

Debt Collateralization, Structured Finance, and the CDS Basis

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Abstract

We study how the ability to use risky debt as collateral in funding markets affects the CDS basis. We use a general equilibrium model with heterogeneous agents, collateralized financial promises, and multiple states of uncertainty. We show that a positive basis emerges when risky assets and their derivative risky debt contracts can be used as collateral for additional financial promises. Additionally, because a risky asset can always serve as collateral for more promises than its derivative debt contracts can, the basis for a risky asset will always differ from the basis for its derivative risky debt.

Keywords: Collateral, securitized markets, cash-synthetic basis, credit default swaps, asset prices, credit spreads.

JEL classification: D52, D53, G11, G12.

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1 Introduction

Credit spreads typically differ from implied default risk, and this “excess bond premium” has substantial information content for explaining fluctuations in economic activity (Gilchrist and Zakrajsek, 2012).¹ Accordingly, fluctuations in credit spreads may be driven by the effective supply of funds offered by financial intermediaries and by the functioning of financial markets. In the years prior to the 2007 recession, the excess bond premium was significantly negative. This period exhibited a proliferation of financial innovations in funding markets. The shadow-banking system oversaw the creation of a variety of structured credit products, including collateralized debt obligations (“CDOs”) and CDO-squareds, as well as the practice of rehypothecation, all of which greatly increased the ability of assets to serve as collateral in financial markets.² Our paper considers how innovations in the use of collateral (such as these) can lead to a negative excess bond premium (“EBP”).

Specifically, our paper shows that the ability to use risky debt as collateral to issue further financial promises can lead to a positive CDS basis, and thus contribute to a negative EBP. The CDS basis is the difference between the spread on a bond and the premium on a credit default swap (CDS) protecting that bond. (The typical convention is $\text{CDS basis} = \text{CDS spread} - \text{bond spread}$.) If the CDS premium accurately captures implied default risk, then the CDS basis would equal the excess bond premium.³ In practice this equivalence is not exact (e.g., CDS contracts include counter-party risk and CDS and bond markets may be partially segmented). Nevertheless, the CDS basis is one factor that contributes to the magnitude of the excess bond premium, and fluctuations in the CDS basis translate into fluctuations in the excess bond premium. Empirically these measures move together very closely (see Figure 1).⁴ Before the crisis, bases on high yield bonds were

¹Firm borrowing costs, and in particular credit spreads, have important implications for economic activity. Credit spreads are the difference in yields between private debt instruments and government securities of comparable maturity. The excess bond premium is the difference between credit spreads and implied default risk. Gilchrist and Zakrajsek (2012) show that shocks to the excess bond premium lead to economically significant declines in consumption, investment, and output. Furthermore, they find that the excess bond premium correlates with the health of highly leveraged financial institutions.

²See Gorton and Metrick (2009); Fostel and Geanakoplos (2012a).

³A credit default swap is a financial contract that provides protection against a specific credit event, such as a loan default. Specifically, the buyer of a CDS contract on a bond receives payment equal to the exact difference between the promised payout of that bond and the actual payout. As such, a CDS can be viewed as insurance on a particular risky asset, which is specified by the CDS contract.

⁴Figure 1 plots the excess bond premium (EBP) and the average CDS basis, defined as CDS premium minus bond premium, for investment grade bonds (IG). Data for the CDS bases are taken from Gârleanu and Pedersen (2011).

significantly positive. Figure 2, from Bai and Collin-Dufresne (2013), shows the significantly positive basis for high-yield bonds pre-crisis (the average HY basis was about 80 basis points during that period).

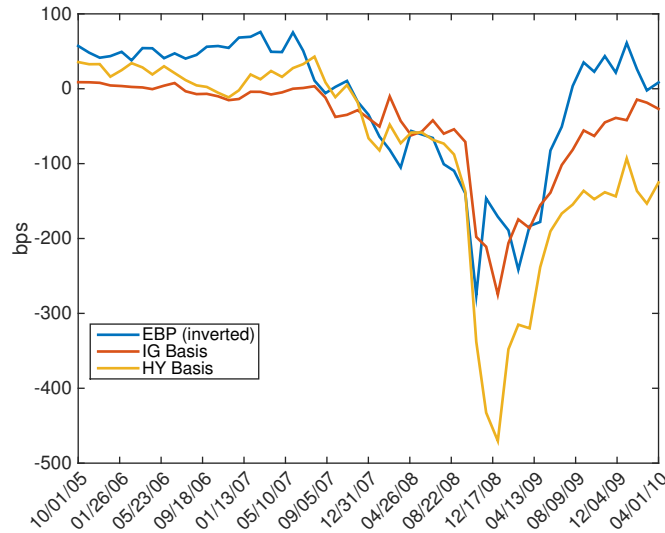


Figure 1: Credit Spreads: Excess Bond Premium (inverted) and the CDS Basis.

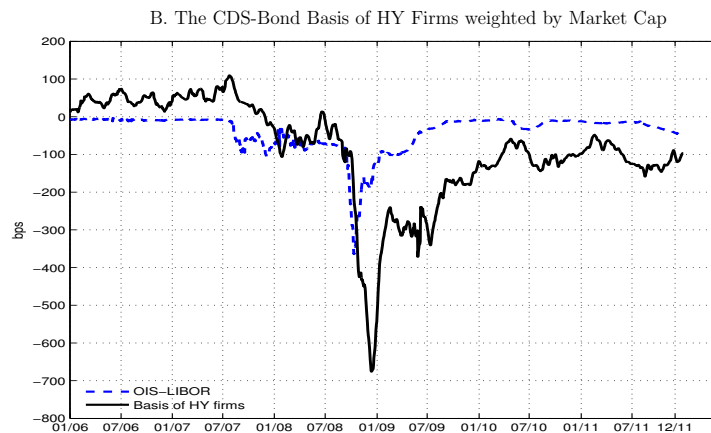


Figure 2: Figure 1B of Bai and Collin-Dufresne (2013): significant positive bases for HY securities before the crisis.

We provide a theoretical model that shows that the CDS basis on a risky asset is positive

Data for the excess bond premium is taken from Gilchrist and Zakrajsek (2012) and plotted in basis points, with the inverted EBP plotted in panel (b). The correlation between the (negative) GZ index and the IG and High Yield (“HY”) bases are 0.963 and 0.938; the correlation between the (neg.) EBP and the IG and HY bases are 0.855 and 0.803.

when derivative debt backed by the risky asset can be used as collateral to issue further promises. To establish our results, we consider a general equilibrium model with heterogeneous agents and collateralized borrowing following Fostel and Geanakoplos (2012a), which we extend to multiple states of nature, implying that in equilibrium agents trade both safe and risky debt contracts. As a result, risky debt contracts can be used non-trivially to back further debt contracts, a process we refer to as “debt collateralization.”⁵ Thus, our contribution is to introduce debt collateralization into a multi-state extension of Fostel and Geanakoplos (2012a) and to derive the implications for the CDS basis.

Importantly, the debt collateralization financial environment can reflect innovative financial structures such as senior-subordinated tranches, structured credit facilities, or collateralized debt obligations (see Gong and Phelan, 2016, for more detail on the equivalence between capital structure and the ability to use debt as collateral).⁶ Equivalently, the CDS basis on a risky asset is positive when the risky asset is tranching into a senior-subordinated capital structure. We then extend our analysis by introducing a CDS on risky *debt* backed by the asset. We show that in equilibrium the basis on the underlying collateral always exceeds the basis on the derivative debt, which is itself backed by the risky asset.⁷ This is because relative to its derivative debt contracts, the risky asset always has a greater degree to which it can serve as collateral. This prediction has implications for the CDS-CDX basis, which we discuss in greater detail in Section 5

Our results imply that fluctuations in how financial markets treat collateral, and in the ability

⁵With only two states, in equilibrium only risk-free debt is traded (Fostel and Geanakoplos, 2015). Our results easily generalize to multiple states of nature; we consider only three states in the baseline model to keep the analysis tractable. Geerolf (2015) introduces debt pyramiding in a model in which debt perceived as risk-free is used to issue other debt contracts that are also perceived to be risk-free.

⁶Consider a simple, stylized version of a deal with senior, mezzanine, and equity tranches all with face-values of 1, and suppose the value of the collateral could take values of 1, 2, or 3. The senior bonds would get paid 1 for sure; the mezzanine bond would get paid 1 in only when the collateral is worth 2 or 3, and zero otherwise; and the equity would get paid 1 only in the best state, and zero otherwise. This structure can be equivalently implemented with leveraged investments in the collateral, in the debt backed by the collateral, *and in debt backed by the debt backed by collateral*. The equity investor is effectively buying the collateral with leverage, promising to repay 2 units and defaulting whenever the collateral is worth 2 or less. The mezzanine investor is effectively buying *the promise from the equity investor* and using *this promise as collateral* to borrow 1 from the senior investor. The senior investor buys this promise from the mezzanine investor. This investment scheme exactly replicates the payoffs to the tranches, giving (the mezzanine) investors the ability to use debt as collateral to make new promises. In practice the payoffs to ABS tranches are complicated by timing of prepayments and how principal payments are allocated to the different tranches.

⁷We think of the underlying risky asset as a financial asset such as a corporate bond or a mortgage-backed security, and we think of the risky debt as a tranche issued by the collateral. This exercise is only meaningful in a multi-state model in which risky debt is traded in equilibrium. In binomial economies, all equilibrium debt is risk-free and so a CDS on debt is redundant.

of financial markets to provide funding, should lead to fluctuations in the CDS basis and therefore in the excess bond premium. The most common explanation for positive CDS bases is that physically settled CDS contain a cheapest-to-deliver (“CTD”) option that increases the premium of the CDS contract (Blanco et al., 2005; De Wit, 2006). Blanco et al. (2005) find that the CTD option is most prevalent for European entities because U.S. CDSs have been subject to a Modified Restructuring definition since May 11, 2001, which reduces the value of the delivery option.⁸ Additional technical considerations of CDS contracts and bond trading can increase the basis.⁹ Our theory implies that variations in the extent to which funding markets can use debt as collateral (or to which structured finance implicitly allows debt to be used as collateral) ought to correspond to variations in the CDS basis. In contrast, funding markets for derivative debt securities ought to have no direct effect on the value of the CTD option.

Our result is consistent with the notion that a decrease in the excess bond premium reflects an increase in the effective risk-bearing capacity of the financial sector (through the ability to use assets to borrow) and an expansion in the supply of credit (reflecting increased willingness to lend against certain assets).¹⁰ Since many financial intermediaries (particularly highly leveraged ones) rely heavily on collateralized markets to finance operations, the condition of funding markets is closely tied to the health of financial institutions. Thus, we provide a mechanism explaining why the excess bond premium correlates with the health of the financial sector, as shown by Gilchrist and Zakrajsek (2012).¹¹ Our theoretical mechanism can rationalize the existence of positive and negative bases and generates predictions that align with changes in the CDS basis during the past decades. During ebullient times when many assets can serve as collateral, bases can be positive, while during crisis times when funding markets limit collateral to only the highest quality assets, bases can turn negative. During the height of the crisis, pessimism and increased uncertainty

⁸Blanco et al. (2005) argue that it is almost impossible to value this option analytically since there is no benchmark for the post-default behavior of deliverable bonds.

⁹e.g., CDS premia are floored at zero, CDS restructuring clause for technical default, bonds trading below par (De Wit, 2006).

¹⁰See, for example, the theoretical work of He and Krishnamurthy (2013), Adrian and Boyarchenko (2012), Brunnermeier and Sannikov (2014), and the financial accelerator mechanisms emphasized by Bernanke and Gertler (1989) and Kiyotaki and Moore (1997).

¹¹A non-zero basis on a bond is of particular significance for the firm issuing that bond. A positive basis indicates that the bond is “expensive,” since a combination of the bond and the CDS costs more than other comparable safe assets. In this case, it is easy for the firm to borrow and finance its operations by issuing bonds. (The excess bond premium is negative.) When there is a negative basis, the bond is “cheap” and the firm has reduced funding capacity. (The excess bond premium is positive.)

limited the ability of many assets to serve as collateral (see Gorton and Metrick, 2012), and the basis for most securities become negative.¹² Figure 3 plots CDO issuance since 2001. Indeed, before the crisis CDO issuance was high when the HY CDS basis was positive.

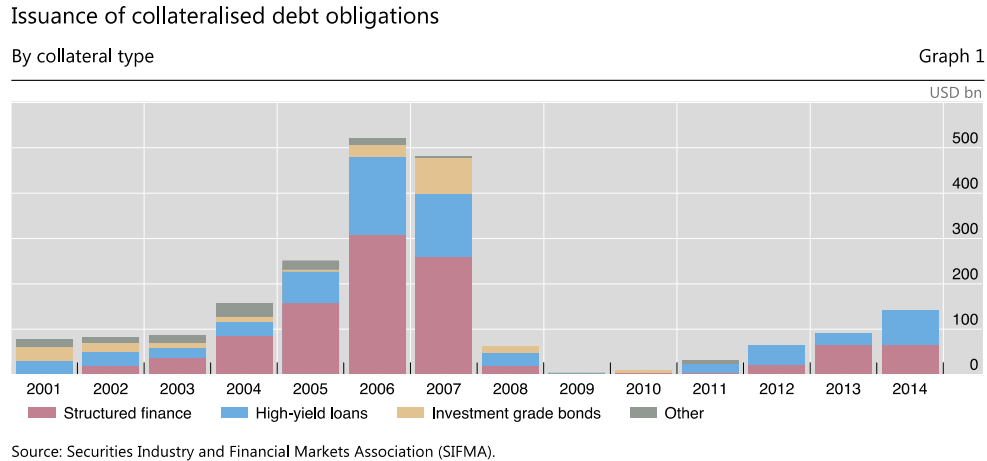


Figure 3: Debt Collateralization: CDO Issuance. (Source: Antoniadis and Tarashev (2014))

The rest of the paper is outlined as followed. The remainder of this section discusses the related literature. Section 2 presents the basic general equilibrium model with collateralized CDS and debt contracts. Section 3 derives the main results regarding how the basis varies with the financial environment. Section 4 introduces CDS on debt contracts and derives results for the “double basis.” Section 5 discusses empirical implications and suggestive evidence. Section 6 concludes.

