). When η is low, the "bailout component" of the policy is most salient, in expected present value terms, thus increasing welfare. If the financial sector is currently well capitalized, then central bankers would decide against having monetary policy systematically respond to the financial sector. When the economy is presently at a high η, the costs of LAW/Put policies (primarily in lowering asset prices for high η), outweigh the benefits from stability. The baseline model without inflation costs provides a lower-bound on the costs of active monetary policy.

D Timing the Put: Keep Powder Dry?

Cutting rates during a crisis can substantially improve stability by providing cheap funding for banks, enabling them to quickly rebuild equity. Should central banks, therefore, cut rates early to avoid entering a crisis? Or should central banks "keep their powder dry" by waiting to cut but then cutting quickly? In a standard linearized model, what often matters the most is the level of rates, not the change in the policy rule. In contrast, because our model features non-linear dynamics, *changes* in rates matter more for some variables than the overall level of rates.

In a standard New Keynesian model, the optimal timing depends on the risk of hitting the zero-lower bound ("ZLB"). Reifschneider and Williams (2000) find that when the ZLB is nowhere in view, one can afford to move slowly and take a "wait and see" approach to gain additional clarity about potentially adverse economic developments. But when interest rates are in the vicinity of the ZLB, central banks ought to "vaccinate" against further ills, acting quickly to lower rates at the first sign of economic distress. Our model provides complementary insights with regards to using monetary policy to target financial stability, which is not identical to the standard focus of aggregate stabilization. We find that whether the central bank should "keep their powder dry" or not depends on the extent to which the central bank can cut rates during a financial crisis.[19]

To analyze the effect of cutting early and slow compared to cutting early and late, we consider state-dependent policies with two thresholds. Above η^{LAW} rates are held constant at i^{LAW}; below η^{Put}, rates are held constant at i^{Put}; between η^{LAW} and η^{Put}, rates change linearly between i^{LAW} and i^{Put}. Formally, rates take the form in equation (15).

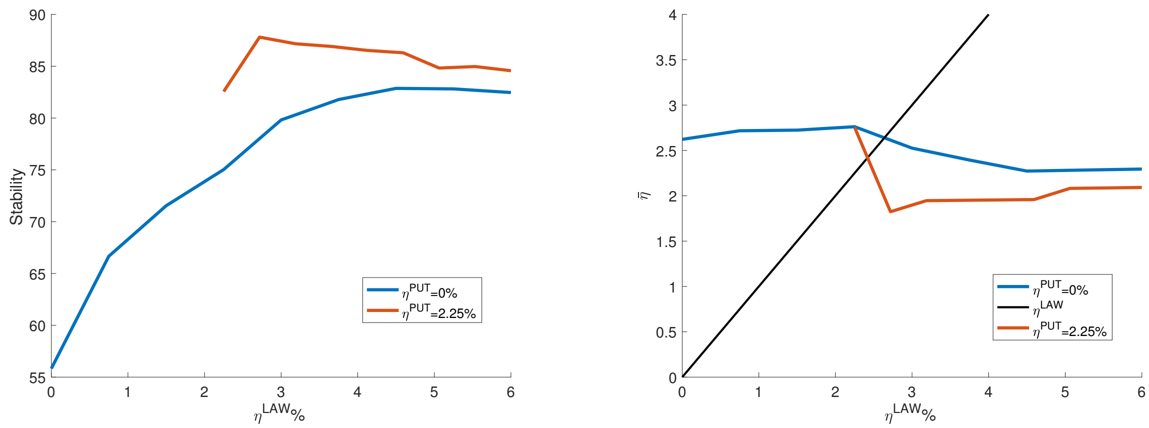
We fix the lower threshold η^{Put} and consider how stability varies with η^{LAW}: higher η^{LAW} corresponds to "early" rate cuts in the sign of financial-sector distress, but shallower rate cuts, while a lower η^{LAW} corresponds to "late" but fast rate cuts. We set $i^{LAW} = 4\%$ and $i^{Put} = 0\%$. We consider two values, $\eta^{Put} = 2\%$ and $\eta^{Put} = 0\%$. When $\eta^{Put} = 2\%$, rates are held at zero for almost the entire crisis region. When $\eta^{Put} = 0\%$, rates are always positive converging to zero on at the very worst part of a crisis (if $\eta = 0$).

Figure 16(a) plots stability as a function of η^{LAW}. How stability varies with η^{LAW} depends critically on η^{Put}. When $\eta^{Put} = 2.25\%$ so that rates will be brought to zero just before a crisis occurs, stability is improved by delaying rate cuts until nearly as late as possible (until 2.75%)—but then cutting quickly to zero. As the red line illustrates, stability is greatest when η^{LAW} is very close to η^{Put}, corresponding to late but fast cuts. In this case, the change in rates is larger, which leads to greater changes in stability in a crisis, allowing more stability globally. Waiting too long,

[19]Models of "information effects" of Fed policy have the same result that cutting once but big is better than small and frequent, see for example Campbell et al. (2019).

however, will hurt stability (intuition below).

