Collateral Constraints, Tranching, and Price Bases

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Abstract

Tranching an asset increases its basis; tranching a CDS, as occurs with the CDX index, increases the basis on the underlying asset. We consider a general equilibrium model with collateralized financial promises and multiple states of uncertainty to study how allowing an asset to back multiple financial contracts (i.e., tranching) affects price bases. A positive basis emerges when risky assets and their derivative contracts can be used as collateral for financial promises. We provide an empirical test of our theory using inclusion in the CDX and find that inclusion in the CDX increases the CDS basis.

Keywords: Collateral, securitized markets, cash-synthetic basis, credit default swaps, asset prices, credit spreads.

JEL classification: D52, D53, G11, G12.

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1 Introduction

In the years prior to the 2007 recession, the shadow-banking system produced a legion of structured credit products including asset-backed securities ("ABS"), collateralized debt obligations ("CDOs"), and CDO-squareds. These structured credit products tranched collateral into multiple securities, and reused these tranches as an additional layer of collateral to issue more securities. Such financial innovations, along with the practice of rehypothecation, greatly increased the ability of assets to serve as collateral.[1] This increased capacity sharply reversed during the financial crisis (see Gorton and Metrick, 2012), but recovery led to the resumed expansion of funding markets and the issuance of CDOs. Figure 1 plots CDO issuance from 2001 to 2014.

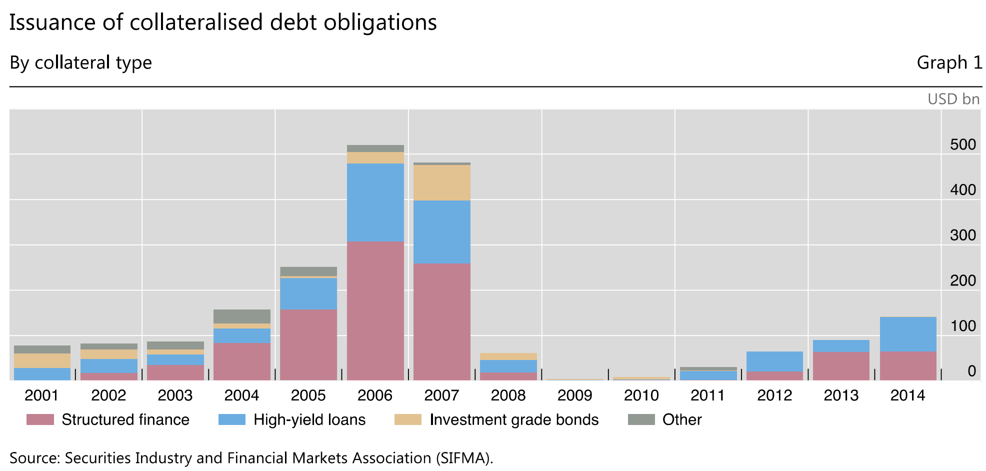

Figure 1: CDO Issuance. Source: Antoniades and Tarashev (2014).

Our paper considers how innovations in the use of collateral such as these can affect investment decisions and the prices of portfolios with identical cashflows—i.e., price *bases*. As one example, consider the CDS basis, which is the difference between the spread on a bond and the premium on a credit default swap (CDS) protecting that bond. The typical convention is CDS basis equals CDS spread minus bond spread. CDS bases before the crisis, especially on high yield ("HY") bonds, were significantly positive with an average HY basis of about 80 basis points. This means that a (nearly risk-free) portfolio of a HY bond with a CDS would be *more expensive* than purchasing a similar maturity US Treasury. During the crisis the basis became negative, and the post-crisis

[1] See Gorton and Metrick (2009); Fostel and Geanakoplos (2012a)

financial recovery has led to a normalization of the CDS basis around 0.

We provide a theoretical model that shows that the basis on a risky asset is positive when it can be tranched or when derivative contracts backed by the risky asset can be used as collateral to issue further promises (pyramiding). Thus, all else equal, tranching and pyramiding increases bases. We consider a general equilibrium model with heterogeneous agents and collateralized borrowing following Geanakoplos and Zame (2014). Issuing financial contracts requires using assets as collateral, but cross-netting frictions may limit the number of contracts that a single asset can back. We suppose that some assets can be used to back multiple contracts simultaneously, and this could occur in at least two ways. First, the asset could be use as collateral directly to issue several securities at once (tranching). Second, a financial promise backed by the asset could be used as collateral to back further financial promises (pyramiding). In either case, the original asset is explicitly or implicitly used to back multiple distinct financial contracts. We show that, when considering portfolios with identical payoffs, tranching or pyramiding an asset leads to a positive basis on the underlying asset, which is consistent with the stylized empirical facts.

Our theory has implications for how inclusion of bonds in CDX indices affects CDS bases. Importantly, the CDX index can be tranched into indices with multiple "attachment points," thus increasing the ability of the underlying assets to serve as collateral. Our theory yields two predictions. First, tranching the CDX should create a positive CDS-CDX basis, which is consistent with the data. Second, inclusion in the CDX should increase the CDS basis on underlying bonds, since the CDX index tranches CDS contracts. We provide an empirical test of this prediction using difference-in-differences for contracts included/excluded from the CDX index and show that inclusion increases the CDS-bond basis, consistent with our theory.

Related literature

Our model introduces tranching and pyramiding into a model of collateral equilibrium based on Geanakoplos (1997, 2003) and Geanakoplos and Zame (2014). Geanakoplos and Zame (2013) discuss how using promises to back further promises (what they call pyramiding) can potentially allow the market to achieve efficient allocations, though the central finding is that even with pyramiding, equilibrium is robustly inefficient. Gottardi and Kubler (2015) implicitly assume that all financial securities serve as collateral. Provided the financial markets are sufficiently rich in terms of the specification of payoffs and of collateral requirements, any Arrow-Debreu equilibrium allocation with limited pledgeability can also be attained at a collateral-constrained financial market equilibrium and debt pyramiding can replicate tranching. In contrast to this rich environment, we focus on limited tranching and pyramiding across assets. We show that the possibility of pyramiding changes the set of contracts issued so that investors will never issue a promise that cannot later be used as collateral. As a result, pyramiding increases the basis on the underlying asset.

Our paper relates to the literature on collateral equilibria in models with multiple states (see Simsek, 2013; Toda, 2015; Gottardi and Kubler, 2015; Phelan, 2015; Gong and Phelan, 2019; Phelan and Toda, 2019). Araujo et al. (2012) examine the effects of default and collateral on risk sharing. Cao (2010, 2017) and Cao and Nie (2017) study how collateral constraints and incomplete markets affect asset price volatility and amplification (see also Brumm et al., 2015). Ellis et al. (2019) study tranching as security design motivated by diverse beliefs. Darst and Refayet (2018) study credit default swaps in equilibrium. Using a binomial model, Fostel and Geanakoplos (2012a) provide an example where the basis is negative (when the risky asset cannot be used as collateral or can be leveraged but cannot be tranched). In their model, tranching refers to using an asset to issue state-contingent contracts, and it is sufficient for an asset to back a single contract at a time. In contrast, our definition of tranching is using an asset to back multiple contracts simultaneously. Since we consider multiple states of uncertainty, both tranching and pyramiding are meaningful innovations in our setting, and we produce positive bases.

Our insight about the role of collateral to determine the basis is closely related to Shen et al. (2014), which proposes a collateral view of financial innovation driven by the cross-netting friction. Shen et al. (2014) show that derivatives allowing investors to "carve out" risks emerge to conserve collateral, and as a result the price of a risky asset is always less than the price of a portfolio replicating it with derivatives (negative basis). Their result follows because the risky asset requires "too much" collateral for agents to isolate the risks they want. In our model, we derive the same result when the risky asset cannot be used as collateral, but in contrast we show that the sign of the basis can flip (the risky asset can be expensive) when the risky asset and its derivative debt contracts can be used as collateral. We extend their original insight by considering when the risky asset can in fact "require less collateral" than alternatives. In our model, tranching and pyramiding are ways of stretching collateral, which is similar to their insight that financial innovation is a response to scarce collateral (see also Gottardi et al., 2019, regarding collateral re-use). Our theory rooted in collateral can explain positive bases by emphasizing financial innovations that stretch collateral.

Most theoretical papers explain why non-zero bases can persist once deviations occur. This literature relies on limits of arbitrage conditions in the market to explain the existence of non-zero basis: a "shock" occurs that causes CDS and bond premia to diverge, and the basis persists because arbitrageurs cannot fully arbitrage the difference. Of these limits to arbitrage conditions, the most commonly cited is the existence of limits in firms' funding capacity, which prevents firms from conducting enough trades to eliminate the basis. With this interpretation, differences in cross-sectional bases at different points in time point to variations in funding capacity across firms. Notably, the literature focuses on explaining when bond premia exceed CDS spread, as occurs during crises, but does not typically explain the reverse phenomena, which we do. Shleifer and Vishny (2011) show that fire-sale models can explain failures of arbitrage in markets featuring large differences in prices of very similar securities.