Related literature

Our model introduces debt collateralization into a model of collateral equilibrium with CDS based on builds on Fostel and Geanakoplos (2012a). The literature of collateral equilibrium was pioneered by Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014). In addition to their work on asset prices, Fostel and Geanakoplos (2012a) use a binomial model to provide an example where the equilibrium basis is negative (specifically, when the risky asset cannot be used as collateral, or when the asset can be leveraged but it cannot be tranching or used as collateral to issue CDS).

¹²For motivations for why financial markets may *decrease* the available set of assets serving as collateral, see the informational explanations in Dang et al. (2009); Gennaioli et al. (2013); Gorton and Ordoñez (2014).

Our analysis builds on their examples by classifying precisely the conditions necessary for either a positive or negative basis to occur. Specifically: (i) we introduce debt collateralization and show that a positive basis occurs in equilibrium, (ii) we show that there is always a difference between the basis on the asset and the debt, and (iii) we more precisely characterize the basis when the risky asset can be leveraged (with multiple states a positive basis can emerge even in this case, which does not occur with two states). Our paper also relates to the literature on collateral equilibria in models with multiple states (Simsek, 2013; Toda, 2015; Gottardi and Kubler, 2015; Phelan, 2015; Gong and Phelan, 2016). Several papers study credit default swaps in equilibrium (see Banerjee and Graveline, 2014; Danis and Gamba, 2015; Darst and Refayet, 2016). Geerolf (2015) shows that debt pyramiding increases leverage and asset prices.

Our insight about the role of collateral to determine the basis is closely related to Shen et al. (2014), which proposes a collateral view of financial innovation driven by the cross-netting friction. Shen et al. (2014) show that derivatives allowing investors to “carve out” risks emerge to conserve collateral, and as a result the price of a risky asset is always less than the price of a portfolio replicating it with derivatives (negative basis). Their result follows because the risky asset requires “too much” collateral for agents to isolate the risks they want. In our model, we derive the same result when the risky asset cannot be used as collateral. However, in contrast, we show that the sign of the basis can flip (the risky asset can be expensive) when the risky asset and its derivative debt contracts can be used as collateral. In essence, we extend their insight by considering when the risky asset can in fact “require less collateral” than alternatives. In their terminology, debt collateralization is a financial innovation designed to conserve collateral. Our theory rooted in collateral can explain positive bases by emphasizing financial innovations that stretch collateral.

Most theoretical papers explain why non-zero bases can persist once deviations occur. Notably, the literature focuses on explaining when bond premia exceed CDS spread, as occurs during crises, but does not typically explain the reverse phenomena, which we do. This literature relies on limits of arbitrage conditions in the market to explain the existence of non-zero basis: a “shock” occurs that causes CDS and bond premia to diverge, and the basis persists because arbitrageurs cannot fully arbitrage the difference. Of these limits to arbitrage conditions, the most commonly cited is the existence of limits in firms’ funding capacity, which prevents firms from conducting enough trades to eliminate the basis. With this interpretation, differences in cross-sectional bases

at different points in time point to variations in funding capacity across firms. Shleifer and Vishny (2011) show that fire-sale models can explain failures of arbitrage in markets featuring large differences in prices of very similar securities. Gârleanu and Pedersen (2011) provide a model where margin constraints can lead to pricing differences between two identical financial securities.¹³ Oehmke and Zawadowski (2015) show that a negative basis emerges when transaction costs are higher for bonds than for CDS. In our paper, negative bases can persist when risky assets are imperfect collateral, and positive bases can persist *even when agents can short assets* because the efficient use of collateral is to buy CDS rather than to short assets.

Many authors in the empirical literature on CDS and the CDS basis have identified factors that partially explain the behavior of the CDS basis. Blanco et al. (2005) argue that the bond market lags behind the CDS market in determining the price of credit risk, causing short-run deviations in prices; long-run deviations arise from imperfections in contract specification of CDSs, which cause the CDS price to be an upper-bound on credit risk, and measurement errors, which understate the true credit spread. Nashikkar et al. (2011) show that bonds of firms with a greater degree of uncertainty are expensive (i.e., the basis is positive), which they claim to be consistent with limits to arbitrage theories. Choi and Shachar (2014) argue that a negative basis emerged during the 2008 financial crises because the limited balance sheet capacity of dealer banks prevented corporate bond dealers from trading aggressively enough to close the basis. Bai and Collin-Dufresne (2013) empirically test explanations for the violation of the arbitrage relation between cash bond and CDS contracts and conclude that the basis is larger for bonds with higher frictions, which include trading liquidity, funding cost, counterparty risk, and collateral margin. Zhu (2004) finds that the CDS market moves ahead of the bond market in terms of price adjustment because the two markets respond differently to changes in credit conditions, and this timing may explain the existence of non-zero bases in the short run.

Finally, we stress that our results about collateral quality provide only one explanation of

¹³Specifically, negative shocks to fundamentals cause margin constraints to become binding and differences in margin requirements can then cause the basis to deviate from zero. Our analysis and results differ from Gârleanu and Pedersen (2011) in several ways. First, in Gârleanu and Pedersen (2011), a basis only occurs when negative shocks cause a funding-liquidity crisis and losses for leveraged agents. In our model, non-zero bases are due to the financial environment (assets used as collateral), not the presence of a funding-liquidity crisis. Second, we show that the basis between two assets does not only depend on the margin requirements of the assets themselves, but also on the margin requirements for derivative debt contracts collateralized by the assets. Our model demonstrates that agents choose to buy and sell different assets precisely because of the different promises they can make with these assets.

fluctuations in the basis. Our results can begin to explain some of the time-series variation within a collateral class (corresponding to fluctuations in CDO issuance and other structured finance) and some of the cross-sectional difference across classes. Empirical evidence by Bai and Collin-Dufresne (2013) document substantial cross-sectional dispersion in the basis during the crisis among bonds of similar collateral quality (similar investment grade). Nevertheless, the basis depends on many things besides implied collateral quality. For example, the liquidity explanation clearly also matters, as does segmentation between CDS and bond markets.¹⁴ One can consider our explanation as having an effect *in addition* to what liquidity premia would imply. In addition, there have been many other apparent arbitrages that behaved similar to the CDS basis, but for which our story does not apply directly (e.g., cash-futures, mortgage rolls, fed funds, swap spreads, covered interest parity). One might suppose that the ability to use different assets as collateral affects the balance sheet costs of financial institutions, and thus the costs of “limits to arbitrage,” which would affect the sizes of these arbitrages.

2 General Equilibrium Model with Collateral

This section presents the basic general equilibrium model with collateralized borrowing, which is a multi-state extension of Fostel and Geanakoplos (2012a) with the addition of giving agents the ability to use financial contracts as collateral to issue further promises.

Time, Assets, and Investors

Consider a two-period, three-state general equilibrium model with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, M, D\}$ with a root $s = 0$ at time $t = 0$ and three states of nature $s = U, M, D$ at time 1, occurring with probabilities $\gamma_U, \gamma_M, \gamma_D$ respectively.

There are two assets, X and Y , which produce dividends of the consumption good at time 1. Asset X is risk-free, producing (as a normalization) 1 unit of the consumption good in every final state. Asset Y is risky, producing $d_U^Y = 1$ unit in state U (a normalization), $d_M^Y < 1$ units in state M , and $d_D^Y < d_M^Y$ in state D . We think of asset Y as a financial asset—such as a corporate bond, a pool

¹⁴Note that when the CDS market leads the bond market, this would lead to a positive widening in the basis during crises, which is the opposite of what broadly occurred during the recent crisis.

of mortgages, or an asset-backed security—rather than a physical asset like a house or the assets of a firm. To simplify notation, we denote $d_M^Y = M$ and $d_D^Y = D$, denoting the state and the payoff by the same variable. Asset payoffs are shown in Figure 4.

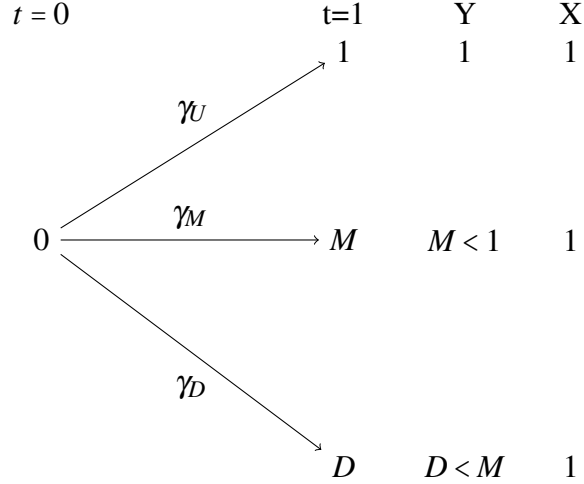


Figure 4: Payoff tree of assets X and Y in three-state world

We suppose that agents are uniformly distributed in $(0, 1)$, that is they are described by Lebesgue measure. (We will use the terms “agents” and “investors” interchangeably.) At time 0, each investor is endowed with one unit of each asset X and Y . Agents also have endowments of e_s^h units of consumption good in period-1 in state s . Agents consume only in period 1, and they have concave utility over consumption. Agents have expected utility

$$U^h(c_U, c_M, c_D) = \gamma_U u_h(c_U) + \gamma_M u_h(c_M) + \gamma_D u_h(c_D),$$

where c_s is consumption in state s , $u_h(c)$ is increasing and concave, and agents have common priors consistent with objective probabilities.

We suppose that endowments at $t = 1$ are sufficiently large compared to the dividends from the assets so that agents’ future marginal utilities are exogenously given by their endowments and preferences. Define $\gamma_s(h) = \gamma_s u_h'(e_s^h)$ to be the marginal utility for agent h in state s . We specify that $\gamma_U(h)$ and the ratio $\gamma_M(h)/(\gamma_M(h) + \gamma_D(h))$ are monotonically increasing in h . This implies that investors with high h have uniformly higher marginal utility for consumption in states in which the asset payoff is higher. (For example, investors can be risk averse with endowments satisfying a

monotone likelihood ratio.)

One can equivalently think of our agents as risk-neutral with subjective probabilities $\gamma_s(h)$, in which high h investors would be uniformly more optimistic about the risky asset's dividend payoffs. The second condition implies that the subjective conditional probability of state M , given that U does not occur, is increasing in h . Hence, both conditions imply that a higher h indicates more optimism. In this case, one can interpret investors as having expected utility $U^h(c_U, c_M, c_D) = \gamma_U(h)c_U + \gamma_M(h)c_M + \gamma_D(h)c_D$.

Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. We explicitly incorporate repayment enforceability problems, and we suppose that collateral acts as the only enforcement mechanism.¹⁵ At time 0, agents trade financial contracts. A financial contract $j = (A^j, C^j)$, consists of a promise $A^j = (A_U^j, A_M^j, A_D^j)$ of payment in terms of the consumption good, and an asset C^j serving as collateral backing the promise. The lender has the right to seize the predetermined collateral as was promised. Therefore, upon maturity, the financial contract yields $\min\{A_S^j, d_S^{C^j}\}$ in state S . Note that agents must own collateral before making promises.

The financial contracts that are central to our analysis are debt contracts and the credit default swap. Debt contracts, denoted j_ℓ , have non-contingent promises $A^\ell = (\ell, \ell, \ell)$. Without loss of generality, we suppose that all debt contracts are collateralized by one unit of the risky asset Y .¹⁶ Debt contracts with promises $\ell \leq D$ are fully collateralized (never default) and are therefore risk free. Debt contracts with $\ell \leq M$ will default in state D but deliver the promise ℓ in states U and M .

A CDS contract on the risky asset Y , which we denote CDS_Y , pays $1 - d_S^Y$ in state s , which is the difference between the maximum payout of Y and the actual payout of Y . To simplify the analysis, we require that each unit of the CDS contract be fully collateralized so that any agent selling the CDS_Y contract is able to repay his obligations regardless of which state is realized.¹⁷

¹⁵We exclude cash flow problems. For an extensive analysis on the implications on asset prices, leverage and production arising from the distinction see Fostel and Geanakoplos (2015, 2016). Crucially, in our model all safe assets truly are safe; assets perceived to be risk-free do not suddenly become risky. See Gennaioli et al. (2013) for a model in which a crisis occurs when “safe assets” are not truly safe.

¹⁶In equilibrium, selling a non-contingent promise backed by X as collateral would be equivalent to selling a fraction of X .

¹⁷This restriction is not without loss of generality for the equilibrium regime, though our main results continue to hold. As will be clear from the analysis that follows, if agents could sell “partially collateralized CDS,” then in

The safe asset X can serve as collateral for CDS. Since CDS_Y pays $(0, 1 - M, 1 - D)$ in states (U, M, D) , and because $(1 - D)$ is the maximum payout for each unit of the CDS_Y contract, every unit of CDS_Y must be collateralized by $(1 - D)$ units of X . Alternatively, an agent holding one unit of X can sell $\frac{1}{1-D}$ units of CDS_Y . When Y can serve as collateral for CDS, one CDS_Y contract must be backed by $\frac{D}{1-D}$ units of Y ; alternatively, $\frac{1}{D}$ units of Y can back $\frac{1}{1-D}$ units of CDS_Y . We let J^Y and J^X be the set of promises j backed by Y and X respectively. Thus, to start $J^X = (CDS_Y, (1 - D)X)$. Later we will introduce a CDS on risky debt contracts (specifically on j_M), which will expand J^X .

Agents are allowed to use debt contracts $j \in J^Y$ to issue financial promises in the form of debt or CDS. We refer to this process as “debt collateralization.” First, we allow agents to trade contracts of the form $j_\ell^1 = (\ell, j_M)$. This contract specifies a non-contingent promise (ℓ, ℓ, ℓ) backed by the risky debt j_M acting as collateral (the restriction to j_M is without loss of generality).¹⁸ The contract j_M delivers $d_s^{j_M} = (M, M, D)$, and the payoff to j_ℓ^1 in each state is the minimum of the promise ℓ and the payoff of the debt contract j_M (i.e., $\min\{\ell, d_s^{j_M}\}$). Note that the act of holding j_M and selling the contract j_D^1 is equivalent to buying j_M with leverage promising D , yielding a payoff of $(M - D, M - D, 0)$. Second, we allow agents to use safe debt j_D to issue CDS, which is the contract $(CDS_Y, (1 - D)j_D)$, and this contract has identical payoffs to CDS backed by X . Denote the set of contracts backed by j_M and j_D by J^1 . Note that even in this scenario, all financial contracts are ultimately collateralized by either X or Y .