The results are quite different when rates are constrained to be positive during crises. When $\eta^{Put} = 0\%$, then it is better for stability to cut rates sooner; maximal stability occurs when η^{LAW} is around 4.5%, much higher than was true in the previous case. In this case, because rates will not hit zero unless a terrible crisis occurs, cutting rates earlier means lower rates everywhere—in a crisis, and before the crisis. Thus, it's important in this case to begin cutting rates early *in order* to get rates low enough to provide support for the financial sector.



(a) Timing and Stability (b) Changes in $\bar{\eta}$

Figure 16: Stability and endogenous changes in the crisis given timing of rate cuts.

The intuition is provided in Figure 16(b), which plots how the crisis threshold $\bar{\eta}$ varies with η^{LAW} in both cases. The black line depicts when η^{LAW} precisely equals the crisis threshold $\bar{\eta}$. When the blue or red line fall below the black line, then policy cuts rates before the crisis. Remember that in the crisis region $\eta < \bar{\eta}$ households produce good 1, depressing the returns on bank assets and weakening the automatic stabilizing mechanisms in the economy. When $\eta^{Put} = 2.25\%$ so that rates will be held at zero throughout the crisis, waiting to cut rates (lower η^{LAW}) stabilizes the economy, and so in general the crisis region endogenously *shrinks* ($\bar{\eta}$ decreases for the red line). Keeping the powder dry endogenously makes the crisis occur at a later stage in the financial cycle. In contrast, when $\eta^{Put} = 0\%$ so that rates are positive in a crisis, the economy is always less stable, and so endogenously the crisis occurs earlier ($\bar{\eta}$ is always higher). But furthermore, when η^{LAW} is low so that rates are held high for longer, the crisis occurs even earlier in the cycle, and can even occur while rates are still held at i^{LAW}. In this case, waiting to cut makes the crisis more likely, consistent with the intuition that a rate cut would provide "vaccination" against a crisis occurring. The same effect occurs when $\eta^{Put} = 2.25\%$ and rates are not cut until the last moment, which is why the red line spikes up when η^{LAW} equals η^{Put}.

It should be noted that in this case the crisis is very shallow—indeed, between $\eta = 1.95\%$ and 2.25% bank leverage spikes so much that banks *alone* invest in good 1 so that the economy is not in crisis. There are actually two regions in which households invest in good 1: below $\eta = 1.95\%$ and between $\eta = 2.25\%$ and 2.9%. In this case, $\bar{\eta}$ is not a robust measure of when a crisis occurs. However, the measure of stability, measuring the frequency of all states in which households specialize, does indeed fall.

E Comparison with Macroprudential Policy

In the debate about whether monetary policy should be used to address financial stability, one of the key considerations is the extent to which macroprudential policy measures ("MaP") can be used instead. In this section we consider two types of MaP policies: leverage constraints and equity injections (or tail risk insurance).

E.1 Leverage Constraints

Informed by the analysis in Phelan (2016), we consider how the results with monetary policy would compare to the effects with MaP. Specifically, Phelan (2016) considers the effects of leverage limits on financial stability. While leverage limits have a greater effect on stability, the welfare benefits are quantitatively much smaller.

First, the mechanics of how MaP and monetary policy affect stability are quite different. When leverage limits bind, intermediation falls and so flow outcomes suffer. At the same time, leverage limits increase banks' investment returns (when they bind), and so banks rebuild equity faster, thus improving stability. Thus, MaP provides a tradeoff between current outcomes (worsened) and dynamic stability (improved). Furthermore, since Phelan (2016) finds that leverage limits are actually likely to bind following losses (i.e., balance sheet leverage is countercyclical), MaP provides a time-inconsistency problem because regulators would be tempted to relax leverage limits following bad shocks. In contrast, the monetary policy can increase the rate of equity growth for banks by decreasing funding costs and liquidity premia. Crucially, lower rates encourage higher leverage (when rates are lower) and so accommodative monetary policy improves flow allocations. Thus, using monetary policy to target financial stability is closer to time-consistent, given the transmission channel and mechanism in our model.

Second, the quantitative implications of MP and MaP appear to be quite different. Leverage limits have quantitatively larger effects on financial stability, more effectively shifting mass toward high η states. Indeed, very stringent leverage limits (assuming they are effective) can lead to extremely stable financial sectors almost without bound, whereas there appears to be a bound to how much monetary policy can improve stability.