Gârleanu and Pedersen (2011) provide a model where margin constraints can lead to pricing differences between two identical financial securities. Negative shocks to fundamentals cause margin constraints bind and differences in margin requirements cause the basis to deviate from zero. Our analysis and results differ from Gârleanu and Pedersen (2011) in several ways. First, in Gârleanu and Pedersen (2011), a basis only occurs when negative shocks cause a funding-liquidity crisis and losses for leveraged agents, while in our model non-zero bases are due to the financial environment (assets used as collateral), not the presence of a funding-liquidity crisis. Second, we show that the basis between two assets depends not only on the margin requirements of the assets themselves but also on the margin requirements for derivative debt contracts collateralized by the assets. Relatedly, Oehmke and Zawadowski (2015) show that a negative basis emerges when transaction costs are higher for bonds than for CDS. In our paper, negative bases can persist when risky assets are imperfect collateral, and positive bases can persist *even when agents can short assets* because the efficient use of collateral is to buy CDS rather than to short assets.

2 General Equilibrium Model with Collateral

This section presents the basic general equilibrium model with a rich set of collateralized financial contracts subject to cross-netting frictions. Appendix A imposes additional restrictions on preferences, endowments, and available contracts in order to characterize equilibrium in more detail.

Time, Assets, and Households

We consider a two-period model with time $t = 0, 1$ and $N > 2$ states at $t = 1$. Uncertainty is represented by a tree with a node s_0 at $t = 0$ and states $n \in \mathcal{N} = \{1, \ldots, N\}$ at $t = 1$.

There is a single consumption good and Z fundamental assets, indexed by $z \in \mathcal{Z} = \{1, \ldots, Z\}$, which produce dividends of the consumption good at $t = 1$. For a generic asset $z \in \mathcal{Z}$, let d_n^z be the dividend of asset z in state n. Each asset trades for a price p_z at $t = 0$, and we denote the vector of asset prices at $t = 0$ by p.

There are H agents, denoted by superscript $h \in \mathcal{H} = \{1, \ldots, H\}$. Each agent is endowed with $\bar{\theta}^h \in \mathbb{R}_+^Z$ assets at $t = 0$ and $e^h \in \mathbb{R}_+^N$ consumption goods at $t = 1$. Agents consume at $t = 1$ and the preference ordering of agent h is represented by a utility function $u^h : \mathbb{R}^N \to \mathbb{R}$, defined over consumption $c^h = (c_1^h, \ldots, c_N^h) \in \mathbb{R}_+^N$. We denote asset holdings at $t = 0$ by $\theta^h = (\theta_1^h, \ldots, \theta_Z^h) \in \mathbb{R}_+^Z$, implying no short sales of assets.

The characteristics of agent h are summarized by a utility function and endowment vectors $(u^h, \bar{\theta}^h, e^h)$ satisfying standard assumptions. Throughout the analysis we suppose the set of agents $\mathcal{H}$ is a finite but very large set with a rich degree of heterogeneity coming from preferences or endowments.[2]

Financial Contracts and Collateral

The heart of our analysis involves contracts and collateral. Agents trade financial contracts at $t = 0$. A financial contract $j = (A^j, C^j)$, consists of a promised payment $A^j = (A_n^j)_{n \in \mathcal{N}}$ in terms of the consumption good at $t = 1$, and an asset C^j serving as collateral backing the promise. Since collateral is the only enforcement mechanism, the financial contract yields $\min\{A_n^j, d_n^{C^j}\}$ in state n.

[2]One condition that would generate sufficient heterogeneity would be a hazard rate dominance over marginal utilities across states with a sufficiently large set of agents. See Appendix A for an example.

Agents must own collateral in order to make promises.

Let $\mathcal{J} = \{1, \ldots, J\}$ be the set of all possible financial contracts. Throughout the analysis we suppose the set of contracts $\mathcal{J}$ is a finite (guaranteeing equilibrium existence) but very large set. In particular, we suppose that there are no restrictions on the set of contingent promises available so that agents can issue contracts j with any set of promised payoffs A^j. Because promises can be state-contingent, without loss of generality we can restrict attention to promises $A_n^j \leq d_n^{C^j}$ since promising more is redundant given default.

Each contract $j \in \mathcal{J}$ trades for a price π_j. We denote contract holdings of $j \in \mathcal{J}$ by φ_j, where $\varphi_j > 0$ denote *sales* and $\varphi_j < 0$ denote *purchases*. The sale of a contract corresponds to borrowing the sale price and the purchase of a promise is equivalent to lending the price in return for the promise. A position of $\varphi_j > 0$ units of a contract requires ownership of φ_j units of the collateral, whereas the purchase of such contracts does not require ownership of the collateral.

We suppose that assets are potentially subject to a cross-netting friction as follows. Asset z can back up to M_z contracts simultaneously, subject to payment enforceability, i.e., z can back M_z contracts $j_1, \cdots, j_{M_z}$ if $\sum_m^{M_z} A_n^{j_m} \leq d_n^z$ for all n. When $M_z > 1$ we say that asset z can be *tranched* into multiple securities.[3] Given a portfolio of financial promises φ, we define collateral requirement for asset z as follows. Let $\varphi(z)$ be the contract sales with $C^j = z$. Let $\chi_z(\varphi)$ to be the minimum holding of asset z required to satisfy the collateral requirement for $\varphi(z)$. In other words, $\chi_z(\varphi)$ is such that each unit of z backs at most M_z contracts and the contracts are able to deliver the promised payments backed by the collateral.

Discussion Even though promises can be state-contingent, an economy in which fundamental assets can back only one contract at a time cannot implement an Arrow-Debreu equilibrium with limited pledgeability (see Geanakoplos and Zame, 2014; Gottardi and Kubler, 2015). Generally, if a contract pays in $K \leq N$ states, then the issuer of the contract retains payments in at least $N - K$ states. Collateral constraints *require* that in equilibrium some agents must hold "bundles" of Arrow-Debreu securities, which is not required with complete markets—in other words, collateral

[3] Note that using an asset z to issue a single contract splits the payoffs to z into two sets of contingent claims: those cash flows defined by the contract, and the residual cash flows that accrue to the holder of z after making the contract payments. Thus, issuing a single contract is akin to tranching z into two sets of payoffs. In contrast, we are interested in when an asset can be tranched into at least 3 sets of cash flows.

constraints prevent the complete splitting of asset payoffs into Arrow-Debreu securities. Because of this, tranching is not redundant in equilibrium precisely because it increases the set of contingent payoffs that can be backed by an asset.

We take the financial environment as exogenous. Calza et al. (2007) show the importance of institutions for determining what assets are permitted as collateral and how they are treated. Fostel and Geanakoplos (2012a) argue that the natural progression is for an asset to back non-contingent claims, then contingent claims (tranching), and finally for CDS on the asset (backed by other collateral) to emerge last. For for informational explanations regarding why financial markets may change the available set of assets serving as collateral, see also Dang et al. (2011); Gennaioli et al. (2013); Gorton and Ordoñez (2014).

Budget Set

Without loss of generality, we normalize the price of consumption to be 1 in all states of the world. Given asset and contract prices at time 0, each agent chooses asset holdings and trades contracts j to maximize utility, subject to the budget set

$$B^h(p,\pi) = \Big\{ (\theta,\varphi,c) \in R_+^Z \times R^J \times R_+^N :$$

$$\sum_{z \in Z} \theta_z p_z \leq \sum_{j \in \mathcal{J}} \varphi_j \pi_j, \tag{1}$$

$$\chi_z(\varphi) \leq \theta_z, \forall z \in \mathcal{Z}, \tag{2}$$

$$c_n = \sum_{z \in Z} \theta_z d_n^z - \sum_{j \in \mathcal{J}} \varphi_j \min\{A_n^j, d_n^{C^j}\} \Big\}. \tag{3}$$

Equation (1) states that expenditures on assets (purchased or sold) cannot be greater than the resources borrowed by selling contracts. Equation (2) is the collateral constraint for contracts backed by fundamental assets, requiring that agents must hold sufficient assets to collateralize the contracts they sell. Equation (3) states that in the final states, consumption must equal dividends of the assets held minus debt repayment. Recall that a positive φ_j denotes that the agent is selling a contract or borrowing π_j, while a negative φ_j denotes that the agent is buying the contract or lending π_j. Short selling of fundamental assets is not possible ($\theta_z \geq 0$).

Collateral Equilibrium

Definition 1. *A* Collateral Equilibrium *in this economy is a set of asset prices, contract prices, asset purchases, contract trades, and consumption decisions all by agents,* $((p,\pi),((\theta^h,\varphi^h,c^h)_{h\in\mathcal{H}})) \in (R_+^Z \times R_+^J)\times(R_+^Z \times R^J \times R_+^N)^{\mathcal{H}}$*, such that*

1. $\sum_{\mathcal{H}} \theta^h = \sum_{\mathcal{H}} \bar{\theta}^h$,
2. $\sum_{\mathcal{H}} \varphi_j^h = 0,\ \forall j \in \mathcal{J}$,
3. $(\theta^h,\varphi^h,c^h) \in B^h(p,\pi), \forall h$,
4. $(\theta,\varphi,c) \in B^h(p,\pi) \Rightarrow U^h(c) \leq U^h(c^h), \forall h$.