The set of contracts available for trade is $J = J^Y \cup J^1 \cup J^X$. We denote the sale of $|\varphi_j|$ units of a promise $j \in J$ when $\varphi_j > 0$ and the purchase of $|\varphi_j|$ units of the contract when $\varphi_j < 0$. The sale of a contract corresponds to borrowing the sale price and the purchase of a promise is equivalent to lending the price in return for the promise. The sale of $\varphi_j > 0$ units of a contract requires ownership of φ_j units of that asset, whereas the purchase of such contracts does not require ownership.

When an asset is allowed to serve as collateral to back financial contracts, we say that it has “collateral value” (CV). Fostel and Geanakoplos (2008) show that the price of an asset can be decomposed into the sum of its “payoff value” (PV) and its CV to any agent who holds the asset.

equilibrium some agents would sell CDS collateralized by only $1 - M$ units of X , which would yield the CDS buyers a payoff of $(0, 1 - M, 1 - M)$ and the sellers a payoff of $(1 - M, 0, 0)$. The first payoff would be attractive to “high pessimists” and the second payoff would be attractive to the most optimistic agents, and is equivalent to buying Y and promising M , which we consider in the sections with leverage.

¹⁸We could let any contract $j \in J^Y$ serve as collateral; however, Gong and Phelan (2016) show that in equilibrium only j_M will be traded and thus only j_M will serve as collateral.

Formally, the PV is the the normalized expected marginal utility of its future payoff for the agent; the CV is a function of the asset's collateral capacity and how much the agent values liquidity. Thus, the collateral value represents the asset's marginal contribution to the agent's liquidity. In general, when an asset can be used as collateral, its price exceeds the payoff value. When an asset cannot act as collateral, the CV is always zero.

We take the financial environment as exogenous for modeling tractability, but one should understand variations in the financial environment as reflecting endogenous changes in the ease with which agents can use different assets as collateral. For example, informational issues could explain why assets or their derivatives cannot be used effectively as collateral (see e.g., Dang et al., 2009; Gorton and Metrick, 2012; Gorton and Ordoñez, 2014).

Budget Set

Each contract $j \in J$ trades for a price π^j . An investor can borrow π^j by selling contract j in exchange for a promise to pay A^j tomorrow, provided that he owns C^j . We normalize by the price of asset X , taking it to be 1 in all states of the world. Thus, holding X is analogous to holding cash without inflation. We let p denote the price of the risky asset Y . Given asset and contract prices at time 0, each agent decides how much X and Y he holds and trades contracts φ_j to maximize utility, subject to the budget set

$$B^h(p, \pi) = \{(x, y, \varphi, x_U, x_M, x_D) \in R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ \times R_+ :$$

$$(x - 1) + p(y - 1) \leq \sum_{j \in J} \varphi_j \pi^j \quad (1)$$

$$\sum_{j \in J^X} \max(0, \varphi_j) \leq x, \quad \sum_{j \in J^Y} \max(0, \varphi_j) \leq y, \quad \sum_{j \in J^1} \max(0, \varphi_j) \leq \varphi_{j_M} \quad (2)$$

$$c_s = x + yd_s^Y - \sum_{j \in J} \varphi_j \min(A_s^j, d_s^{C^j}). \quad (3)$$

Equation (1) states that expenditures on assets (purchased or sold) cannot be greater than the resources borrowed by selling contracts using assets as collateral. Equation (2) is the collateral constraint, requiring that agents must hold the sufficient number of assets to collateralize the contracts they sell, which includes positions in risky debt contracts used as collateral for further promises. Equation (3) states that in the final states, consumption must equal dividends of the assets

held minus debt repayment. Recall that a positive φ_j denotes that the agent is selling a contract or borrowing π^j , while a negative φ_j denotes that the agent is buying the contract or lending π^j . Thus there is no sign constraint on φ_j . Due to pledgeability concerns, agents cannot take negative positions in assets (i.e., $y \geq 0$ and $x \geq 0$); however, we later allow for collateralized short selling of the risky asset by letting agents issue a promise replicating Y backed by X as collateral. Our results are robust to allowing this form of short selling.

Collateral Equilibrium

A collateral equilibrium in this economy is a price of asset Y , contract prices, asset purchases, contract trades, and consumption decisions all by agents,

$((p, \pi), (x^h, y^h, \varphi^h, x_U^h, x_M^h, x_D^h)_{h \in (0,1)}) \in (R_+ \times R_+^J) \times (R_+ \times R_+ \times R^{J^X} \times R^{J^Y} \times R_+ \times R_+ \times R_+)^H$ such that

1. $\int_0^1 x^h dh = 1$
2. $\int_0^1 y^h dh = 1$
3. $\int_0^1 \varphi_j^h dh = 0 \quad \forall j \in J$
4. $(x^h, y^h, \varphi^h, x_U^h, x_D^h) \in B^h(p, \pi), \forall h$
5. $(x, y, \varphi, x_U, x_D) \in B^h(p, \pi) \Rightarrow U^h(x) \leq U^h(x^h), \forall h$

Conditions 1 and 2 are the asset market clearing conditions for X and Y at time 0 and condition 3 is the market clearing condition for financial contracts. Condition 4 requires that all portfolio and consumption bundles satisfy agents' budget sets, and condition 5 requires that agents maximize their expected utility given their budget sets. Geanakoplos and Zame (2014) show that equilibrium in this model always exists under the assumptions made thus far.

3 A Model of the CDS Basis

We first consider when agents can use X as collateral to issue CDS_Y and can also use Y as collateral to issue debt contracts and to issue CDS_Y . This case, which we refer to as the leverage economy, has been considered by Fostel and Geanakoplos (2012a) in a two-state economy. We then consider when agents can use risky debt contracts backed by Y as collateral to issue debt contracts, which is the debt collateralization economy.

We define the CDS basis as the difference between the CDS price and the bond price:¹⁹

$$\text{Basis}_Y = \pi_C^Y - (1 - p). \quad (4)$$

Note that the payout of holding one unit of X is equivalent to holding one unit of Y and one unit of CDS_Y . Thus, the basis can be equivalently defined to be the difference in the price of these two options: $\text{Basis}_Y = (p + \pi_C^Y) - 1$, or $p + \pi_C^Y = 1 + \text{Basis}_Y$. We use the term ‘‘cash-synthetic asset’’ to refer to a portfolio consisting of equal units of Y and CDS_Y since this option, like holding X , is completely risk-free.

The main result of this section is that the basis is positive in an economy with debt collateralization (Proposition 4). Section 3.4 discusses our results when agents can sell short Y by issuing a collateralized financial promise replicating Y . Equilibrium conditions for all economies are in Appendix B and all proofs are in Appendix C.

3.1 Leverage Economy: $C^j \in \{X, Y\}$

Consider when the risky asset Y can be used as collateral to issue debt contracts and CDS_Y . In particular, one unit of Y can back a non-contingent debt promise (ℓ, ℓ, ℓ) , or $\frac{1-D}{D}$ units of Y can back one (fully collateralized) CDS contract. This is due to the fact that the CDS pays $1 - D$ in the same state when Y pays D .

The results of Fostel and Geanakoplos (2012a,b) characterize which contracts will be traded in equilibrium in an economy with only debt contracts, and these results allow us to characterize equilibrium with CDS . In an economy with debt contracts and without leverage limits, two debt contracts are traded in equilibrium: $j_D = D$ and $j_M = M$, with prices π^D and π^M respectively. The contract j_D delivers (D, D, D) , while j_M delivers (M, M, D) . Unlike the safe promise j_D , the delivery of j_M depends on the realization of the state at time 1. Therefore, j_M is risky and has price $\pi^M < M$. The interest rate for j_M is strictly positive and is given by $i_M = \frac{M}{\pi^M} - 1$, and is endogenously determined in equilibrium.

First, note that holding $1 - D$ units of Y and selling D units of CDS contracts yields $(1 - D, M -$

¹⁹Defining in this order preserves the standard notation, defined based on spreads (which move inversely with prices) so that a positive basis indicates that the bond is ‘‘expensive.’’

$D,0$), which is the same payoff as holding one unit of Y and selling the promise j_D . Second, holding $(1-D)$ of X and selling one unit of CDS_Y also yields the same payoff as holding one unit of Y and selling the promise j_D . We denote buying Y and selling CDS by Y/CDS_Y , buying Y and selling j_D by Y/j_D , and buying X and selling CDS by X/CDS_Y , where all positions are appropriately scaled to be fully collateralized: Y/CDS_Y costs $(1-D)p - D\pi_C^Y$; Y/j_D costs $p - \pi^D$; X/CDS_Y costs $1 - D - \pi_C^Y$. Since all positions yield the same cash flows, investors will choose the positions which are cheapest. An immediate implication is that the equilibrium basis is non-negative.

Proposition 1. *In an economy with $J^X = (CDS_Y, (1-D)X)$ and in which Y can serve as collateral for debt contracts and for CDS contracts, the basis on Y is non-negative. In other words, X and Y can both serve as collateral for debt and CDS contracts, $\pi_C^Y + p \geq 1$. Furthermore, in equilibrium no agent will trade Y/j_D and no agent will buy j_D*

If the basis were negative, then agents would prefer to use Y as collateral to issue CDS over using X , and so no agent would hold X . In fact, we can say more: if the basis is zero, then X/CDS_Y are equivalent Y/CDS_Y and both will be traded in equilibrium; when the basis is strictly positive then X/CDS_Y is cheaper and no agent will trade Y/CDS_Y in equilibrium. Accordingly, equilibrium in the leverage economy can be described by three marginal investors h_1, h_2, h_3 . Investors $h > h_1$ buy the risky asset Y and issue risky debt. Investors with $h \in (h_2, h_1)$ issue CDS contracts, using either X or Y as collateral. Investors with $h \in (h_3, h_2)$ buy risky debt, and the remaining investors buy CDS.

Lemma 1. *In the leverage economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) Y/j_M , (2) $X/CDS_Y \equiv Y/CDS_Y$, (3) j_M , (4) and CDS_Y . When the basis is zero, then a fraction of Y is used for Y/CDS_Y , but no agents trade Y/j_D when the basis is positive.*

That the four positions exist in equilibrium is immediate. Figure 5 shows the equilibrium regime. Arrows point from lender to borrower. In this economy, pessimists lend to optimists.

Notice that we could implement this equilibrium if we let any safe asset—specifically, j_D in addition to X —be used as collateral to back CDS_Y . Whether or not Y can back CDS_Y , equilibrium would be unchanged. In equilibrium, if the basis is zero, then agents will trade Y/j_D , and every

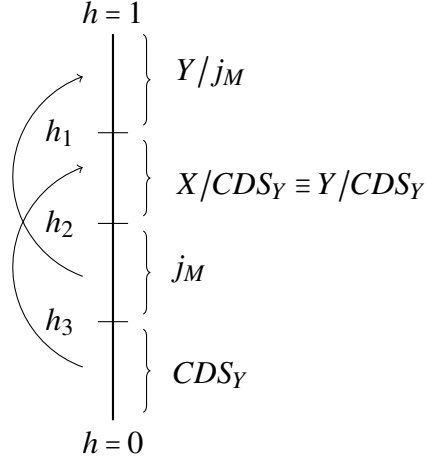


Figure 5: Equilibrium with leverage and CDS_Y backed by X . Buyers of CDS_Y fund moderates holding X/CDS_Y . Agents purchasing j_M lend to optimists.

agent that buys j_D will use it as collateral to sell CDS_Y (just as they do with X). Thus, $\pi^D = D$, and the following positions will be equivalent: $X/CDS_Y, Y/j_D, j_D/CDS_Y$. The risky asset Y would implicitly back CDS_Y because it would be used to back safe debt which was used to back CDS_Y . This observation motivates Section 3.2 on using debt as collateral.

Before considering debt collateralization, we note that limiting leverage (i.e., restricting the set of contracts backed by Y) decreases the basis. If Y is imperfect collateral, perhaps because of regulations or because financial markets have concerns arising from informational issues, then the basis will be negative. The following propositions extend to results in Fostel and Geanakoplos (2012a) to multi-state economies. The details of equilibrium in these environments are provided in the appendix, which also provide further analysis of how leverage limits affect the basis.

Proposition 2. *In an economy with $J = J^X = (CDS_Y, (1-D)X)$, the basis on Y is negative. In other words, when CDS_Y is the only financial contract in the economy, $\pi_C^Y + p < 1$.*

Proposition 3. *Suppose X can issue CDS_Y but Y cannot, and Y can issue debt contracts where the highest leverage allowed is $\bar{\ell} \leq D$. Then the basis is negative, $p + \pi_C^Y < 1$.*

3.2 Debt Collateralization Economy: $C^j \in \{X, Y, j_M\}$

We now suppose that agents can use debt backed by Y as collateral. We refer to this financial innovation as “debt collateralization” and show that this results in a positive basis. Specifically,

without loss of generality, we let agents use the debt contract j_M as collateral to make secondary non-contingent promises, following Gong and Phelan (2016). Additionally, agents can use safe debt as collateral for CDS_Y . None of the following results depend on which assets can serve as collateral for CDS_Y . (We prove in the appendix that all of our results still hold even when we allow agents to use Y and j_M to back the CDS.) The following proposition describes the equilibrium basis in this regime.

Proposition 4. *In the economy with debt collateralization and CDS_Y backed by X , there is a positive basis on the risky asset. In other words, $p + \pi_C^Y > 1$ and $Basis_Y > 0$.*

Allowing j_M to serve as collateral for financial contracts increases the collateral value of j_M , since agents buying j_M are also buying the ability to sell j_D^1 . This increases π^M in equilibrium. Since agents can leverage their purchases of Y by borrowing π^M , this implies that agents can now buy Y with higher leverage, raising the equilibrium demand for Y . Gong and Phelan (2016) show that debt collateralization increases the collateral value of Y because Y can be used to issue j_M and therefore inherits some of the increase in the collateral value of j_M . Thus the risky asset Y now has two “levels” of collateralization—the first from allowing Y to back debt contracts, and the second from allowing these debt contracts to back further contracts. The collateral value of X does not change due to the fact that it can still issue only one contract, CDS_Y . All of these forces combined increases the price of Y relative to the price of X and result in a positive basis. These results allow us to characterize the equilibrium regime.