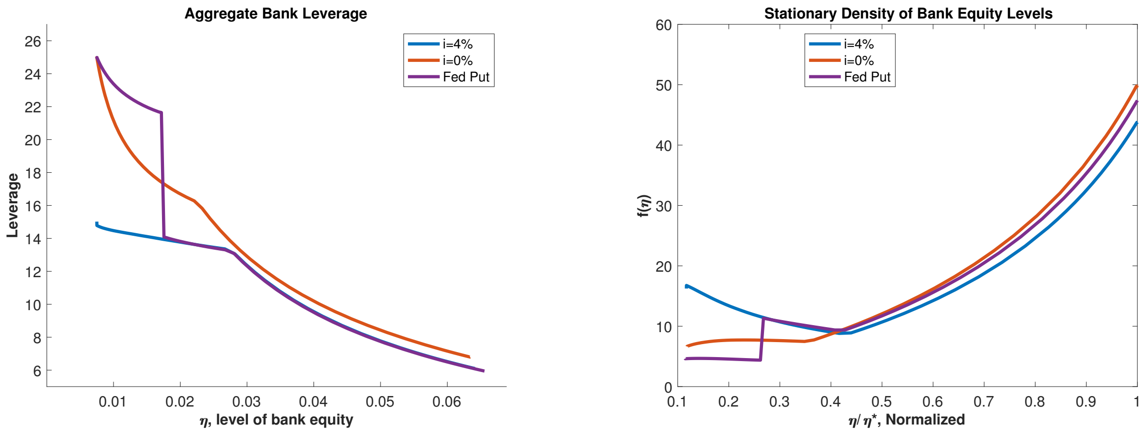
Nonetheless, monetary policy appears to have quantitatively much larger effects on welfare. Decreasing interest rates improves stability and flow outcomes at the same time, while MaP improves stability at the cost of flow outcomes. Using MaP to improve welfare is more difficult, and so it appears that the potential welfare gains from MaP are negligible compared to the potential gains from monetary policy. As an example, consider when interest rates are 0% and imposing leverage limits of 12. The welfare gain evaluated at η^* is roughly 0.04%, and evaluated under the ex-ante measure it is 0.35%. In contrast, compared to holding rates at 4% everywhere, cutting rates to zero below 2.75% improves welfare by 0.87% evaluated at η^* and by 1.9% evaluated under the ex-ante measure. Even with some inflation costs of $\varepsilon_\pi = 40$, this Fed Put improves welfare by 0.17% evaluated at η^* and by 0.69% evaluated under the ex-ante measure. As an extreme example, with no inflation costs, lowering interest rates from 4% to 0% increases welfare by 7.16% evaluated at η^* and by 7.92% under the ex-ante measure.

Finally, MaP does not incur inflation losses arising from deviating from the target rate. Indeed, since MaP can substantially improve stability, it appears that using MaP in conjunction with monetary policy could decrease the inflation losses from deviations in monetary policy. Since flow inflation costs occur predominantly at low η, and since MaP decreases the likelihood of the economy entering low η regions, the expected discounted inflation losses decrease with MaP is also used. However, the quantitative significance of these changes appear to be negligibly small.

E.2 Equity Injection

Another MaP policy to consider is an equity injection if capitalization of the financial sector hits some level $\underline{\eta}$. Such a policy of "tail risk insurance" automatically recapitalizes the banking sector, preventing further equity losses and also enabling banks to take on more risk even if capitalization is near $\underline{\eta}$. Brunnermeier and Sannikov (2014) find that such a policy is particularly effective at stabilizing the economy, without providing moral hazard incentives. We consider how such a policy influences the role of monetary policy. We find that a policy of equity injection can act as a substitute for active monetary policy. If the injection is sufficiently aggressive (high $\underline{\eta}$), then active monetary policy is unnecessary to prevent crises or to improve stability.

Figure 17 plots stability and leverage with an equity injection at $\underline{\eta} = 0.75\%$. Compared to the baseline model, there are several key differences with equity injections. First, the behavior of leverage for low η can be quite different. Without equity injection, leverage with constant rates was almost the same regardless of the level of rates, but that is not so with the equity injection policy. Leverage increases by much more when rates are constant at 0%, converging to the level of leverage under the Fed Put (where rates drop to 0%). Second, the endogenous crisis region $\bar{\eta}$ is much more sensitive to the overall level of rates: $\bar{\eta} = 2.82\%$ when rates are held at 4%, and $\bar{\eta} = 2.8\%$ under the Fed Put, but $\bar{\eta} = 2.37\%$ when rates are constant at 0%—a substantial decrease. As a result, the economy with rates held constant at 0% is much more stable than compared to the economy without equity injection. Indeed, the Fed Put only stabilizes "within the crisis region." It makes the economy better off by speeding up recovery from the very worst state; it does not make staying in good states much more likely. Stability is actually greatest for the constant at 0% economy, with stability of 88.1%, compared to 84.6% and 75.5% for the economies with the Fed Put and with rates constant at 4%. Thus, with an equity injection, a Fed Put improves stability compared to holding rates constant at 4%, but leaning against the wind in good times does not improve stability.



(a) Leverage

(b) Stationary distribution, normalized

Figure 17: Equilibrium stability and leverage with equity injection (tail risk insurance).

These results are even more stark if the equity injection is more aggressive. If injection occurs at $\underline{\eta} = 1.5\%$, then there is *no* crisis with rates held at 0%. A crisis occurs below $\bar{\eta} = 2.87\%$ if rates are constant at 4% and below $\bar{\eta} = 2.32\%$ if rates are constant at 2%.

Thus, there is a degree of substitutability between equity injections and monetary policy. An aggressive equity injection makes active monetary policy unnecessary and can completely prevent a crisis. If equity injections are less aggressive (lower $\underline{\eta}$), then active monetary policy (lower rates) can prevent a crisis.