Condition 1 is the asset market clearing conditions at time 0 and condition 2 is the market clearing condition for financial contracts. Condition 3 requires that all portfolio and consumption bundles satisfy agents' budget sets, and condition 4 requires that agents maximize their expected utility given their budget sets. By the same arguments made in Geanakoplos and Zame (2014), equilibrium in this model exists under the assumptions made thus far.

Our baseline model imposes minimal restrictions on preferences and available contracts. In Appendix A we characterize equilibrium in an economy where agents are restricted to trading non-contingent debt contracts and credit-default swaps. Even though agents cannot trade a full set of contingent contracts, the results on the basis are identical in this setting.

3 Theoretical Results

We now provide the theoretical results regarding investment choices and price bases. Throughout we consider results "generically" (e.g., for an open and full-measure set of individual endowments). We define the basis on an asset z as the difference between the price p_z of the asset and the closest price $p(\theta)$ of a replicating portfolio θ:

$$\text{Basis}_z = p_z - p(\theta). \tag{4}$$

Defining in this order preserves the standard notation based on bond spreads (which move inversely with bond prices) so that a positive basis indicates that the bond is "expensive."

3.1 Results with Tranching

First, in equilibrium, any asset that can be tranched into multiple contracts will be tranched.

Proposition 1 (Tranching)**.** *Suppose asset z can be used to issue $M_z > 1$ contracts simultaneously. Then, generically, no agent will buy z and issue fewer than M_z contracts. In equilibrium agents that hold z will issue M_z contracts.*

Proof. First, suppose $M_z = 2$ (the argument immediately generalizes by induction). Consider a contract j_1 with price π_1 such that $A^{j_1} < d_n^z$ for some n. Suppose z can also back a second contract j_2 with price π_2, so that $A_n^{j_1} + A_n^{j_2} \leq d_n^z$ for all n, with strict inequality for some n. In other words, an agent can hold z, issue both contracts, and receive positive payoffs in at least one state. Let $\mu^h = (\mu_1^h, \ldots, \mu_N^h)$ denote agent h's marginal utility in each state. If an investor prefers to hold z and issue j_1, then generically we have

$$\frac{\mu^h \cdot (d^z - A^{j_1})}{p_z - \pi_1} > \frac{\mu^h \cdot (d^z - A^{j_1} - A^{j_2})}{p_z - \pi_1 - \pi_2},$$
$$\frac{\mu^h \cdot (d^z - A^{j_1})}{p_z - \pi_1} > \frac{\mu^h \cdot (A^{j_2})}{\pi_2},$$

where the numerator of each term is the marginal utility of the investment and the denominator is the price of the investment (and the inequalities are strict generically). The first line says that holding z and issuing j_1 is preferred over issuing both contracts, and the second line says that holding z and issuing j_1 is preferred over holding j_2. Multiplying we have

$$\mu^h \cdot (d^z - A^{j_1})(p_z - \pi_1 - \pi_2) > \mu^h \cdot (d^z - A^{j_1} - A^{j_2})(p_z - \pi_1),$$
$$\mu^h \cdot (d^z - A^{j_1})\pi_2 > \mu^h \cdot (A^{j_2})(p_z - \pi_1),$$

and adding these two inequalities yields

$$\mu^h \cdot (d^z - A^{j_1})(p_z - \pi_1) > \mu^h \cdot (d^z - A^{j_1})(p_z - \pi_1),$$

which is a contradiction since all payoffs and marginal utilities are positive. Since some agent must hold z in equilibrium, any agent holding z will issue 2 contracts. By the same argument, any investor that holds z in equilibrium would prefer to issue M_z contracts over any fewer number.

□

Compared to holding an asset and issuing a single contract, any agent would either prefer to issue multiple contracts or to invest directly in one of those contracts. Since some agent must hold the asset in equilibrium, any agent holding the asset will completely tranche it. With this result, we can now state our first asset pricing result.

Lemma 1. *Consider two assets z_1 and z_2 with identical payoffs and prices p_1 and p_2, but suppose that z_1 cannot be tranched while z_2 can be tranched. Then $p_2 > p_1$.*

Proof. Consider a contract j_1 backed by z_1 that is traded in equilibrium at a price π_1. An investor can receive the payoffs $d^1 - A^{j_1}$ by using either asset as collateral to issue the payoffs A^{j_1}. But by Proposition 1 no investor holding z_2 would choose to issue j_1 alone. Since by assumption some investor is holding z_1 to yield the cash flows $d^1 - A^{j_1}$, but no investor would want to hold z_2 to hold those same cash flows, it must be that the cost of those cash flows is cheaper when holding z_1, i.e.,

$$p_1 - \pi_1 < p_2 - \pi_1.$$

□

This result is basic but also fundamental. As is well-known, the price of an asset depends on the value of its payoffs as well as its ability to serve as collateral (Fostel and Geanakoplos, 2008).[4] Lemma 1 reflects this basic insight with cross-netting. An asset that can be tranched is more valuable collateral than an asset that cannot be. Importantly, however, the collateral value of tranching extends beyond considering two identical *assets*. Portfolios of assets that generate the same payoffs—or even whose payoffs are *complements*—will generally have a basis if there are differences in how assets can be tranched.

The most common basis is the CDS basis, which compares the price of a risk-free asset to the price of a "cash-synthetic portfolio" consisting of a risky asset and a CDS on the asset. The CDS pays the difference between the maximum promised payoff and the realized dividend on the asset. Let the dividends for an asset Y be ordered $s_1 < \cdots < s_N$ with $s_N = 1$ the maximum dividend as a

[4] Fostel and Geanakoplos (2008) define the payoff value of an asset j to an agent i as $PV_j^i \equiv \sum_{s \in S} d_s^j \left(\frac{\mu_n^i}{\mu_0^i} \right)$, where μ_s^i is the marginal utility of agent i for state s.

normalization. A CDS on Y delivers $1 - s_n$ units in n. Let X be a risk-free asset delivering 1 for sure, i.e., $d_n^X = 1$ for all n, with price p_X. Issuing a CDS using risk-free asset X requires $1 - s_1$ units of X and grants the issuer $s_n - s_1$ units in n. If the asset Y can be tranched but X cannot be, then Y will have a positive basis.

Proposition 2 (CDS Basis)**.** *Consider two assets X and Y described above. In equilibrium, either (i) X is used to issue a CDS on Y, with price π_C, and $Basis_Y = \pi_C^Y - (p_X - p_Y) > 0$, or (ii) CDS are not traded.*

Proof. Suppose CDS are traded. Using X to issue the CDS yields $s_n - s_1$ and costs $(1 - s_1)p_X - \pi_C$. However, an investor can yield $s_n - s_1$ by holding Y and issuing a single risk-free contract that promises s_1 in each state, which costs $p_Y - s_1 p_X$. But since Y has capacity to be tranched further, any investor holding Y would strictly prefer to issue additional contracts, and so $p_Y - s_1 p_X > (1 - s_1)p_X - \pi_C$. Rearranging yields the result. If the basis is otherwise, then no agent would choose to use X to issue CDS and would choose instead to use Y to issue contracts. □

Proposition 2 says that tranching an asset increases its CDS basis. There are two ways to create a risk-free portfolio: hold X for a cost p_X, or hold Y and a CDS together for a cost of $p_Y + \pi_C$. When Y can be tranched, it is strictly cheaper to get a risk-free portfolio by holding X, which cannot be tranched. Indeed, in equilibrium an investor that chooses to hold Y would use it to issue tranches would therefore not choose to also hold a CDS.

The CDS basis is just one example of a basis comparing a cash and a synthetic asset. A swap basis compares the cost of an asset to its corresponding derivative. Suppose X can be used to issue a promise j_Y with payoffs $s_1, \ldots, s_N$, which are the payoffs to Y. Let π_Y be the price of the contract backed by X. Then the basis on Y would compare the price of Y to the price π_Y of the contract.

Corollary 1 (Swap Basis)**.** *In equilibrium, either (i) X is used to issue a derivative contract replicating Y, with price π_Y and $Basis_Y = p_Y - \pi_Y > 0$, or (ii) the derivative replicating Y is not traded.*

The result follows by the exact same logic. Any investor who wants the cash flows for Y can buy the contract for π_Y or can buy Y for p_Y. But any investor that buys Y will tranche it into multiple securities. Even if the contract j_Y can be tranched, the price of Y will be higher so long as

Y can back more tranches than the derivative j_Y can. This observation motivates the next section with pyramiding.