Corollary 1. *In the economy with debt collateralization and CDS_Y backed by X , it is cheaper to hold X/CDS_Y than Y/j_D . Thus, no agent will hold Y/j_D . That is, $(1 - D) - \pi_C^Y < p - D$.*

Lemma 2. *In this economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) Y/j_M , (2) $X/CDS_Y \equiv j_D^1/CDS_Y$, (3) j_M/j_D^1 , and (4) CDS_Y . This characterization of equilibrium is not dependent on which assets can be used to issue CDS_Y . In fact, the equilibrium regime does not change even if we allow agents to use Y and j_M to back the CDS,*

It is clear from earlier results that the above four positions must exist in equilibrium. Figure 6 depicts the equilibrium regime. There are three marginal buyers. Arrows demonstrate the lender-borrower relationship in this economy, pointing from lenders to borrowers. Compared to the

leverage economy, there is no longer a clean lending relationship, with pessimistic investors always lending to more optimistic agents. In addition to the usual lending flows, in this equilibrium we also see relatively optimistic agents (those holding the safe asset and selling CDS) lending to more pessimistic agents (those holding the risky debt contract) by buying the safe debt contract issued by the pessimists. This occurs because the safe debt issued by these pessimists can be leveraged to make an even more optimistic trade. (This is a form of financial entanglement.)

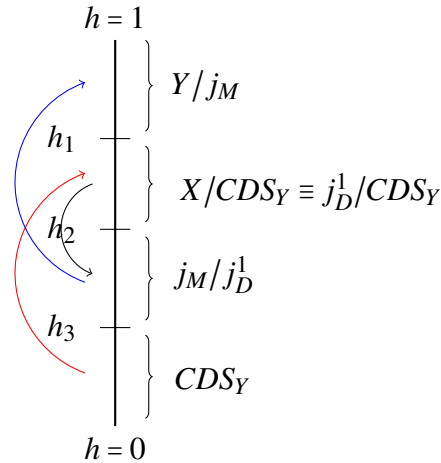


Figure 6: Equilibrium with debt collateralization and CDS_Y backed by X . Regime features financial entanglement.

Our results yield two key insights regarding how collateral affects the basis. First, the cash-synthetic basis is a measure of the differential “collateral values” between risky and safe assets. Importantly, the collateral value of a risky asset does not only depend on the extent to which it can be used as collateral, but also on the extent to which downstream debt contracts backed by the asset can be used as collateral. In other words, the asset’s collateral value depends on the collateral value of derivative debt. When risky bonds can be used as collateral, and debt contracts backed by risky bonds can also be used as collateral for financial contracts, the bond premium is less than the corresponding CDS premium and the excess bond premium is negative.

Allowing risky debt to serve as collateral implicitly raises the degree to which the underlying asset can serve as collateral, since the same asset directly and indirectly backs a greater degree of promises. Thus, our analysis highlights that the existence of a non-zero basis implies, in addition to the other factors identified in the literature to contribute to bases, a difference between the collateral

value of safe and risky assets. The positive basis emerges because Y can be used to issue financial promises with positive collateral value. Accordingly, if the collateral value of the derivative debt contracts decreases, then the basis for Y should decrease.

Second, agents value assets based on their abilities to provide payoffs in different states, not just based on the original payoffs of the assets. Assets with the same payoffs but that can be used as collateral for different promises allow agents to isolate payoffs in different states. Thus, agents choose to buy assets that best allow them to isolate payoffs in states in which their marginal utilities are higher. As a result, agents may not “trade against” the basis even though there is an apparent arbitrage opportunity, but trade to receive their most preferred state-contingent payoffs.²⁰

3.3 Numerical Example

While our results hold across parameters and are not quantitative, a numerical example is helpful to fix ideas. We let beliefs be $\gamma_U(h) = h$, $\gamma_M(h) = h(1 - h)$, and let payoffs be $d_M^Y = 0.3$ and $d_D^Y = 0.1$. Table 1 compares equilibrium with no leverage, leverage, and debt collateralization. When debt backed by Y can be used to back further debt contracts, the basis is positive since Y now has two levels of collateralization. Our results explicitly demonstrate that the basis does not only depend on whether Y can be used as collateral—it is also intrinsically linked to the collateral value of “downstream” promises backed by Y .

Table 1: Equilibrium with No Leverage, Leverage, and Debt Collateralization

	No Leverage	Leverage	Debt Collateralization
p	0.447	0.508 ↑	0.529 ↑
π_C^Y	0.513	0.492 ↓	0.491 ↓
π^M	–	0.204	0.224 ↑
Basis $_Y$	-0.040	0 ↑	0.020 ↑

An agent in the no-leverage regime could choose to buy the cash-synthetic asset consisting of a portfolio of Y and CDS_Y —at a lower price than X while earning the same return—but this

²⁰This insight is especially important when balance sheet considerations imply that a small arbitrage may not be worth undertaking given the costs of balance sheets. Thus, investors may prefer a risky investment with large upside potential over an arbitrage for only several basis points. For evidence based on deviations from covered interest rate parity see Du et al. (2016).

portfolio is less valuable to agents because it cannot be used as collateral to back financial contracts. Thus, the cash-synthetic asset does not provide agents the ability to isolate payoffs in a state of the world. Similarly, every investor in the debt collateralization economy could sell Y and CDS_Y to buy X at a price lower than the cash-synthetic asset. However, in equilibrium, no agent chooses to do so because the value of “downstream” contracts backed by X is lower than those backed by Y , and it is also cheaper for the agent to buy X while selling the CDS_Y contract.

In fact, a *positive* basis could emerge in a leverage economy when there is a strong demand to use Y to issue risky debt, rather than to use Y to issue CDS , which is the equivalent leveraging with safe promises.²¹ However, if agents could sell partially collateralized CDS , then a zero-basis would re-emerge because issuing a partially collateralized CDS is equivalent to Y/j_M . Thus, the positive basis emerges with the restriction that CDS be fully collateralized because X is “constrained” in the set of promises it can make while Y is not. See Figure 11 for comparative statics regarding positive bases with leverage.

3.4 Economies with Short Selling

Thus far we have been silent about the possibility of short sales. One could understandably worry that, given the literature on limits to arbitrage, ignoring short selling would be a central driver of our results. We now show that this is not the case. In this section we provide agents the ability to sell short Y and we show that in general agents will *not* choose to do so. The intuition for our result is that to bet against Y , a collateral-efficient strategy is to buy CDS (requiring no collateral) rather than to sell short the asset.

Let agents now be allowed to issue a contract promising $(1, M, D)$, which we call a Y -promise. This Y -promise is collateralized by 1 unit of X and costs π_{short}^Y . Note that buying X and issuing a Y -promise is a collateralized short position in Y , which costs $1 - \pi_{short}^Y$ and delivers $(0, 1 - M, 1 - D)$, which is exactly the payoff to a CDS . Thus, agents can bet against Y by either buying CDS or by shorting Y . However, a unit of X can issue more CDS than Y -promise: one CDS is backed by $1 - D$

²¹To see this, consider the following comparative static for the economy above. Redistribute wealth from agents $h < h_3$ to agents $h > h_1$. For small redistribution, the only equilibrium variable affected would be η , the fraction of Y used to back CDS_Y , and thus the supply of CDS . Taking wealth from agents $h < h_3$ would decrease demand for CDS , and increasing wealth for agents $h > h_1$ would increase demand for Y/j_M . A large enough redistribution would require $\eta = 0$, at which point marginal agents and prices would change, at which point the basis could be positive so that agents trading X/CDS_Y would not trade Y/CDS_Y .

units of X as collateral while selling Y -promise requires one unit of X . This is precisely what we mean when we say that buying the CDS to bet against Y is collateral efficient.²²

We now reinforce our previous results by showing that our results hold even when short sales are allowed.

Lemma 3 (Shorting with no leverage). *Suppose that Y cannot be used as collateral and agents can short Y by issuing collateralized Y -promises. In equilibrium, agents do not issue Y -promises and the basis is negative.*

Since neither Y nor Y -promises can be used as collateral, then investors are indifferent between buying Y or the Y -promise, and so if the Y -promise is traded in equilibrium it must be that $\pi_{short}^Y = p$. Since buying X and issuing a Y -promise delivers the same payoffs as buying a CDS, a Y -promise will be issued in equilibrium only if $\pi_C^Y \geq 1 - \pi_{short}^Y$, implying that $p + \pi_C^Y \geq 1$ (positive basis). But this cannot be the case in equilibrium because the most optimistic investors would then have a greater incentive to *issue* CDS because this position is relatively cheap. The intuition for the result is immediate: when Y cannot be used as collateral, the basis is negative (Y is cheap) and so investors do not want to sell short the already-cheap asset, but those who wish to bet against it do so by buying CDS.

Lemma 4 (Shorting with leverage). *Suppose that Y can be used as collateral to issue debt contracts and agents can short Y by issuing collateralized Y -promises. In equilibrium, the basis on Y is non-negative, as it was without short sales.*

Now, because Y can be used as collateral while the Y -promise cannot, it must be that $\pi_{short}^Y \leq p$. First, suppose that short sales do occur in equilibrium. As we just argued, agents are only willing to issue Y -promises (to short Y) if the basis is non-negative since a negative basis implies it is cheaper to buy CDS. Thus, the presence of short-sales imply a non-negative basis. In particular, the equilibrium regime would feature a set of agents buying Y promises with these agents lying between those using X to issue CDS and those buying the risky debt.

²²An alternative modeling strategy follows Bottazzi et al. (2012) by explicitly requiring agents to borrow the asset Y at a funding cost in order to sell it short in the market. This “box constraint” is how short sales are done in reality. They show that a binding box constraint leads to a liquidity premium (bonds are special in repo), increasing the cost of shorting. Our setup will deliver a similar result—the Y -promise may trade at a discount to Y , implying that shorting Y entails a funding cost.

Second, if short sales do not occur, then the equilibrium regime is exactly as discussed in the previous section so the basis is non-negative.

Lemma 5 (Shorting with debt collateralization). *Suppose that Y can be used as collateral to issue debt, which can be used as collateral to issue further promises (debt collateralization). Agents can short Y by issuing collateralized Y -promises, but these promises cannot be used as collateral. In equilibrium, the basis on Y is strictly positive.*

The result follows from the same argument as the previous lemma.

The restriction that Y -promises cannot be completely collateralized as the underlying asset can reflect either (i) direct limitations in borrowing underlying assets to short or (ii) the fact that assets that are used in CDOs or other structured securities cannot be frictionlessly replicated to be used in these same structures.²³ These restrictions are empirically relevant given the assets we have in mind (corporate bonds, mortgage- and asset-backed securities, etc.)

4 A Model of the Double Basis

The risky debt j_M behaves like the risky asset Y since it has the same payoff as Y in the M and D states. Accordingly, we introduce a CDS on the risky debt j_M and now consider the CDS basis for the risky debt j_M and to study the relationship between this basis and the basis for the risky asset. We think of the basis on j_M as corresponding to the basis on ABS or CDO tranches, rather than the basis on the underlying pool of collateral. As before, we define the basis on the risky debt, denoted Basis_M , as the difference between the spread on the debt CDS and the bond spread, $\text{Basis}_M = \pi^M - (M - \pi_C^M)$.

The main result of this section is that the basis on the risky debt j_M is always lower than the basis on the underlying risky asset Y . In other words, the basis on the most upstream collateral (the risky asset Y) is greater than the basis on downstream contracts (the risky debt j_M). This occurs because the risky asset Y can always back at least one more level of debt contracts than the risky debt can back, and so the debt has a lower collateral value. We use the term “double basis” to refer to this phenomenon of two unequal bases occurring in equilibrium for assets with correlated

²³Technically, the result holds when the Y -promise can be collateral but the debt backed by the Y -promise cannot be, implying that a risky promise backed by the Y -promise would be different from the risky promise backed by Y .

payoffs. The results for the basis on the risky asset Y all continue to hold in this environment with CDS_M . In a model with $N > 3$ states of uncertainty and with debt collateralization, both bases can be positive when debt can be used to back debt a sufficiently high number of times.

4.1 The Double Basis in a Leverage Economy

The CDS on j_M pays the difference between the promised delivery of j_M and the actual return: $(0, 0, M - D)$ in states (U, M, D) . We use CDS_M to denote the contract and π_C^M to denote its price. Notice that for this economy, the CDS on j_M is functionally an Arrow security for state D . We specify that each unit of this CDS contract must be fully collateralized by the safe asset X . That is, any agent selling the promise must hold the sufficient amount of collateral so that he can always deliver the amount promised, so a unit of this CDS contract must be backed by $M - D$ units of X . Equivalently, one unit of X can be used to back $\frac{1}{M-D}$ unit of CDS_M , and we use X/CDS_M to represent the act of holding X and selling the maximum amount of CDS_M .

We first examine the basis on the CDS_Y contract and the basis on the CDS_M contract in an economy with leverage. We also let the safe debt j_D serve as collateral for both CDS_Y and CDS_M . We specify that both CDS contracts must be fully collateralized by either X or j_D so that any agent who makes the CDS promise can always deliver the contractual amount. In this economy, the set of tradable financial contracts is $J = J^Y \cup J^X \cup J^1$, where $J^Y = \{j_M, j_D\}$ which are the non-contingent debt contracts, $J^X = \{(CDS_Y, X), (CDS_M, X)\}$, and $J^1 = \{(CDS_Y, j_D), (CDS_M, j_D)\}$. The following proposition characterizes the equilibrium basis for the CDS_M contract in this economy.

Proposition 5. *In an economy with leverage, and CDS contracts CDS_Y and CDS_M , which are backed by safe assets, the basis on the risky debt is negative and the basis on the risky asset is non-negative. That is, $\pi^M + \pi_C^M < M$ and $p + \pi_C^Y \geq 1$.*

The intuition for this result is similar to the one provided in the previous section—in this economy, j_M has no collateral value because agents are not allowed to leverage the risky debt that they hold. However, X is allowed to issue CDS_M , which gives X higher collateral value relative to j_M . This results in a negative basis on the risky debt. The negative basis occurs because agents buy the safe asset in order to issue CDS contracts. Because the combination of CDS_M and j_M does not provide agents with this ability, the cash synthetic asset made of CDS_M and j_M naturally has a

lower price than X . Thus, agents buying X have fundamentally different motivations from agents buying j_M or CDS_M : investors buy X to increase payoffs in the upstate, while investors purchasing j_M or CDS_M are betting on either the middle state or the down state, respectively.

Figure 7 illustrates the equilibrium regime with the direction of the arrow indicating the direction of funding. In general, pessimists lend to optimists in this economy. The most pessimistic agents buy the CDS_M promise from moderates, thereby lending to agents holding X/CDS_M . Agents who are slightly less pessimistic hold CDS_Y , funding those who hold X/CDS_Y . Moderates buying the risky debt contracts lend to the most optimistic agents in the economy, who are buying Y while making the j_M promise. However, financial entanglement occurs between agents who hold X/CDS_Y , Y/j_D or X/CDS_M ; the safe debt contracts, j_D are being bought by agents who hold X . Thus, within (h_1, h_2) , agents are (potentially) lending to each other, and agents in (h_2, h_3) are also lending to those in (h_1, h_2) .

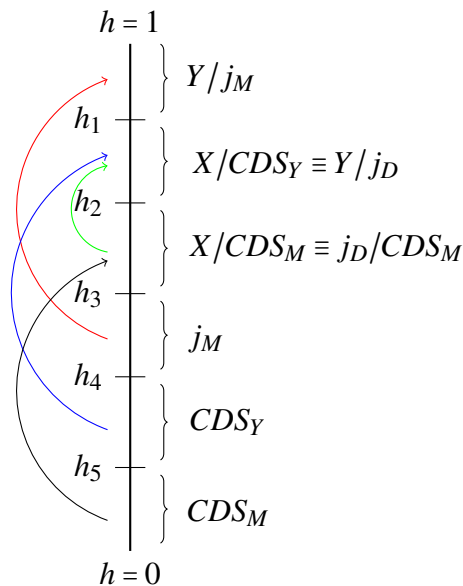


Figure 7: Equilibrium with CDS_Y and CDS_M (backed by X). No debt collateralization.

4.2 The Double Basis in a Debt Collateralization Economy

We now re-introduce j_M into the set of assets that can serve as collateral for other financial contracts. Since j_M delivers at least d_D^Y in every state of the world, the j_D^1 contract is safe. We allow agents to use j_D^1 as collateral to back both CDS_Y and CDS_M . Thus, $J = J^Y \cup J_X \cup J^1$, where

$J^1 = \{(CDS_M, j_D^1), (CDS_Y, j_D^1), (j_D^1, j_M)\}$. Note that using $(M - D)$ units of X to sell one unit of CDS_M has the same payout as buying one unit j_M , leveraged with safe debt. The following proposition characterizes the equilibrium basis on CDS_M

Proposition 6. *In an economy with debt collateralization, CDS_Y , and CDS_M , the basis on the risky debt is zero and the basis on the risky asset is positive.*

Note that $\text{Basis}_M = 0$ implies that $\text{Basis}_Y > 0$ since Y always has one more level of collateralization than j_M . Allowing j_M to serve as collateral implicitly raises the collateral value of Y , and causes $\text{Basis}_Y > 0$. The basis on the most upstream collateral is greater than the basis on downstream contracts. Figure 8 depicts the equilibrium regime and shows the direction of funding between agents. The borrower-lender relationships are similar to those in the previous regime. However, agents who are buying safe assets and selling the CDS_Y contract are now lending to more pessimistic investors holding the risky debt contract with leverage. This occurs because the safe debt issued by the moderates can be used as collateral to issue CDS_Y , which is a riskier position.

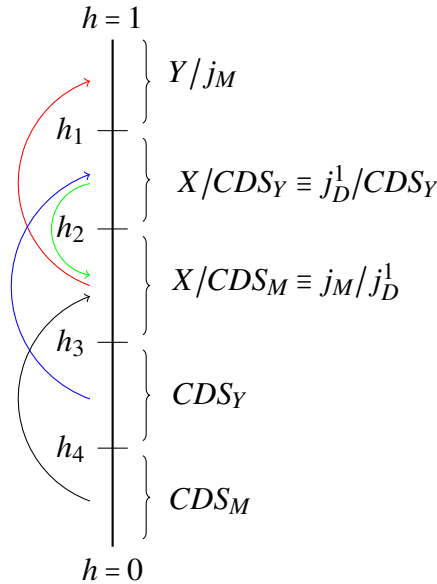


Figure 8: Equilibrium with CDS_Y , CDS_M , and Debt Collateralization.

Table 2 compares the prices and bases in the CDS_M regime with leverage and the CDS_M regime with equilibrium. The price of the risky asset increases because debt backed by Y can now serve as collateral. The price of risky debt increases because agents can now buy the debt with

leverage, increasing demand for risky debt. Furthermore, allowing j_M to serve as collateral for non-contingent debt contracts increases the supply of safe assets in the economy. Since safe assets are used to issue both CDS_Y and CDS_M , the supply of both these CDS contracts increase, resulting in a lower π_C^Y and π_C^M . While in this 3-state economy Basis_M can never be positive because j_M can be collateralized at most once, we can obtain a positive basis on both the risky debt in a four-state model in which downstream debt contracts can be used to back multiple layers of debt. See Appendix B for an example, and see Gong and Phelan (2016) for a theoretical characterization of debt collateralization with $N > 3$ states.

Table 2: Double-Basis Equilibrium with Leverage and Debt Collateralization

	Leverage	Collateralization
p	0.502	0.527 ↑
π^M	0.196	0.223 ↑
π_C^Y	0.498	0.491 ↓
π_C^M	0.090	0.077 ↓
Basis_Y	0	0.018 ↑
Basis_M	-0.014	0 ↑

5 Empirical Implications

Our analysis offers a few testable implications regarding fluctuations in bases. In this section we discuss these implications, some suggestive evidence supporting our theory, and empirical considerations for more careful tests by future research.

First, our theory predicts that debt collateralization increases the CDS basis. Thus, variations in (i) the extent to which funding markets use debt as collateral, or (ii) to which structured finance implicitly allows debt to be used as collateral, ought to correspond to variations in the CDS basis. This implication is distinct from the prediction of the cheapest-to-deliver mechanism, in which funding markets for derivative debt contracts have no direct effect on the positive basis. This implication is also distinct from the prediction of the “CDS market leads the bond market” mechanism, in which the basis fluctuates with credit risk.

There are two sets of facts that provide suggestive evidence for the predictions of our model.

First, the predictions of our model are broadly consistent with the stylized facts regarding the prevalence and collapse of CDO and structured finance issuance as well as the time series behavior of average bases (see Figures 1–3). Rauh and Sufi (2010) show that low-credit-quality firms are more likely to have a multi-tiered capital structure with subordinated debt. Hence, our model predicts that pre-crisis the HY basis should be larger than IG basis because senior-subordinated capital structures, which implicitly use debt as collateral for debt, increase the basis (post-crisis, funding market freezes disproportionately affected weak collateral, which is why HY bases would turn more negative).²⁴

Second, our result from Section 4 provides a potential explanation for an additional force driving the CDS-CDX basis. The CDS-CDX spread is defined as difference between the average five-year CDS spreads on the 125 constituents of the NA.IG.CDX index and the spread on the NA.IG.CDX index, obtained from Markit. This spread is positive, meaning that the CDS spreads on the underlying constituents is greater than the spread on the CDX index and so it is cheaper to buy protection on the index (pay the premium) than on every individual constituent (see Figure 9).²⁵

Importantly, the CDX index is tranching into synthetic “index CDO tranches”: in addition to buying (or selling) protection on the overall level of the CDX index, investors can also buy protection on the first 3% of losses among the 125 constituents, or losses between 3 and 7%, and so on with attachment points at 10, 15, and 30 percent of losses. The CDX tranches correspond to downstream contracts backed by the underlying constituent assets. Accordingly, the overall CDX index spread captures the spreads on the CDX tranches. Our theory predicts that the basis on the most upstream collateral—namely, the 125 constituent single name CDS contracts—should be greater than the basis on downstream contracts—namely, the index tranches. Accordingly the CDS basis on the constituents ought to exceed the basis on the CDX, implying a higher CDS spread on the constituents. This is exactly what we observe in the data.²⁶ Furthermore, when collateral is most scarce, this basis ought to widen, as occurred during the crisis.²⁷

²⁴Accordingly, credit ratings could serve as an instrument for capital structure/debt collateralization for empirical studies.

²⁵Liquidity conditions provide another explanation. Junge and Trolle (2015) argue that CDX-CDS basis measures the overall liquidity of the CDS market. According to this theory, widening of the CDS-CDX basis would reflect deterioration in liquidity in CDS relative to liquidity in CDX.

²⁶We are grateful to Nina Boyarchenko for her comments on this topic.

²⁷Undoubtedly, limits to arbitrage are important for explaining difficulties in exploiting the apparent arbitrage trade

CDX-CDS Basis

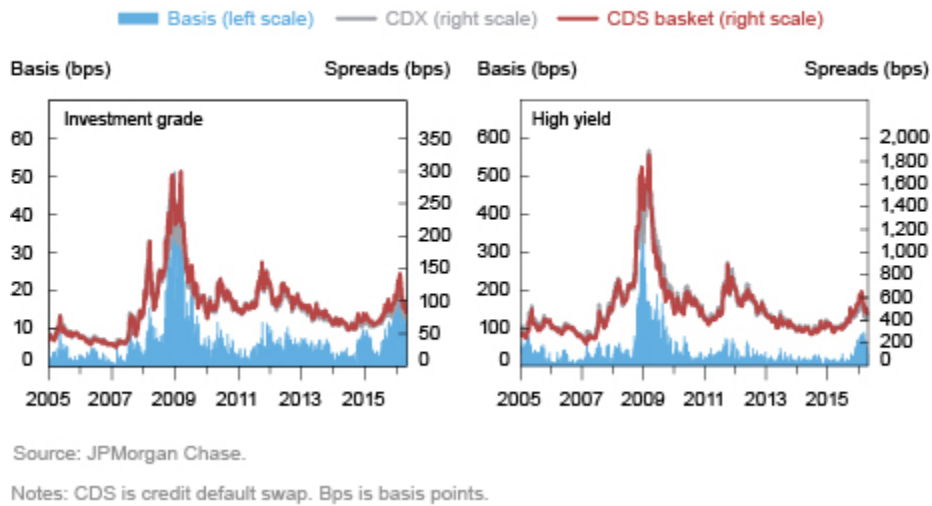


Figure 9: CDS-CDX basis. Source: Boyarchenko et al. (2017)

While our analysis is primarily theoretical, our model also yields a few quantitative implications regarding how the basis depends on the asset payoffs in each financial environment. Figure 10 plots the basis (multiplied by 100) with debt collateralization varying the payoffs M and D . We parameterize beliefs as before (results are qualitatively the same for other belief structures). The comparative statics provides the following main qualitative results, which are interesting testable implications for our model. With debt collateralization the basis is more positive when tail risk is larger (when D is small and M is large). Debt collateralization endogenously shifts equilibrium so that investors purchase the asset only with the riskiest contract. When M and D are very different, leveraging the asset with a safe promise is not very valuable. Since debt collateralization endogenously increases the fraction of investors issuing expensive promises to buy the asset, with substantial tail risk, the collateral value of Y substantially. Thus, variations in tail risk ought to correspond to variations in the size of the CDS basis.

Carefully testing all the empirical implications of our model requires care. Ideally, the econometrician would want data on terms and availability for financing of risky assets, as well as data on financing for debt backed by these risky assets. This would include haircuts for collateralized loans, at

of buying protection on the CDX index (pay the premium) and selling protection on the underlying 125 names (receive the higher premium). Our theory suggests that non-arbitrageur investors would trade instead in particular tranches in order isolate precisely the risk profile they desire. For example, see Longstaff and Rajan (2008) for an analysis of how each tranche corresponds to different levels of systemic/correlated default risk.

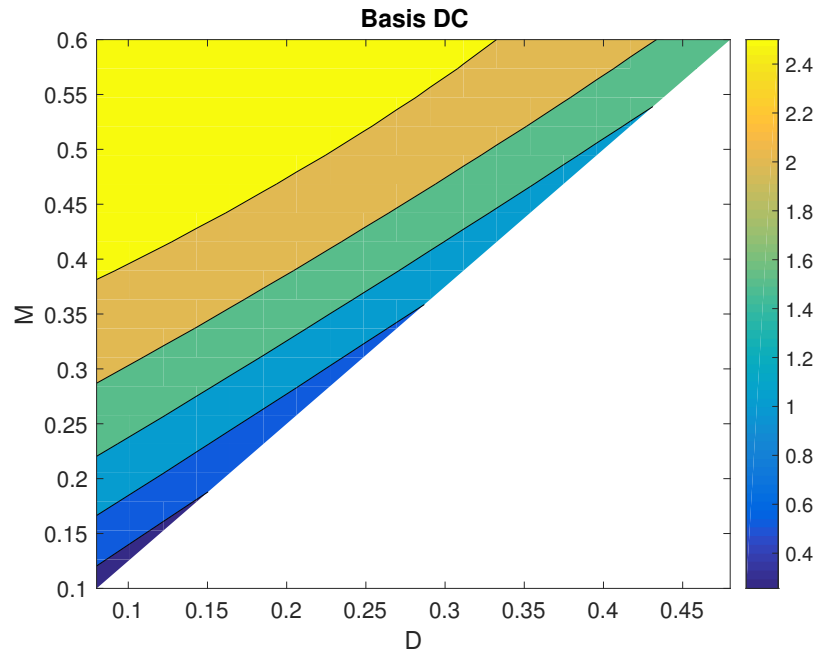


Figure 10: Comparative Statics with debt collateralization: basis (times 100) varying payoffs M , D .

different maturities, on underlying securities in structured finance pools and any margins available to finance tranches. Alternatively, tranches collateralized into CDOs or CLOs could have margins imputed based on the terms of the structured finance deal. The econometrician would then want exogenous variation in loan terms (margins, etc.) to explain fluctuations in the basis. However, as the earlier cited literature argues, the availability of collateral is endogenous. For example, Gorton and Ordoñez (2014) suggest that opaque assets are ideal collateral, or that CDOs are created intentionally opaquely in order to create collateral. Thus, assets that are used as collateral, and for which risky debt contracts can be used as collateral (packaged into CDOs), may be underlying assets that are *less* liquid but which are more liquid (in the sense used in Gorton and Ordoñez (2014)) as a package, implying that other informational frictions are also present in determining the basis.