More interesting is when we consider that the CDS contracts themselves can be tranched into further promises. Suppose now that assets z_1 and z_2 have the same collateral capability (i.e., can be tranched the same), but suppose that the CDS on z_2 can be tranched while the CDS on z_1 cannot. In this context, tranching the CDS could mean breaking it into one security that pays in some states and another security that pays in others. The ability to tranche CDS on z_2 will increase the basis on z_2.

Corollary 2. *Suppose that z_2 and z_1 can be collateralized the same but suppose that the CDS on z_2 can be tranched. Then the basis on z_2 is greater than the basis on z_1. Furthermore, in equilibrium, the CDS on z_1 will not be traded.*

The intuition is similar. Since the assets z_1 and z_2 are identical in terms of payoffs and collateral they must have the same price in equilibrium. However, the CDS on z_2 is superior collateral to the CDS on z_1 and so its price must be higher. Thus, the basis on z_2 must exceed the basis on z_1. However, both CDS are issued by using X as collateral. An investor holding X and issuing CDS on z_1 or on z_2 would receive the same payoffs but would strictly prefer to issue the CDS on z_2 since selling the CDS on z_2 earns him more money. Thus, the CDS on z_1 would be priced but not traded in equilibrium.

The second part of this result is particularly important because it links to the literature on liquidity and CDS trading (e.g., Oehmke and Zawadowski, 2015). Our model provides a novel explanation for why differential degrees to which CDS are included in structured finance products would affect liquidity. In our model, CDS which are poor collateral are not even issued. While the result is very stark given the stylized nature of our model, the insight is clearly more general: the ability to tranche a contract or to use a contract as collateral, will affect the issuance and trade in that contract. Investors will naturally issue contracts which are superior collateral. This result is similar to Fostel and Geanakoplos (2016), who show that investment in risky assets increases when the asset can be used as collateral.

3.2 Results with Pyramiding

We now consider using contracts as collateral for further contracts, as occurs with the creation of CDOs and other structured products. In reality, both of these innovations (tranching and pyramiding) occur and often occur simultaneously. ABS are tranched capital structures in the underlying collateral (within our definition of structured finance), and CDOs are tranched capital structures in which the underlying collateral are ABS tranches. Similarly, index CDO tranches fit within our definitions, since underlying collateral (CDS) are tranched simultaneously into multiple indices corresponding to different loss levels.

3.2.1 Contracts and Pyramiding

We now suppose that assets can back at most one financial contract at a time, but we consider the possibility of contract collateralization (pyramiding), which allows financial contracts to be used as collateral to issue further promises. As we will discuss, there is an essential equivalence between tranching and pyramiding when promises can be any contingent promise. But when promises are restricted to a limited set of contingencies (for example debt), pyramiding can be used to create contingencies otherwise unavailable (see Gong and Phelan, 2019, for an analysis of the state-contingencies created via pyramiding).

We introduce multiple levels of pyramiding inductively. Level-0 contracts are promises using one unit of a fundamental asset as collateral, with the set of contracts denoted by $\mathcal{J}^0$. Note as before that an economy with level-0 contracts only cannot implement an Arrow-Debreu equilibrium because a fundamental asset can back only one contract at a time. Because of this, allowing level-0 contingent contracts to serve as collateral is not redundant in equilibrium precisely because it increases the collateral capacity of the underlying asset. Contract collateralization effectively allows a fundamental asset to serve as collateral for *multiple* contracts—the asset directly backs the level-0 contract, and indirectly backs level-1 contracts, etc.

Level-0 debt contracts in $\mathcal{J}^0$ can be used as collateral to issue further contingent promises.

Definition 2. *We say the* first level of collateralization *is the creation of promises j^1 using $k^0 \in \mathcal{J}^0$ as collateral. Denote the set of contracts at the first level of contract collateralization by $\mathcal{J}^1$. We write $j^1(k^0) = (A^{j^1}, k^0)$ to denote the contract that is traded when an agent holds k^0 as collateral* *and promises to pay A^{j^1}. We denote the act of holding k^0 and selling j^1 by k^0/j^1.*

For a contract k^0 to be meaningful collateral for a promise A^{j^1} it must be that $A^k_n \geq A^{j^1}_n$ because otherwise the payoff to k^0 would always be less than the promise (equality for all n would render the new promise redundant). Thus, in what follows we will only consider when agents use meaningful collateral to make new promises. The payoffs to $j^1_m(k^0)$ are the same whenever k^0 is sufficient collateral, and so we can denote the price of a contract $j^1_m(k^0)$ by π^1_m.

In general, level L contract collateralization is to promise a non-contingent payment using a level $L-1$ debt as collateral.

Definition 3. *We say the L-th* level of contract collateralization *is the creation of contracts j^L using $k^{L-1} \in \mathcal{J}^{L-1}$ as collateral. Denote the set of contracts at the L-th level of collateralization by $\mathcal{J}^L$. We write $j^L(k^{L-1}) = (A^{j^L}, k^{L-1})$ to denote the contract that is traded when an agent holds $k^{L-1} \in J^{L-1}$ as collateral and promises to pay $A^{j^L}_n$ in state n. The contract delivers* $\min\{A^{j^L}_n, A^{k^{L-1}}_n\}$ *in state n.*

With meaningful collateral, the payoff of any contract is defined by the promise, and we use π^L_m to denote the price of any security $j^L_m(k^{L-1}) \in \mathcal{J}^L$. With L levels of collateralization, the set of financial contracts is given by $\mathcal{J} = \mathcal{J}^0 \cup \mathcal{J}^1 \cup \cdots \cup \mathcal{J}^L$. Thus, each additional level of collateralization involves the creation of new contracts and allows all previously existing contracts to be purchased with leverage (by issuing new contracts). The budget set now includes the constraint

$$\sum_{j=j^l_n(j^{l-1}_k) \in \mathcal{J}^l} \max\{0, \varphi_{j^l_n(j^{l-1}_k)}\} \leq \varphi_{j^{l-1}_k} \, \forall l \in 1, \ldots, L, \tag{5}$$

which is the collateral constraint for contracts backed by contracts, up to L levels, which is a parameter of the financial environment.

3.2.2 Theoretical Results

First, there is an essential equivalence between tranching and using contracts as collateral. To see this, consider using an asset Y to issue level-0 contract j^0 which is then used as collateral to issue a level-1 contract j^1. The owner of j^0 yields payoffs $A^{j^0} - A^{j^1}$. Define a contract j^{01} backed by the asset Y and making promised payoffs $A^{j^{01}} = A^{j^0} - A^{j^1}$. Then Y can simultaneously be tranched into a contract paying A^{j^1} as well as $A^{j^{01}}$. Thus, tranching can implement the same state-contingent payoffs created when Y issues a contract, and then that contract is used as collateral.

However, when contracts are limited in their set of state-contingencies, it is no longer true that tranching can directly implement the same payoffs. As an example, if all contracts are restricted to be non-contingent (debt), then pyramiding creates contingencies via default. Replicating the payoffs created by pyramiding requires tranching an asset into a senior-subordinated capital structure (i.e., contingent payoffs). The equivalence between senior-subordinated tranching and equilibrium payoffs when debt can be used as collateral is completely general (Gong and Phelan, 2019).

With tranching we proved that any investor buying an asset will use it to issue the maximum number of tranches. When contracts can be used as collateral, any investor buying an asset will use it to issue contracts that can be used as collateral for further promises, which generalizes a similar result found in Gong and Phelan (2019).

Proposition 3 (Pyramiding). *Suppose asset z can be used to issue contracts that can be used as collateral. Then, generically, no agent will buy z and issue a contract that cannot be used as collateral. In equilibrium, agents holding fundamental assets will issue contracts that can be pyramided to issue more contracts.*

Proof. Consider a contract j_1 with price π_1 such that $A^{j_1} < d_n^z$ for some n. Consider a second contract j_2 with price π_2 that can serve as collateral for j_1, and the payoffs to j_2 are not identical to the payoffs for z. In other words, an agent can hold z, issue j_2, and receive positive payoffs in at least one state, and another agent can similarly hold j_2, issue j_1, and receive positive payoffs in some states.

If an investor strictly prefers to hold z and issue j_1, then generically we have

$$\frac{\mu^h \cdot (d^z - A^{j_1})}{p_z - \pi_1} > \frac{\mu^h \cdot (d^z - A^{j_2})}{p_z - \pi_2},$$
$$\frac{\mu^h \cdot (d^z - A^{j_1})}{p_z - \pi_1} > \frac{\mu^h \cdot (A^{j_2} - A^{j_1})}{\pi_2 - \pi_1},$$

where the numerator of each term is the marginal utility of the investment and the denominator is the price of the investment. The first line says that holding z and issuing j_1 is preferred over issuing j_2, and the second line says that holding z and issuing j_1 is preferred over holding j_2 and issuing

j_1. Multiplying we have

$$\mu^h \cdot (d^z - A^{j_1})(p_z - \pi_2) > \mu^h \cdot (d^z - A^{j_2})(p_z - \pi_1),$$
$$\mu^h \cdot (d^z - A^{j_1})(\pi_2 - \pi_1) > \mu^h \cdot (A^{j_2} - A^{j_1})(p_z - \pi_1),$$

and adding these two inequalities yields

$$\mu^h \cdot (d^z - A^{j_1})(p_z - \pi_1) > \mu^h \cdot (d^z - A^{j_1})(p_z - \pi_1),$$

which is a contradiction. Thus, the agents either prefers issuing the contract j_2 which can be used as collateral or investing in j_2 and issuing the contract j_1.