6 Conclusion

In the context of firm borrowing costs, the CDS basis (which strongly correlates with the excess bond premium) has important implications for both firm funding capacity and economic activity. We present a theoretical model that relates the extent to which financial markets can effectively use assets as collateral to the CDS basis on those bonds. In particular, we show that the basis is positive when agents can use risky debt contracts as collateral to issue financial promises. Structured finance that uses pools of collateral to issue senior-subordinated capital structures will produce positive bases on the underlying collateral, and thus financing these assets will be cheap (excess bond premium is negative). We also prove that when multiple CDS contracts are traded in an economy with debt collateralization, the bases on the CDS contracts must be different as each level of has a different collateral value.

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Appendices

A Additional Financial Environments

A.1 No Leverage: $C^j = X$

Consider the scenario in which agents cannot use Y as collateral to issue debt contracts. Formally, $J^Y = \emptyset$ and $J = J^X = (CDS_Y, (1 - D)X)$ is the only financial contract available for trade. We denote the act of holding X and selling the maximum allowable amount of CDS_Y by X/CDS_Y . In this regime, agents can take any of the following positions: (i) X/CDS_Y (hold X and sell CDS_Y), (ii) buy Y , (iii) buy X or the cash-synthetic asset made of a portfolio of both Y and CDS_Y , and (iv) buy the financial contract CDS_Y . Notice that the above positions are listed in terms of decreasing optimism/increasing pessimism. An agent who believes that state U is very likely to happen will choose to either buy Y or hold X/CDS_Y , whereas an agent who believes that state D is more likely will want to purchase CDS_Y . Because agents are risk neutral, every agent will choose exactly one of the above positions based on how optimistic they are. The following proposition characterizes equilibrium in this economy.

Lemma 6. *In this regime, no agent chooses to hold safe assets without selling financial contracts. That is, no agent chooses to hold simply X or the cash-synthetic asset made of a portfolio of Y and CDS_Y . In fact, any agent who holds X will also sell the maximum allowable amount of CDS_Y .*

The intuition is straightforward. Any agent who does not want to buy X and sell the CDS must value consumption in state D . This is because selling the CDS means that the agent loses consumption if the down state occurs. Thus, these agents are relatively pessimistic (compared to agents who do choose to sell the CDS) and must therefore be willing to sacrifice consumption in state U for the chance to have even more consumption in state M or D . Since CDS_Y pays

$(0, 1 - M, 1 - D)$, in equilibrium prices must be such an agent will want to invest in CDS_Y rather than hold X . The basis must be negative in this economy (Proposition 2).

In this equilibrium regime, agents choose to hold X rather than the cash-synthetic asset even though the two have equivalent payoffs and the latter is cheaper. While this outcome may seem illogical, the result occurs in equilibrium because neither Y nor CDS_Y can be used as collateral: neither have collateral value. Thus, agents hold X precisely because it allows them to sell the CDS, and therefore isolate payoffs in states U and M . Any agent who chooses to hold the portfolio of Y and CDS_Y cannot isolate payoffs in any states but accepts equal payoffs in every state.²⁸ It is the ability of X to issue financial contracts that gives X a higher price. Combining these results, we obtain the following lemma, which describes equilibrium in this regime.

Lemma 7. *In this economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) X/CDS_Y , (2) Y , and (3) CDS_Y .*

Thus, there are two marginal buyers h_1 and h_2 . The most optimistic agents in the economy $h > h_1$ will sell their endowment of Y to buy X and issue the maximum allowable number of CDS_Y . Moderate agents $h \in (h_1, h_2)$ will sell their endowment of X to buy all the units of the risky asset Y . Pessimists $h < h_2$ will sell their endowment of X and Y to buy the financial contract CDS_Y sold by optimists.²⁹

A.2 Leverage Constraints and Negative Bases

Before investigating how leverage limits affect the basis, we document that for almost all parameters, the basis is zero with full leverage. (Our theoretical result is simply that the basis is non-zero.) Figure 11 plots the basis in leverage economies, with beliefs parametrized by the form $\gamma_U(h) = h^\zeta$ and $\gamma_M(h) = h^\zeta(1 - h^\zeta)$, when beliefs are given by $\zeta = 0.5$ and $\zeta = 1$. The parameter ζ determines the relative frequency of optimists and pessimists in the economy; equivalently, the frequency of

²⁸It is worth contrasting this result with traditional theories that ignore collateral. Traditional theory predicts that the CDS spread should be equal to the bond spread, due to the arbitrage opportunity that would arise otherwise. Even when agents cannot short-sell assets, the spreads should still be equal because agents can always choose buy the cheaper option—either the safe asset or a combination of the risky asset and its CDS.

²⁹This equilibrium could be replicated using two risk-averse agents. The more optimistic/risk-tolerant investor would hold a portfolio of X selling CDS and also some of the risky asset Y . The more pessimistic/risk-averse investor would hold a portfolio of CDS and the risky asset Y . Thus, this investor would be partially invested in a cash-synthetic asset.

pessimists can be interpreted as the relative demand for assets that pay in bad states (negative-beta assets), perhaps from hedging needs or risk aversion. High ζ corresponds to relatively more pessimists and low ζ to more optimists (with $\zeta > 1$, γ 's are convex; $\zeta < 1$, concave).

In general the basis is zero, but as noted earlier the basis can be positive. In these cases, the risky asset Y is not used to issue CDS but is exclusively used to issue risky debt. There is a small range with a positive basis around $M = 0.3, D = 0.08$. This region grows slightly as ζ decreases, but for ζ sufficiently high (for example, $\zeta = 1.5$) the basis is always zero for all payoffs.

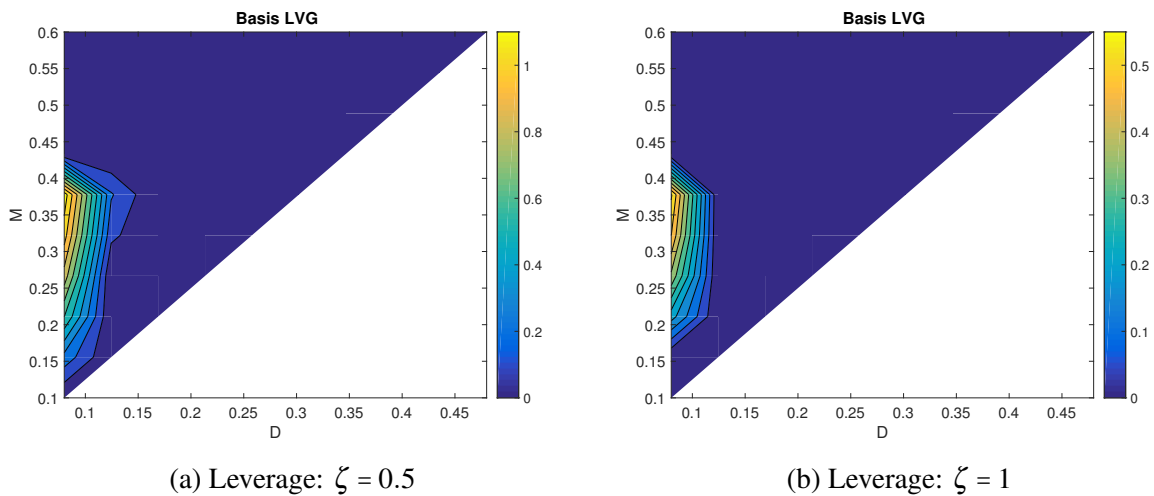


Figure 11: Comparative Statics: basis (times 100) varying payoffs M, D with leverage.

The zero-basis result emerges when Y and X have equal abilities to serve as collateral, albeit to make different promises. However, if Y is imperfect collateral, perhaps because of regulations or because financial markets have concerns arising from informational issues, then the basis will be negative. This follows because if the collateral value of Y decreases, then a negative basis emerges. Suppose that Y can be used to issue debt contracts, but the maximum promise is $\bar{\ell} < M$. That is, one unit of Y can at most back a non-contingent promise $(\bar{\ell}, \bar{\ell}, \bar{\ell})$. Furthermore, Y cannot be used to issue CDS.

When $\bar{\ell} \leq D$, the only debt contract traded is $j_{\bar{\ell}} = \bar{\ell}$ which delivers the promised amount in every state of the world. However, because this safe debt cannot be used to issue CDS, it trades at a discount to X (there is a basis on the safe debt), and so $\pi^{\bar{\ell}} < \bar{\ell}$. Equilibrium in this case is ordered as follows (starting with the most optimistic): agents holding X to issue CDS; agents holding Y and issuing safe debt (the leverage constraint); agents holding safe debt; agents holding CDS.

Furthermore, the basis is negative. While we have not been able to prove so, numerical examples suggest that the basis is monotonic in $\bar{\ell}$ for $\bar{\ell} < D$, with the basis more negative the tighter is the leverage constraint (lower $\bar{\ell}$).

When $D < \bar{\ell} < M$, two debt contracts are potentially traded: the safe contract $j_D = D$ and a risky contract $j_{\bar{\ell}} = \bar{\ell}$. The j_D contract delivers (D, D, D) while the $j_{\bar{\ell}}$ contract delivers $(\bar{\ell}, \bar{\ell}, D)$ because agents default in the down state. Depending on parameters, in equilibrium agents may trade the risky contract only. While we have not been able to prove so in this case, numerical results (below) suggest that in either case the leverage constraint decreases the basis.

Figure 12 plots the basis with beliefs parametrized by the form $\gamma_U(h) = h^\zeta$ and $\gamma_M(h) = h^\zeta(1 - h^\zeta)$, with $D = 0.1$ and $M = 0.3$, solving for the basis as a function of $\bar{\ell}$ and varying the parameter ζ .

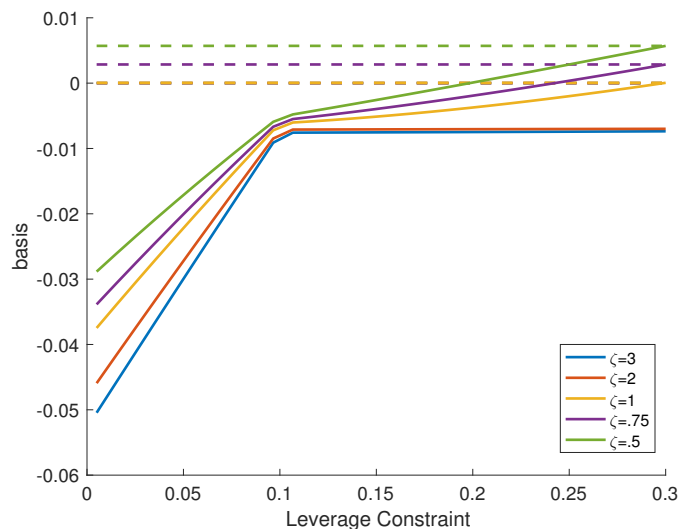


Figure 12: Leverage Constraints and the Basis. Dashed lines are the basis in an economy without leverage constraints and in which Y can be used to issue CDS .

The numerical examples provide two results in addition to our propositions. First, for low ζ (corresponding to high levels of optimism or high marginal utilities in good states), the basis with leverage limits and when Y cannot be used to issue CDS converges to the basis without leverage limits and when Y can be used to issue CDS . In particular, in these cases the restriction that Y cannot issue CDS is not binding when leverage limits are relaxed (note that the basis would actually be positive in this case). In these economies, when $\bar{\ell} > D$ agents trade only risky debt in equilibrium.

However, when ζ is high (corresponding to low levels of optimism or high marginal utilities in bad states), the basis with leverage limits does not converge to the basis when Y can be used to issue *CDS*. In these cases, in equilibrium agents use Y to issue safe debt, and the basis on the asset exactly equals the basis on the safe debt .

Second, when neither safe assets nor Y can back *CDS* contracts, the basis need not be monotonic in $\bar{\ell}$ when $D < \bar{\ell} < M$. In particular, when the economy features a relatively high demand for risk (ζ is low, marginal utilities are high for higher states), the basis is monotonic. However, when the economy features a substantially high demand for negative-beta assets (ζ is high, marginal utilities are high for low states), the basis can decrease as $\bar{\ell}$ increases from D to M . Varying the asset payoffs emphasizes these non-monotonicity results. Figure 13 plots the effects of leverage constraints on the basis, varying ζ , for two different sets of payoffs. When in equilibrium agents do not use Y to issue safe debt, the basis decreases significantly when $\bar{\ell}$ increases beyond D . In panel (a) to the left, for $\zeta = 2, 3$ agents use Y to exclusively issue risky debt. In this case, increasing the leverage limit actually decreases the basis. However, when agents use Y to issue safe debt, there is a basis on safe debt (because it cannot be used to issue *CDS* while X can), and the basis on the asset exactly equals the basis on the safe debt. Panel (b) to the right shows this for $\zeta = 0.75, 1, 2$, and for $\bar{\ell} > .3$ for $\zeta = 2.5$. For $\zeta = 2.5$ the equilibrium regime shifts as leverage constraints rise. For the loosest constraints, agents use Y to issue safe debt, but this is not the case for tighter constraints.