□

Thus, any agent holding z will prefer to issue a contract that can be used as collateral. The following basis results immediately follow.

Lemma 2. *Consider two assets z_1 and z_2 with identical payoffs and prices p_1 and p_2, but suppose that contracts backed by z_1 cannot be used as collateral, while contracts backed by z_2 can be used as collateral. Then $p_2 > p_1$.*

Proposition 4 (CDS Basis with Pyramiding)**.** *Consider a risk-free asset X and risky asset Y, and suppose that contracts backed by Y can be used as collateral while contracts backed by X cannot. In equilibrium, either (i) X is used to issue a CDS on Y, with price π_C and $Basis_Y = \pi_C^Y - (p_X - p_Y) > 0$, or (ii) CDS are not traded.*

Corollary 3 (Swap Basis with Pyramiding)**.** *Suppose that contracts backed by Y can be used as collateral. In equilibrium, either (i) X is used to issue a derivative contract replicating Y, with price π and $Basis_Y = p_Y - \pi > 0$, or (ii) the derivative is not traded.*

Consider two risky assets z_1 and z_2 with identical dividends, and suppose that z_2 can be collateralized more than z_1: if z_1 cannot be used as collateral, then z_2 can be used as collateral to issue contracts (and perhaps the contracts can also be collateral); if z_1 can be used as collateral to issue contracts but these contracts cannot be used as collateral, then contracts backed by z_2 can be used as collateral.

Corollary 4. *Suppose that z_2 can be collateralized more than z_1. Then the CDS basis on z_2 is greater than the basis on z_1.*

The proof is immediate. Since the CDS on z_1 or on z_2 are identical, they must have the same price. But since z_2 is superior collateral to z_1, z_2 has a greater collateral value than z_1 and thus has a higher price than z_1 in equilibrium.

Our results yield two key insights regarding how collateral affects the basis. First, the cash-synthetic basis is a measure of the differential "collateral values" between risky and safe assets. Importantly, the collateral value of a risky asset does not only depend on the extent to which it can be used as collateral, but also on the extent to which downstream contracts backed by the asset can be used as collateral. In other words, the asset's collateral value depends on the collateral value of derivative contracts. When risky bonds can be used as collateral, and contracts backed by risky bonds can also be used as collateral for financial contracts, the bond premium is less than the corresponding CDS premium.

Allowing contracts to serve as collateral implicitly raises the degree to which the underlying asset can serve as collateral, since the same asset directly and indirectly backs a greater degree of promises. Thus, our analysis highlights that the existence of a non-zero basis implies, in addition to the other factors identified in the literature to contribute to bases, a difference between the collateral value of safe and risky assets. The positive basis emerges because the asset can be used to issue financial promises with positive collateral value. Accordingly, if the collateral value of the derivative contracts decreases, then the basis for the asset should decrease.

Second, agents value assets based on their abilities to provide payoffs in different states, not just based on the original payoffs of the assets. Assets with the same payoffs but that can be used as collateral for different promises allow agents to isolate payoffs in different states. Thus, agents choose to buy assets that best allow them to isolate payoffs in states in which their marginal utilities are higher. As a result, agents may not "trade against" the basis even though there is an apparent arbitrage opportunity, but trade to receive their most preferred state-contingent payoffs. This insight is especially important when balance sheet considerations imply that a small arbitrage may not be worth undertaking given the costs of balance sheets. Thus, investors may prefer a risky investment with large upside potential over an arbitrage for only several basis points. For evidence based on deviations from covered interest rate parity see Du et al. (2016).

3.3 Economies with Multiple Bases

We now consider an economy with a single underlying risky asset and consider CDS on it and on derivative debt.

Consider a risky contract j_M, backed by an asset Y, where the maximum payoff to j_M is M. We introduce a CDS on the risky contract j_M and now consider the CDS basis for the risky debt j_M and to study the relationship between this basis and the basis for the risky asset. We think of the basis on j_M as corresponding to the basis on ABS or CDO tranches, rather than the basis on the underlying pool of collateral. As before, we define the basis on the risky contract, denoted Basis_M, as the difference between the spread on the debt CDS and the bond spread, $\text{Basis}_M = \pi^M - (Mp_X - \pi_C^M)$, where the basis is defined with Mp_X to reflect the cost of a risk-free payoff of M. Our main result is that Basis_M and Basis_Y are never equal. We use the term "double basis" to refer to this phenomenon of two unequal bases occurring in equilibrium for assets with correlated payoffs.

Proposition 5. *Consider an economy with CDS contracts CDS_Y and CDS_M, which are backed by safe assets:*

1. *(Leverage) In an economy with level-0 contracts only, the basis on the risky debt is negative and the basis on the risky asset is non-negative. That is, $\pi^M + \pi_C^M < Mp_X$ and $p + \pi_C^Y \geq p_X$.*
2. *(Pyramiding) In an economy with level-1 contracts, the basis on the risky debt is zero and the basis on the risky asset is positive, $\pi^M + \pi_C^M = M$ and $p + \pi_C^Y > p_X$*

The intuition for this result is similar to the intuition provided in the previous section. Without pyramiding, j_M has no collateral value. However, the asset X is allowed to issue CDS_M, which gives X higher collateral value relative to j_M. This results in a negative basis on the risky debt. In the pyramiding economy, $\text{Basis}_M = 0$ implies that $\text{Basis}_Y > 0$ since Y always has one more level of collateralization than j_M. Allowing j_M to serve as collateral implicitly raises the collateral value of Y, and causes $\text{Basis}_Y > 0$. The basis on the most upstream collateral is greater than the basis on downstream contracts. The basis on the risky contract j_M is always lower than the basis on the underlying risky asset Y. In other words, the basis on the most upstream collateral (the risky asset Y) is greater than the basis on downstream contracts (the risky debt j_M). This occurs because the risky asset Y can always back at least one more level of debt contracts than the risky debt can back, and so the debt has a lower collateral value.

It is clear that the previous results on how tranching and pyramiding affect the basis on fundamental assets z also apply in the same way to the basis on derivative contracts. Thus, bases on derivative contracts are measures of the collateral value of those derivatives.

4 Empirical Implications and an Empirical Test

Our analysis offers a few testable implications regarding fluctuations in bases. In this section we first discuss empirical implications, some suggestive evidence supporting our theory, and considerations for more careful tests by future research. We then present an empirical test of one of the key predictions.

4.1 Predictions

Our theory predicts that tranching or pyramiding increases the CDS basis. Thus, variations in the extent to which funding markets use debt as collateral, or to which structured finance implicitly allows debt to be used as collateral, ought to correspond to variations in the CDS basis. This implication is distinct from existing explanations in the literature such as the cheapest-to-deliver mechanism or the "CDS market leads the bond market" mechanism (see discussion below).

There are two sets of facts that provide suggestive evidence for the predictions of our model. First, the predictions of our model are broadly consistent with the stylized facts regarding the prevalence and collapse of CDO and structured finance issuance as well as the time series behavior of average bases. Rauh and Sufi (2010) show that low-credit-quality firms are more likely to have a multi-tiered capital structure with subordinated debt. Hence, our model predicts that pre-crisis the HY basis should be larger than IG basis because senior-subordinated capital structures, which implicitly use debt as collateral for debt, increase the basis (post-crisis, funding market freezes disproportionately affected weak collateral, which is why HY bases would turn more negative).

Our results from Section 3.3 also provides important predictions for CDS contracts that are part of a CDX basis. Importantly, the CDX index is tranched into synthetic "index CDO tranches": in addition to buying (or selling) protection on the overall level of the CDX index, investors can also buy protection on the first 3% of losses among the 125 constituents, or losses between 3 and 7%, and so on with attachment points at 10, 15, and 30 percent of losses. The CDX tranches correspond to downstream contracts backed by the underlying constituent assets. Accordingly, the overall CDX index spread captures the spreads on the CDX tranches. Because the CDX tranches give greater collateral value to the underlying CDS contracts that make up the index, Corollary 2 predicts that the CDS-bond basis should increase for CDS contracts that are added to a CDX index. We provide an empirical test of this prediction in the next section.