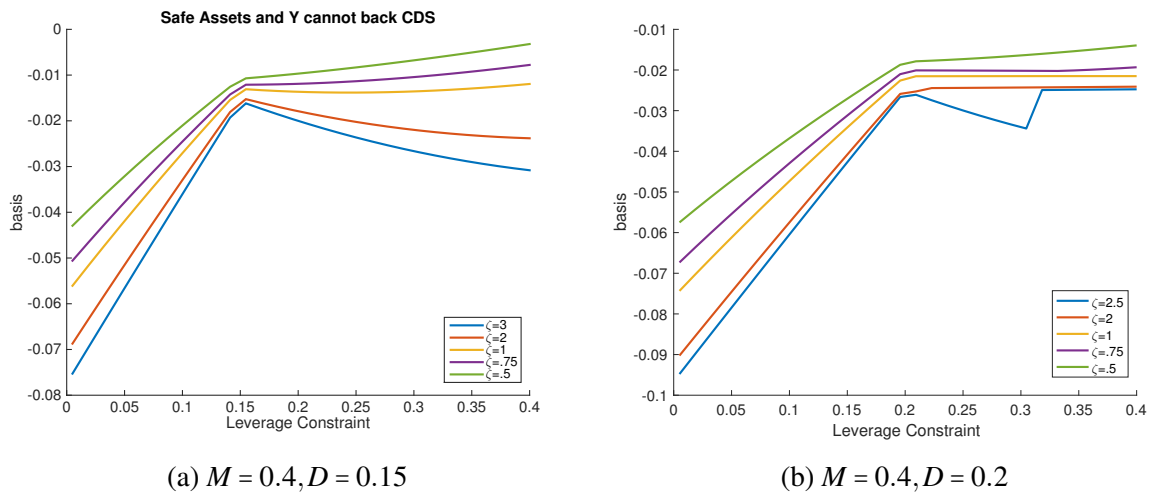


Figure 13: Leverage Constraints and the Basis.

B Equilibrium Conditions

B.1 No Leverage Economy: Section A.1

Marginal investors are indifferent between two different options. Thus, they can be defined by equalizing the expected returns (defined as the expected marginal utility divided by price) on the different investments. Agent h_1 is indifferent between selling the CDS_Y collateralized by X and buying the risky asset Y

$$\frac{\gamma_U(h_1)(1-D) + \gamma_M(h_1)(M-D)}{1-D-\pi_C^Y} = \frac{\gamma_U(h_1) + \gamma_M(h_1)M + \gamma_D(h)D}{p}. \quad (5)$$

Agent h_2 is indifferent between buying Y and buying the financial contract CDS_Y

$$\frac{\gamma_U(h_2) + \gamma_M(h_2)M + \gamma_D(h)D}{p} = \frac{\gamma_M(h_2)(1-M) + \gamma_D(h_2)(1-D)}{\pi_C^Y}. \quad (6)$$

Market clearing for X requires

$$\frac{(1-h_1)(1+p)}{1-\frac{\pi_C^Y}{1-D}} = 1, \quad (7)$$

and market clearing for Y requires

$$\frac{(h_1-h_2)(1+p)}{p} = 1. \quad (8)$$

Equation (7) states that agents buying X , $h \in (h_1, 1)$ will spend all of their endowment, $(1+p)$ to purchase X , which has price 1. With each unit of X they buy, they will also sell $\frac{1}{1-D}$ units of CDS_Y , which has price π_C^Y . The revenue from these sales is used to buy more X . The demand for X is equal to the supply, which is 1. Equation (8) states that agents buying the risky asset Y , $h \in (h_2, h_1)$ will spend all of their endowment on Y , which has price p , and that the amount demanded by these agents must be equal to the unit supply in the economy.

For the beliefs $\gamma_U = 1 - h(1-h)^2 - (1-h)^3$, $\gamma_M(h) = h(1-h)^2$ and $\gamma_D(h) = (1-h)^3$, and $d_M^Y = M = 0.3$ and $d_D^Y = D = 0.1$, the equilibrium is $p = 0.554$, $\pi_C^Y = 0.407$, $\text{Basis}_Y = -.039$.

B.2 Leverage Economy: Section 3.1

With leverage, equilibrium consists of three marginal investors, h_1 , h_2 , and h_3 and the following equations defining the marginal investors. Agent h_1 is indifferent between holding the risky asset with leverage promising M and buying the risky asset with leverage promising D ,

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D) + \gamma_M(h_1)(M-D)}{p-D}. \quad (9)$$

Agent h_2 is indifferent between buying the safe asset to sell CDS_Y and holding the risky debt promising M

$$\frac{\gamma_U(h_2)(1-D) + \gamma_M(h_2)(M-D)}{1-D-\pi_C^Y} = \frac{(\gamma_U(h_2) + \gamma_M(h_2))(M) + \gamma_D(h_2)D}{\pi^M}. \quad (10)$$

Agent h_3 is indifferent between holding the risky debt j_M and buying the CDS_Y contract.

$$\frac{(\gamma_U(h_3) + \gamma_M(h_3))M + \gamma_D(h_3)D}{\pi^M} = \frac{\gamma_M(h_3)(1-M) + \gamma_D(h_3)(1-D)}{\pi_C^Y}. \quad (11)$$

Denote by η the fraction of Y used to back CDS_Y . Market clearing for the risky asset Y requires

$$\frac{(1-h_1)(1+p)}{p-\pi^M} = 1-\eta. \quad (12)$$

Market clearing for risky debt j_M requires

$$\frac{(h_2-h_3)(1+p)}{\pi^M} = \frac{(1-h_1)(1+p)}{(p-\pi^M)}. \quad (13)$$

The market clearing condition for CDS_Y is

$$\frac{h_3(1+p)}{\pi_C^Y} = (1+\eta D) \left(\frac{1}{1-D} \right). \quad (14)$$

Equation (12) states that the amount of risky asset Y demanded by agents $h \in (h_1, 1)$ is equal to the amount of risky assets not backing CDS_Y . Equation (13) states that agents $h \in (h_3, h_2)$ will sell their endowment which has value $1+p$ and buy the risky debt, costing π^M for each unit; this demand

must equal the amount supplied, which is created by the agents $h \in (h_1, 1)$ who sell one unit of j_M for every unit of Y they hold. Finally, Equation (14) states that agents $h \in (0, h_3)$ will sell their endowment to buy CDS_Y , which has price π_C^Y and that this demand is equal to the amount supplied in the economy—a total of $\frac{1}{(1-D)}$ units of CDS_Y are created from the one unit of X and $\frac{D}{1-D}$ units are created from the equilibrium amount η backed by Y .

B.3 Debt Collateralization Economy: Section 3.2

The following equations define marginal investors (given by equalizing expected returns on two investment options) in the debt collateralization economy. Agent h_1 is indifferent between buying Y with leverage promising M and holding X while selling CDS_Y

$$\frac{\gamma_U(h_1)(1-M)}{p - \pi^M} = \frac{\gamma_U(h_1)(1-D) + \gamma_M(h_1)(M-D)}{1-D - \pi_C^Y}. \quad (15)$$

Agent h_2 is indifferent between buying X to sell CDS_Y and buying j_M with leverage D

$$\frac{\gamma_U(h_2)(1-D) + \gamma_M(h_2)(M-D)}{1-D - \pi_C^Y} = \frac{(\gamma_U(h_2) + \gamma_M(h_2))(M-D)}{\pi^M - D}. \quad (16)$$

Agent h_3 is indifferent between buying the risky debt with leverage promising D and buying the CDS

$$\frac{(\gamma_U(h_3) + \gamma_M(h_3))(M-D)}{\pi^M - D} = \frac{\gamma_M(h_3)(1-M) + \gamma_D(h_3)(1-D)}{\pi_C^Y}. \quad (17)$$

Market clearing for the safe asset X requires

$$\frac{(h_1 - h_2)(1+p)}{1 - \frac{\pi_C^Y}{1-D}} = 1. \quad (18)$$

Market clearing for the risky debt j_M implies that

$$\frac{(h_2 - h_3)(1+p)}{\pi^M - D} = 1. \quad (19)$$

Finally, market clearing for CDS_Y requires

$$\frac{h_3(1+p)}{\pi_C^Y} = \frac{1}{1-D}. \quad (20)$$

B.4 Equilibrium Conditions with CDS_M and Leverage: Section 4.1

Marginal investors are given by equalizing expected return on two investment options. There are five marginal investors in equilibrium and they are as follows: agent h_1 is indifferent between buying Y while making the j_M promise and buying Y while making the j_D promise

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D) + \gamma_M(h_1)(M-D)}{1-D-\pi_C^Y}. \quad (21)$$

Agent h_2 is indifferent between buying X leveraged with the CDS_Y contract and buying X leveraged with the CDS_M contract

$$\frac{\gamma_U(h_2)(1-D) + \gamma_M(h_2)(M-D)}{1-D-\pi_C^Y} = \frac{(\gamma_U(h_2) + \gamma_M(h_2))(M-D)}{M-D-\pi_C^M}. \quad (22)$$

Agent h_3 is indifferent between holding X to sell the CDS_M contract and buying the risky debt j_M

$$\frac{(\gamma_U(h_3) + \gamma_M(h_3))(M-D)}{M-D-\pi_C^M} = \frac{(\gamma_U(h_3) + \gamma_M(h_3))M + \gamma_D(h_3)D}{\pi^M}. \quad (23)$$

Agent h_4 is indifferent between buying j_M debt contract and buying the CDS_Y contract

$$\frac{(\gamma_U(h_4) + \gamma_M(h_4))M + \gamma_D(h_4)D}{\pi^M} = \frac{\gamma_M(h_4)(1-M) + \gamma_D(h_4)(1-D)}{\pi_C^Y}. \quad (24)$$

Agent h_5 is indifferent between buying the CDS on the risky asset and the CDS on the risky debt.

$$\frac{\gamma_M(h_5)(1-M) + \gamma_D(h_5)(1-D)}{\pi_C^Y} = \frac{\gamma_D(h_5)(M-D)}{\pi_C^M}. \quad (25)$$

We obtain market clearing conditions by equating the supply and demand for a given asset. For any asset, agents demanding the asset will spend their endowment $(1+p)$ to buy the asset, at

some price either with or without leverage. Market clearing for the safe asset X requires

$$\frac{(h_1 - h_2)(1 + p)}{1 - \frac{\pi_C^Y}{1 - D}} - \left(1 - \frac{(1 - h_1)(1 + p)}{p - \pi^M}\right) + \frac{(h_2 - h_3)(1 + p)(M - D)}{M - D - \pi_C^M} = 1. \quad (26)$$

Market clearing for the risky debt implies

$$\frac{(h_3 - h_4)(1 + p)}{\pi^M} = \frac{(1 - h_1)(1 + p)}{p - \pi^M}. \quad (27)$$

Market clearing for CDS_Y guarantees

$$\frac{(h_4 - h_5)(1 + p)}{\pi_C^Y} = \frac{(h_1 - h_2)(1 + p)}{1 - D - \pi_C^Y} - \left(1 - \frac{(1 - h_1)(1 + p)}{p - \pi^M}\right). \quad (28)$$

Finally, market clearing for CDS_M necessitates

$$\frac{h_5(1 + p)}{\pi_C^M} = \frac{(h_2 - h_3)(1 + p)}{(M - D - \pi_C^M)}. \quad (29)$$

B.5 Equilibrium Conditions with CDS_M and Debt Collateralization: Section 4.2

Marginal investors

- h_1 : indifferent between Y/j_M and X/CDS_Y

$$\frac{\gamma_U(h_1)(1 - M)}{p - \pi^M} = \frac{\gamma_U(h_1)(1 - D) + \gamma_M(h_1)(M - D)}{1 - D - \pi_C^Y}$$

- h_2 : indifferent between X/CDS_Y and X/CDS_M

$$\frac{\gamma_U(h_2)(1 - D) + \gamma_M(h_2)(M - D)}{1 - D - \pi_C^Y} = \frac{(\gamma_U(h_2) + \gamma_M(h_2))(M - D)}{M - D - \pi_C^M}$$

- h_3 : indifferent between X/CDS_M and CDS_Y

$$\frac{(\gamma_U(h_3) + \gamma_M(h_3))(M - D)}{M - D - \pi_C^M} = \frac{\gamma_M(h_3)(1 - M) + \gamma_D(h_3)(1 - D)}{\pi_C^Y}$$

- h_4 : indifferent between CDS_Y and CDS_M

$$\frac{\gamma_M(h_4)(1-M) + \gamma_D(h_4)(1-D)}{\pi_C^Y} = \frac{\gamma_D(h_4)(M-D)}{\pi_C^M}$$

Market Clearing

- Market for Y

$$\frac{(1-h_1)(1+p)}{p-\pi^M} = 1$$

- Market for CDS_Y

$$\frac{(h_3-h_4)(1+p)}{\pi_C^Y} = \frac{(h_1-h_2)(1+p)}{1-D-\pi_C^Y}$$

- Market for CDS_M

$$\frac{h_4(1+p)}{\pi_C^M} = \left(1 + D - \frac{(h_1-h_2)(1+p)(1-D)}{1-D-\pi_C^Y} \right) \left(\frac{1}{M-D} \right)$$

- Market for X and j_M

$$\frac{(h_1-h_2)(1+p)(1-D)}{1-D-\pi_C^Y} + \frac{(h_2-h_3)(1+p)(M-D)}{M-D-\pi_C^M} = 1 + M$$

B.6 Equilibrium when Y and j_D cannot serve as collateral for CDS

Let the set of financial contracts in the economy be given by $J = J^X \cup J^Y$, where J^X consists of CDS_Y backed by X and J^Y consists of non-contingent debt contracts. Note that we no longer allow j_D to back CDS_Y . By Proposition 6, it must be the case that $\pi^D < D$ or no one will want to buy the safe debt. We define the basis on j_D , denoted Basis_D , to be $D - \pi^D = \text{Basis}_D$. Equilibrium features four marginal buyers, $h_1 > h_2 > h_3 > h_4$. All agent $h > h_1$ will hold Y/j_M . Agents $h \in (h_2, h_1)$ will hold a combination of X/CDS_Y and Y/j_D (or just X/CDS_Y if it is cheaper). $h \in (h_3, h_2)$ will sell their endowments to buy j_M and $h \in (h_4, h_3)$ will buy j_D instead. Finally, $h < h_4$ will hold only CDS_Y . Furthermore, we see a double basis in this case—one on the risky asset and one on the *safe debt*. Additionally, when j_D is traded, the basis for Y must be the same as the basis on j_D because

$$p - \pi^D = 1 - D - \pi_C^Y \implies 1 - p - \pi_C^Y = D - \pi^D \implies \text{Basis}_Y = \text{Basis}_D.$$

Note that j_D is not always traded in this equilibrium. Specifically, for low enough values of M , no agent strictly prefers to buy the safe debt. The intuition here is that a lower M raises increases the payout of CDS_Y in the M state, making the CDS a more attractive option for moderate agents who wish to isolate payoffs in state M .