Additionally, the same corollary also provides a potential explanation for an additional force driving the CDS-CDX basis.[5] The CDS-CDX spread is defined as difference between the average five-year CDS spreads on the 125 constituents of the NA.IG.CDX index and the spread on the NA.IG.CDX index, obtained from Markit. Our theory predicts that the basis on the most upstream collateral—namely, the 125 constituent single name CDS contracts—should be greater than the basis on downstream contracts—namely, the index tranches. Accordingly the CDS basis on the constituents ought to exceed the basis on the CDX, implying a higher CDS spread on the constituents. This is exactly what we observe in the data (see Figure 2), meaning that the CDS spreads on the underlying constituents is greater than the spread on the CDX index and so it is cheaper to buy protection on the index (pay the premium) than on every individual constituent.[6]

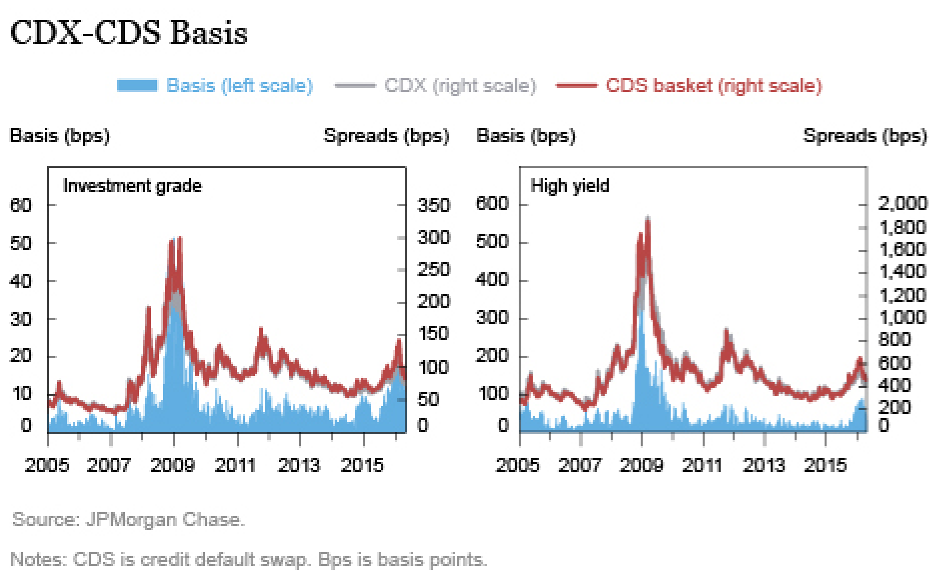

Figure 2: CDS-CDX basis. Source: Boyarchenko et al. (2017)

[5]We are grateful to Nina Boyarchenko for her comments on this topic.

[6]Liquidity conditions provide another explanation. Junge and Trolle (2015) argue that CDX-CDS basis measures the overall liquidity of the CDS market. According to this theory, widening of the CDS-CDX basis would reflect deterioration in liquidity in CDS relative to liquidity in CDX.

Furthermore, when collateral is most scarce, this basis ought to widen, as occurred during the crisis. Undoubtedly, limits to arbitrage are important for explaining difficulties in exploiting the apparent arbitrage trade of buying protection on the CDX index (pay the premium) and selling protection on the underlying 125 names (receive the higher premium). Our theory suggests that non-arbitrageur investors would trade instead in particular tranches in order isolate precisely the risk profile they desire. For example, see Longstaff and Rajan (2008) for an analysis of how each tranche corresponds to different levels of systemic/correlated default risk.

The most common explanation for positive CDS bases is that physically settled CDS contain a cheapest-to-deliver ("CTD") option that increases the premium of the CDS contract. Blanco et al. (2005) find that the CTD option is most prevalent for European entities because U.S. CDSs have been subject to a Modified Restructuring definition since May 11, 2001, which reduces the value of the delivery option. Blanco et al. (2005) argue that it is almost impossible to value this option analytically since there is no benchmark for the post-default behavior of deliverable bonds. Additional technical considerations of CDS contracts and bond trading can increase the basis (e.g., CDS premia are floored at zero, CDS restructuring clause for technical default, bonds trading below par, see De Wit 2006). Our theory implies that variations in the extent to which funding markets can use debt as collateral (or the extent to which structured finance implicitly allows debt to be used as collateral) ought to correspond to variations in the CDS basis. In contrast, funding markets for derivative debt securities ought to have no direct effect on the value of the CTD option.

Many authors in the empirical literature have identified factors that partially explain the behavior of the CDS basis. Zhu (2004) finds that the CDS market moves ahead of the bond market in terms of price adjustment because the two markets respond differently to changes in credit conditions, and this timing may explain the existence of non-zero bases in the short run. Blanco et al. (2005) argue that the bond market lags behind the CDS market in determining the price of credit risk, causing short-run deviations in prices; long-run deviations arise from imperfections in CDS contract specification (the CDS price is an upper-bound on credit risk) and from measurement errors, which understate the true credit spread. Nashikkar et al. (2011) show that bonds of firms with a greater degree of uncertainty are expensive (i.e., the basis is positive), which they claim to be consistent with limits to arbitrage theories. Choi and Shachar (2014) argue that a negative basis emerged during the 2008 financial crises because the limited balance sheet capacity of dealer banks prevented corporate bond dealers from trading aggressively enough to close the basis.

We stress that our results about collateral quality provide only one possible explanation of fluctuations in the basis. Our results can begin to explain some of the time-series variation within a collateral class (corresponding to fluctuations in CDO issuance and other structured finance) and some of the cross-sectional difference across classes. The basis depends on many things besides implied collateral quality: the liquidity explanation also matters, as does segmentation between CDS and bond markets.[7] One can consider our explanation as having an effect *in addition* to what liquidity premia would imply. In addition, there have been many other apparent arbitrages that behaved similar to the CDS basis, but for which our story does not apply directly (e.g., cash-futures, mortgage rolls, fed funds, swap spreads, covered interest parity). One might suppose that the ability to use different assets as collateral affects the balance sheet costs of financial institutions, and thus the costs of "limits to arbitrage," which would affect the sizes of these arbitrages.

4.2 An empirical test

We now provide a rudimentary test of the hypothesis that inclusion in the CDX (and thus being able to be tranched) should increase the CDS basis, which is the prediction of Corollary 2. To directly test the effect that collateralizability has on the CDS-bond basis, we look at changes in the CDS-bond basis for CDS contracts that are removed or added to a Markit CDX index. The two Markit CDX indices we consider are the Markit North American High Yield CDX Index, or the CDX.NA.HY Index and the Markit North American Investment Grade CDX Index, or the CDX.NA.IG Index. Markit tranches the HY and IG indices into five and six tranches, respectively, and allows investors to buy shares of the tranches in addition to buying the entire index. Purchasing a tranche of an asset's cash flows is equivalent to funding the asset with some implicit margin (where the margin is given by the prices of the tranches). As a result, the margin requirement increases for entities that are excluded from an index and decreases for entities that are included. For margin-based asset pricing to be valid, the change CDS-bond basis must be positive (negative) for included (excluded) entities relative to unaffected entities.

The details of our empirical analysis are provided in Appendix C, but we provide a summary

[7]Note that when the CDS market leads the bond market, this would lead to a positive widening in the basis during crises, which is the opposite of what broadly occurred during the recent crisis.

of the methods and results here. We use a difference-in-difference approach to estimate the percentage change in the CDS-bond basis for credit default swaps that are added to or removed from either index over a two-day window, both around the time of announcement and around the time of index roll, using Markit's publicly available record of changes to the CDX.NA.HY index and CDX.NA.IG index from March 2013 to September 2017.

The baseline regression estimation is given by equation (6):

$$\text{basis}_{it} = \beta_1 \cdot (\text{announced}_t) \cdot (\text{added}_i) + \beta_2 \cdot (\text{announced}_t) \cdot (\text{removed}_i) + \gamma \cdot Z_{it} + \varepsilon_{it}, \tag{6}$$

where basis_{it} is the normalized basis for CDS i at time t, where the pre-announcement basis is normalized to be 1. This allows us to estimate the difference in percentages rather than levels.[8] The variable announced_t is an indicator variable that takes a value of 0 before the announcement and a value of 1 after announcement; added_i and removed_i are indicators for whether the CDS has been added to or removed from an index. If both $\text{added}_i = 0$ and $\text{removed}_i = 0$, then the CDS was previously included in the index and had no change in status. Z_{it} consist of a constant term, fixed effect for announcement, fixed effects for addition and removal, year and month fixed effects (the indices are updated twice each year), and indicators for whether the swap switched from one index to another. The coefficient β_1 (β_2) is the difference-in-difference estimator that provides the percentage in the CDS-bond basis for entities that were added to (removed from) an index, relative to swaps that remained on the index. Margin-based asset pricing predicts that $\beta_1 > 0$ and $\beta_2 < 0$.

There are two identifying assumptions. First, the announcement of addition or removal from an index is uncorrelated with other factors that may affect the CDS-bond basis. This is likely satisfied because index inclusion does not reveal new information about the CDS, since the requirements for inclusion are publicly available and the characteristics are easily observable. Furthermore, any revealed information which changes the payoff value of the CDS should also be reflected in an equivalent change in the bond price, so that there is no change in the CDS-bond basis.