B.7 Four-State Example

The setup is as before, but now the set of states is given by $S = (0, S_1, S_2, S_3, S_4)$, where $s = 0$ is the initial state of the world at time $t = 0$. Let the payout of the risky asset Y be $(1, s_2, s_3, s_4)$ in states (S_1, S_2, S_3, S_4) , where $1 > s_2 > s_3 > s_4$. Let j_i be the debt contract promising s_i , and let the price of j_i be π^i . We set $s_2 = 0.5$, $s_3 = 0.3$, $s_4 = 0.1$, and we let beliefs be given by $\gamma_4(h) = (1-h)^3$, $\gamma_3(h) = h(1-h)^2$, $\gamma_2(h) = h^2(1-h)$, $\gamma_1(h) = 1 - \gamma_4(h) - \gamma_3(h) - \gamma_2(h)$, which preserves the properties in the three-state model.

Let there be full debt collateralization in the economy, and let there be a CDS on Y (with price π_C^Y) and a CDS on j_2 (with price π_C^2). We let Basis_α denote the basis on the asset α . In equilibrium, $p = 0.585$, $\pi^2 = 0.339$, $\pi^3 = 0.228$, $\pi_C^Y = .431$, $\pi_C^2 = 0.169$, $\text{Basis}_Y = 0.016$, $\text{Basis}_{j_2} = .009$, and we see a positive basis on both the risky asset and the risky debt.

C Proofs

Proof of Proposition 1. From Lemma 6, the position X/CDS_Y must be traded in equilibrium, otherwise no agent will hold X . Thus X/CDS_Y cannot be more expensive than Y/CDS_Y . Hence, it must be that $1 - D - \pi_C^Y \leq (1 - D)p - D\pi_C^Y$, which simplifies to $1 \leq \pi_C^Y + p$.

In order for any agent to hold j_D , which offers the same payoff as X but which cannot be used as collateral, it must be that $\pi^D < D$ in equilibrium. But since $\pi_C^Y + p \geq 1$, then $p - \pi^D > (1 - D)p - D\pi_C^Y$, which means that agents would strictly prefer to use Y to issue CDS rather than to issue debt. \square

Proof of Lemma 1. First note that because the minimum payout of Y is D and the maximum payout of CDS_Y is $1 - D$, each unit of Y can back $\frac{D}{1-D}$ units of CDS_Y . The payoff of buying one unit of Y and selling $\frac{D}{1-D}$ units of CDS_Y (holding Y/CDS_Y) is $(1, \frac{M-D}{1-D}, 0)$ in states (U, M, D) . However, this

return is equivalent to holding Y and selling j_D , so the choice-set of agents has not been increased by this financial innovation.

Now consider when j_D could also be used to back CDS_Y . Without letting agents use Y to issue CDS_Y , agents holding Y were still able to do this indirectly by selling the promising j_D . One unit of j_D can back $\frac{D}{1-D}$ units of CDS_Y , which is the exactly the amount issued when agents holding Y issue CDS_Y directly. In short, the leverage equilibrium, as we have characterized, does not depend on which assets can back CDS_Y . \square

Proof of Proposition 2. Consider the agent h who is indifferent between holding X/CDS_Y and holding Y . For h , $\mathbb{E}_h[X/CDS_Y] = \mathbb{E}_h[Y]$, thus

$$\frac{\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D}{p} = \frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1-D-\pi_C^Y}. \quad (30)$$

Furthermore, this agent is relatively optimistic and strictly prefers both of these two options to holding the safe asset, X . It follows that

$$\frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1-D-\pi} > 1. \quad (31)$$

Rearranging and simplifying Equation 30, we have that

$$p + \pi_C^Y = (1-D) + \frac{D(1-D-\pi)}{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}.$$

Combining this with 31, $p + \pi_C^Y < (1-D) + D = 1 \implies p + \pi_C^Y < 1$. \square

Proof of Proposition 3. First, suppose $\bar{\ell} = D$. Then in equilibrium investors must be indifferent between X/CDS and Y/j_D (the alternative is investors will hold Y without leverage, in which case the basis is negative per earlier results). Since the payoffs to these positions are the same, the costs of these positions are the same, $1-D-\pi_C^Y = p - \pi_D$, implying the basis is $\pi_D - D$, which is negative since the safe debt cannot be used as collateral while X can. Note that if $D < \bar{\ell} < M$ and safe debt is issued in equilibrium, then the same argument implies the basis is negative.

If $\bar{\ell} < D$ then investors are ordered $X/CDS, Y/j_{\bar{\ell}}, j_{\bar{\ell}}, CDS$. The position $Y/j_{\bar{\ell}}$ pays $(1-D+D-\bar{\ell}, M-D+D-\bar{\ell}, D-\bar{\ell})$. This position can be replicated using X/CDS and buying $\frac{D-\bar{\ell}}{\bar{\ell}}$ units of $j_{\bar{\ell}}$.

Note that the investor indifferent between $Y/j_{\bar{\ell}}$ and $j_{\bar{\ell}}$ is indifferent between buying and selling $j_{\bar{\ell}}$, but strictly prefers $Y/j_{\bar{\ell}}$ over X/CDS . Thus, the position must be cheaper:

$$1 - D + \pi_C^Y + \frac{D - \bar{\ell}}{\bar{\ell}} \pi^{\bar{\ell}} > p - \pi^{\bar{\ell}}.$$

Since $\bar{\ell} < D$ and $\pi^{\bar{\ell}} < \bar{\ell}$, $D \left(\frac{\pi^{\bar{\ell}}}{\bar{\ell}} - 1 \right) < 0$, and the basis satisfies

$$p + \pi_C^Y < 1 - D + D \left(\frac{\pi^{\bar{\ell}}}{\bar{\ell}} - 1 \right) < 1.$$

□

Proof of Proposition 4. First, suppose for contradiction that $1 - D - \pi_C^Y > p - D$. Then, all agents will strictly prefer to hold Y/j_D instead of X/CDS_Y . Furthermore, lemma 6 implies that no one would be willing to hold the safe asset by itself. It follows that no agent will want to hold X , which clearly cannot be an equilibrium.

Now suppose that $1 - D - \pi_C^Y = p - D$. Thus, all agents are indifferent between holding Y/j_D and X/CDS_Y since they are equivalent. However, Gong and Phelan (2016) shows that in an economy with debt collateralization, no agent strictly prefers to hold Y/j_D , which implies then that no agent strictly prefers to hold X/CDS_Y . Again, there must be a Lebesgue-measurable set of agents holding X , so this too cannot be an equilibrium. □

Proof of Corollary 1. From previous theorem, we have that $p + \pi_C^Y > 1 \implies p > 1 - \pi_C^Y \implies p - D > (1 - D) - \pi_C^Y$. Note that $1 - D - \pi_C^Y$ is the cost of holding X/CDS_Y while $p - D$ is the cost of holding Y/j_D . Thus, all agents will choose the cheaper option and hold X while selling the CDS_Y contract. □

Proof of Lemma 2. There are two parts to this proof. First we will show that no one holds Y/CDS_Y . Second, we will show that no agent strictly prefers to hold j_M/CDS_Y . Note that by the previous Lemma 4, we know that $1 - D - \pi_C^Y < p - D$ and hence $p + \pi_C^Y > 1$. Then,

$$\implies (p + \pi_C^Y)(1 - D) > 1 - D \implies p - \frac{D}{1 - D} \pi_C^Y > 1 - \frac{1}{D} \pi_C^Y.$$

So, the cost of holding Y/CDS_Y is higher than the cost of holding X/CDS_Y even though these two positions have equivalent returns. Thus, no agent will choose to hold Y/CDS_Y .

Now, suppose for contradiction that there is an agent, h who strictly prefers to hold j_M/CDS_Y . This means that for investor h , the expected return of j_M/CDS_Y must be greater than the return of X/CDS_Y . Thus,

$$\frac{\gamma_U(h)M(1-D) + \gamma_M(h)(M-D)}{(1-D)\pi^M - D\pi_C^Y} > \frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1-D - \pi_C^Y}. \quad (32)$$

Rearranging this equation and simplifying, we obtain

$$\gamma_U(h)M(1-D) + \gamma_M(h)(M-D) > \quad (33)$$

$$\pi^M(\gamma_U(h)(1-D) + \gamma_M(h)(M-D)) + \pi_C^Y(M-D)(\gamma_U(h) + \gamma_M(h)). \quad (34)$$

Since h strictly prefers j_M/CDS_Y , the expected payout of this position must also be higher than the expected payout of holding j_M/j_D^1 . So,

$$\frac{\gamma_U(h)M(1-D) + \gamma_M(h)(M-D)}{(1-D)\pi^M - D\pi_C^Y} > \frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1-D - \pi_C^Y}. \quad (35)$$

Rearranging and simplifying the above, we obtain

$$-(\gamma_U(h)M(1-D) + \gamma_M(h)(M-D)) > \quad (36)$$

$$-\pi^M(\gamma_U(h)(1-D) + \gamma_M(h)(M-D)) - \pi_C^Y(M-D)(\gamma_U(h) + \gamma_M(h)). \quad (37)$$

Combining Equations (34) and (37) yields $0 > 0$, a contradiction. So, there does not exist a set of agents with positive measure who strictly prefer to sell CDS_Y backed by j_M . \square

Proof of Proposition 5. Consider the agent who is indifferent between holding X/CDS_M and j_M . Since this agent is relatively optimistic, the expected return of both of these two options must be greater than 1. Then, we have that $\mathbb{E}_h[X/CDS_M] = \mathbb{E}_h[j_M] > 1$.

$$\frac{(\gamma_U(h) + \gamma_M(h))(M-D)}{M-D - \pi_C^M} = \frac{(\gamma_U(h) + \gamma_M(h))M + \gamma_D(h)D}{\pi^M} > 1. \quad (38)$$

Rearranging and simplifying 38, we obtain

$$\begin{aligned} (\pi^M + \pi_C^M)(\gamma_U(h) + \gamma_M(h))(M-D) &= D(M-D - \pi_C^M) + (M-D)^2(\gamma_U(h) + \gamma_M(h)) \\ \implies \pi^M + \pi_C^M &= \frac{D(M-D - \pi_C^M)}{(\gamma_U(h) + \gamma_M(h))(M-D)} + M-D. \end{aligned} \quad (39)$$

Combining the above with Equation 38, it follows that $\pi^M + \pi_C^M < M$. \square

Proof of Proposition 6. Because X/CDS_M is equivalent to j_M/j_D^1 , any equilibrium in this economy must feature a zero basis on j_M ($\text{Basis}_M = 0$). A positive basis, $\text{Basis}_M > 0$ would imply that j_M/j_D^1 is expensive relative to X/CDS_M and no agent would want to buy j_M . This is not an equilibrium because optimists who want to isolate payoffs in state U would be willing to sell j_M at a lower price, driving the basis toward 0. A negative basis, $\text{Basis}_M < 0$ is not an equilibrium because this implies j_M/j_D^1 is cheap relative to X/CDS_M and CDS_M is never issued as a result. However, extreme pessimists who want to isolate payoffs in state D and would therefore be willing to buy CDS_M even at a higher price, driving the basis toward 0. \square

Proof of Lemma 6. Suppose some agent strictly prefers to hold only the safe asset X without selling any financial contracts. Let $\mathbb{E}_h[a]$ denote the expected return on holding the position a . Then there exists some agent h such that $\mathbb{E}_h[X] > \mathbb{E}_h[X/CDS]$. This implies that:

$$1 > \frac{\gamma_U(h) + \gamma_M(h) \left(\frac{M-D}{1-D} \right)}{1 - \frac{\pi_C^Y}{1-D}} \implies (1-D) - \pi_C^Y > \gamma_U(h)(1-D) + \gamma_M(h)(M-D). \quad (40)$$

Additionally, since h strictly prefers to hold X , it must be the case that $\mathbb{E}_h[X] > \mathbb{E}_h[CDS_Y]$, implying

$$1 > \frac{\gamma_M(h)(1-M) + \gamma_D(h)(1-D)}{\pi_C^Y} \implies \pi_C^Y > \gamma_M(h)(1-M) + \gamma_D(h)(1-D). \quad (41)$$

Note that adding together equations 40 and 41 implies the following contradiction:

$$(1-D) > (1-D)(\gamma_U(h) + \gamma_M(h) + \gamma_D(h)) \implies (1-D) > (1-D).$$

Thus, no agent ever prefers to hold X . By risk neutrality, it also follows that any agent who

chooses to sell CDS will sell as many units of CDS as they can.

To see that no agent is willing to hold the cash-synthetic asset, suppose for contradiction that some agent h , strictly prefers the cash-synthetic asset. That is, $\mathbb{E}_h[Y + CDS_Y] > \mathbb{E}_h[Y]$. Then,

$$\frac{1}{p + \pi_C^Y} > \frac{\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D}{p} \implies p > (p + \pi_C^Y)(\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D). \quad (42)$$

Additionally, we must also have, $\mathbb{E}_h[Y + CDS_Y] > \mathbb{E}_h[CDS_Y]$, which means

$$\frac{1}{p + \pi_C^Y} > \frac{\gamma_M(h)(1 - M) + \gamma_D(h)(1 - D)}{\pi_C^Y} \implies p > (p + \pi_C^Y)(\gamma_M(h)(1 - M) + \gamma_D(h)(1 - D)). \quad (43)$$

Combining the above two inequalities yields the following contradiction:

$$p + \pi_C^Y > (p + \pi_C^Y)(\gamma_U(h) + \gamma_M(h) + \gamma_D(h)) \implies p + \pi_C^Y > p + \pi_C^Y.$$

□