[8] We use percentage changes because different bonds exhibit a great degree of heterogeneity in the magnitude of the CDS-bond basis. In our sample, the largest bases in absolute value was over 1000 basis points, while the smallest was .5 basis points. CDS contracts with large bases typically were much more volatile in levels. Proceeding with the estimation in percentages reduces the amount of noise. The details of the normalization method can be found in the appendix.

Second, identification requires common trends across the group—that is, in the absence of announcement, the percentage change in the CDS-bond basis for swaps that were added, removed, or unaffected would have been the same. Since swaps that are included on the index or added to the index have relatively high liquidity and are traded on a frequent basis, nothing fundamentally changes around the announcement date other than information about the swap's inclusion.

Table 1: The last two specifications include controls for the month and year, as well as indicators for whether the entity switched indices. The month and year controls are not shown in the table.

	Dependent variable: Normalized CDS basis (percent changes)			
	announcement	roll	announcement	roll
	(1)	(2)	(3)	(4)
switch to HY			0.127**	78.159**
			(0.063)	(38.689)
switch to IG			0.050	17.602
			(0.101)	(43.579)
announced×add	0.187**	−0.128	0.183**	−29.946
	(0.072)	(0.085)	(0.073)	(32.320)
announced×remove	−0.071	−0.116	−0.081	−21.873
	(0.073)	(0.086)	(0.075)	(33.869)
Observations	662	658	662	658
R^2	0.031	0.025	0.045	0.035
Adjusted R^2	0.023	0.018	0.027	0.023
Note:			*p<0.1; **p<0.05; ***p<0.01	

The result of our baseline procedure is given in Table 1. We find that the announcement of the addition of a CDS to an index is associated with an increase in the CDS-bond basis by about 18 percentage points, relative to entities that are unaffected (consistent with our theory). In the appendix, we also explicitly test inclusion relative to exclusion (rather than being unaffected) and find that the change in the CDS-bond basis was 26 percentage points higher for those included than for those excluded. Furthermore, we show that there is no statistically significant percentage change in the CDS-bond basis upon the roll date across the groups.

We also consider the alternative hypothesis that our results are driven by liquidity values, not

Figure 3: Trade Counts for CDS contracts on IG firms in 2017, with the March and September CDX roll dates highlighted

collateral. It is possible that CDS contracts that are added to an index become more liquid as a result of inclusion, and the increase in the liquidity premium increases only the CDS spread and not the bond spread. However, while trade volumes spike on the roll date of the index, this increase in trade volume is temporary and there is no significant increase in trade volume around the time of the announcement (see Figures 3 and 4). Additionally, since there is no significant change in the CDS-bond basis around the roll date, this suggests that liquidity is not the driving force behind changes in bases. Without a doubt liquidity is an important determinant of asset prices and basis behavior, as is well established in the literature.

In Appendix C, we try to eliminate confounding variables from behavioral responses by market participants and estimate a triple-difference estimation, comparing addition to the HY index to addition to the IG index. The difference between these two indices consist only of (i) credit

Figure 4: Trade Counts for CDS contracts on HY firms in 2017, with the March and September CDX roll dates highlighted

rating of the firm, which is publicly known prior to announcement (ii) the number of swaps in each index (100 in HY vs 125 in IG) and (iii) the tranching structure of the two indices. While the first difference should not result in any changes to the CDS-bond basis, the latter have implications for the implicit margin requirement and therefore should translate into differences in the percentage change of the CDS-bond basis. We find that inclusion to the HY index (rather than the IG index) has significant implications in the movement of the CDS-bond basis. Our results therefore suggest that collateral values, driven by index inclusion, may also be an important determinant.

5 Conclusion

We present a theoretical model that relates the extent to which financial markets can effectively use assets as collateral to the CDS basis on those bonds. In particular, we show that the basis is positive when either (i) an asset can be tranched into multiple securities or (ii) agents can use risky debt contracts as collateral to issue financial promises. Structured finance that uses pools of collateral to issue senior-subordinated capital structures will produce positive bases on the underlying collateral, and thus financing these assets will be cheap. We also prove that when multiple CDS contracts are traded in an economy with tranching or pyramiding, the bases on the CDS contracts must be different as each level of has a different collateral value. We provide empirical evidence for our theory using inclusion/exclusion in CDX indices to show that the behavior of the CDS basis is consistent with our theory.

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Appendices

A Three-State Model with Collateral and CDS

This section presents a multi-state extension of Fostel and Geanakoplos (2012a) with the addition of tranching or pyramiding. The restriction to debt requires imposing more structure on preferences and endowments. In this environment, we are able to completely characterize equilibria, as well as derive similar results regarding how structured finance affects the basis. We first present the three-state model and then characterize equilibria with different financial regimes. Proofs are in Appendix B.

A.1 The Model

Time, Assets, and Investors

We consider a two-period, three-state model with time $t = 0, 1$. Uncertainty is represented by a tree $S = \{0, U, M, D\}$ with a root $s = 0$ at $t = 0$ and three states of nature $s = U, M, D$ at $t = 1$.There are two fundamental assets, X and Y, which produce dividends of the consumption good at time 1. Asset X is risk-free, producing (as a normalization) 1 unit of the consumption good in every final state. Asset Y is risky, producing $d_U^Y = 1$ unit in state U (a normalization), $d_M^Y < 1$ units in state M, and $d_D^Y < d_M^Y$ in state D. We think of asset Y as a financial asset, such as a corporate bond, a pool of mortgages, or an asset-backed security, rather than a physical asset like a house or the assets of a firm. With a slight abuse of notation we let M, D be the dividends in states M, D with $D < M < 1$. Asset payoffs are shown in Figure 5.

We suppose that agents are uniformly distributed on $(0, 1)$, that is they are described by Lebesgue measure. (We will use the terms "agents" and "investors" interchangeably.) Agents are risk-neutral and have linear utility in consumption c at time 1. Each agent $h \in (0, 1)$ assigns subjective probability $\gamma_s(h)$ to the state s, and beliefs $\gamma_s(h)$ are continuous in h. The expected utility of agent h is

$$U^h(c) = \gamma_U(h)c_U + \gamma_M(h)c_M + \gamma_D(h)c_D,$$

where c_s is the consumption in state s. At $t = 0$, each investor is endowed with 1 unit of each asset

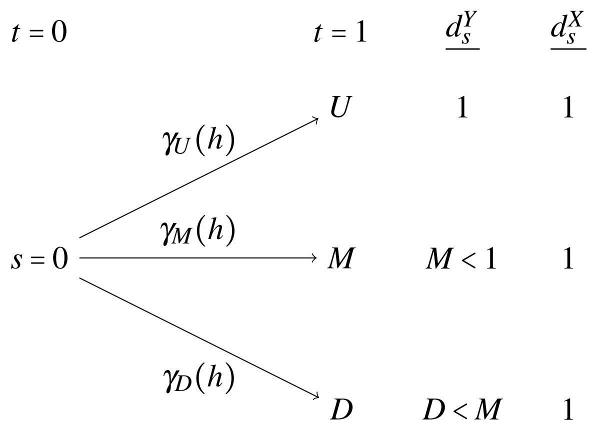


Figure 5: Payoff tree of assets X and Y in three-state world.

X and Y.

To ensure that in equilibrium investors' positions are sorted by their level of optimism, we suppose hazard rate dominance (see also Simsek, 2013; Gong and Phelan, 2019), which we can write as

$$\gamma_U(h)+\gamma_M(h) \text{ and } \frac{\gamma_U(h)}{\gamma_U(h)+\gamma_M(h)} \text{ are increasing in } h. \tag{A1}$$

High h investors believe that state D is unlikely and that, conditional on the state being at least M, state U is relatively likely. This setup is equivalent to a model with finitely many heterogeneous risk-averse agents, where endowments and preferences are such that marginal utilities or "hedging needs" are monotonic and uniformly increasing by state.

Financial Contracts and Collateral

We restrict attention to debt contracts and CDS. Debt contracts, denoted j_ℓ, promise non-contingent payments (ℓ,ℓ,ℓ). Without loss of generality, we suppose that all debt contracts are collateralized by one unit of the risky asset Y (selling a non-contingent promise backed by X as collateral would be equivalent to selling a fraction of X). Debt contracts with promises $\ell \leq D$ are fully collateralized (never default) and are therefore risk free. Debt contracts with $D < \ell \leq M$ will default in state D but deliver the promise ℓ in states U and M.

A CDS contract on the risky asset Y, denoted by CDS_Y, pays $1-d_s^Y$ in state s (the difference between the maximum payout of Y and the actual payout of Y). To simplify the analysis, we require that each unit of the CDS contract be fully collateralized so that any agent selling the CDS_Y contract is able to repay his obligations regardless of which state is realized.[9] The safe asset X can serve as collateral for CDS. Since CDS_Y pays $(0, 1-M, 1-D)$, every unit of CDS_Y must be collateralized by $(1-D)$ units of X. (Alternatively, an agent holding one unit of X can sell $\frac{1}{1-D}$ units of CDS_Y.) When Y can serve as collateral for CDS, one CDS_Y contract must be backed by $\frac{D}{1-D}$ units of Y; alternatively, $\frac{1}{D}$ units of Y can back $\frac{1}{1-D}$ units of CDS_Y. We let J^Y and J^X be the set of promises backed by Y and X respectively. Thus, to start $J^X = (CDS_Y, (1-D)X)$. Later we will introduce a CDS on risky debt contracts (specifically on j_M), which will expand J^X.

Definition 4. *Debt collateralization is the process by which agents use debt contracts $j \in J^Y$ to issue financial promises in the form of debt or CDS. An economy with debt collateralization is one in which agents are allowed to use any debt contract as collateral.*

We allow agents to trade contracts of the form $j^1_\ell = (\ell, j_M)$. This contract promises a non-contingent payment (ℓ, ℓ, ℓ) backed by the risky debt j_M acting as collateral. The restriction to j_M is without loss of generality; we could let any contract $j \in J^Y$ serve as collateral, but in equilibrium only j_M will be traded and thus only j_M will serve as collateral (Gong and Phelan, 2019). The contract j_M delivers $d^{j_M}_s = (M, M, D)$, and the payoff to j^1_ℓ in each state is $\min\{\ell, d^{j_M}_s\}$. Note that the act of holding j_M and selling the contract j^1_D is equivalent to buying j_M with leverage promising D, yielding a payoff of $(M-D, M-D, 0)$. We also allow agents to use safe debt j_D to issue CDS, which is the contract $(CDS_Y, (1-D) j_D)$, and this contract has identical payoffs to CDS backed by X. Denote the set of contracts backed by j_M and j_D by J^1.

In our model, variations in the financial environment are the drivers of variations in CDS bases. These variations can reflect changes in how assets or contracts are used as collateral or changes in how assets are tranched in securitized markets. Before proceeding with the theoretical analysis, we explain this equivalence in greater detail. To fix ideas, let $M = 0.3$ and $D = 0.1$.

[9] This restriction is not without loss of generality for the equilibrium regime, though our main results continue to hold. As will be clear from the analysis that follows, if agents could sell "partially collateralized CDS," then in equilibrium some agents would sell CDS collateralized by only $1-M$ units of X, which would yield the CDS buyers a payoff of $(0, 1-M, 1-M)$ and the sellers a payoff of $(1-M, 0, 0)$. The first payoff would be attractive to "high pessimists" and the second payoff would be attractive to the most optimistic agents, and is equivalent to buying Y and promising M, which we consider in the sections with leverage.

Consider when debt contracts can be used as collateral, and consider the following equilibrium regime. Some investors buy the risky asset Y with maximum leverage, issuing a risky debt contract that promises $M = 0.3$. This debt contract will default in state D, and thus the payoff is $(0.3, 0.3, 0.1)$. The investors that bought Y and issued the contract would be left with payoffs $(0.7, 0, 0)$. Another set of investors would buy this risky debt with leverage, issuing a risk-free debt contract that promises $D = 0.1$. The investors in risky debt would be left with payoffs $(0.2, 0.2, 0)$.

In total, investors in the economy will hold the following set of payoffs, $(0.7, 0, 0)$; $(0.2, 0.2, 0)$; $(0.1, 0.1, 0.1)$, all of which are ultimately backed by the payoffs to Y. These payoffs are exactly what would occur if Y were tranched into senior-subordinated tranches. The most senior tranche would be guaranteed to pay in every state, and thus could deliver $D = 0.1$. The mezzanine tranche would default in state D but would otherwise be able to deliver 0.2. The subordinated, or equity, tranche would deliver the residual payment in state U alone, delivering 0.7.

A.2 Baseline Results

Note that the payout of holding one unit of X is equivalent to holding one unit of Y and one unit of CDS_Y. Thus, the basis can be equivalently defined to be the difference in the price of these two options: $\text{Basis}_Y = (p + \pi_C^Y) - 1$, or $p + \pi_C^Y = 1 + \text{Basis}_Y$. We use the term "cash-synthetic asset" to refer to a portfolio consisting of equal units of Y and CDS_Y since this option, like X, is completely risk-free.

We first characterize the basis in an economy without short selling. We consider when agents can (1) use X as collateral to issue CDS_Y; (2) use Y as collateral to issue debt contracts and to issue CDS_Y; and (3) use debt contracts to issue debt and CDS_Y. We refer to (2) as the leverage economy[10] and (3) as the debt-collateralization (or structured finance) economy. This section presents the main theoretical results, and later sections provide the complete characterizations of equilibria.

Limiting leverage (i.e., restricting the set of contracts backed by Y) decreases the basis. If Y is imperfect collateral, perhaps due to regulations or because financial markets have concerns arising from informational frictions, then the basis will be negative. If the risky asset Y can be used as collateral to issue debt contracts and CDS_Y, then the basis is nonnegative. The following

[10]This case has been considered by Fostel and Geanakoplos (2012a) in a two-state economy.

proposition extends the results in Fostel and Geanakoplos (2012a) to multi-state economies.

Proposition 6. *Suppose that the only financial contracts agents can trade are debt and a CDS on Y. Then,*

1. *(No leverage) If only X can serve as collateral for financial contracts, then agents will issue CDS_Y backed by X and the basis on Y is negative, $\pi_C^Y + p < 1$.*
2. *(Leverage) If X and Y can serve as collateral for financial contracts, then in the following cases*
   - *(a) if there are limits on the collateral ability of Y so that Y cannot issue CDS_Y and Y can only issue safe debt, then the basis is negative $p + \pi_C^Y < 1$.*
   - *(b) if there are no limits on the collateral ability of Y (Y can issue CDS_Y and any kind of debt), then the basis is non-negative $\pi_C^Y + p \geq 1$.*
3. *(Debt Collateralization) If X, Y, and debt can serve as collateral for financial contracts, then the basis is positive $\pi_C^Y + p > 1$.*

Here is the intuition for the results. The price of an asset can be decomposed into the sum of its "payoff value" (PV) and its "collateral value" (CV) to any agent who holds the asset. The PV is an agent's normalized expected marginal utility of the future dividends; the CV measures the asset's value of the collateral capacity of the asset, which is also how much the agent values liquidity.[11] When an asset can be used as collateral, its price generally exceeds the payoff value. When an asset cannot act as collateral, the CV is always zero. When the risky asset Y cannot be used as collateral at all (case 1) or for CDS (case 2), then X is superior collateral and then Y trades at a negative basis to X. When Y can be used as collateral without constraint, then X does not have greater collateral capacity and so the basis disappears. Indeed, since CDS must be fully collateralized whereas Y could be used to issue risky debt (which might default), X has a limited collateral capacity compared to Y and so Y may trade at a premium.

Finally, with structured finance as in the third case, debt backed by Y can be used as collateral. This increases the collateral value of j_M (since agents buying j_M have the ability to sell j_D^1),

[11] Fostel and Geanakoplos (2008) define the PV of an asset j to an agent i as $PV_j^i \equiv \sum_{s \in S} \gamma_s^i d_s^j \left(\frac{du^i(c_s^i)}{dc}\right) / \left(\frac{du^i(c_0^i)}{dc}\right)$, where u^i is the utility of agent i and γ_s^i is the subjective probability the agent assigns to state s.

increasing π^M in equilibrium. Since agents can leverage their purchases of Y by borrowing π^M, agents can now buy Y with higher leverage, raising the equilibrium demand for Y. Debt collateralization increases the collateral value of Y because Y can be used to issue j_M and therefore inherits some of the increase in the collateral value of j_M. Thus the risky asset Y now has two "levels" of collateralization—the first from allowing Y to back debt contracts, and the second from allowing these debt contracts to back further contracts. The collateral value of X does not change because it can still issue only one contract, CDS_Y. In other words, Y back all the same contracts that X can, but Y can also back contracts that can be further collateralized downstream.[12] These forces increase the price of Y relative to the price of X and result in a positive basis.

A.3 No Leverage: $C^j = X$

We first characterize equilibrium with no leverage before considering when Y can serve as collateral.

Consider the scenario in which agents cannot use Y as collateral to issue debt contracts. Formally, $J^Y = \varnothing$ and $J = J^X = (CDS_Y, (1-D)X)$ is the only financial contract available for trade. We denote the act of holding X and selling the maximum allowable amount of CDS_Y by X/CDS_Y. In this regime, agents can take any of the following positions: (i) X/CDS_Y (hold X and sell CDS_Y), (ii) buy Y, (iii) buy X or the cash-synthetic asset made of a portfolio of both Y and CDS_Y, and (iv) buy the financial contract CDS_Y. Notice that the above positions are listed in terms of decreasing optimism/increasing pessimism. An agent who believes that state U is very likely to happen will choose to either buy Y or hold X/CDS_Y, whereas an agent who believes that state D is more likely will want to purchase CDS_Y. Because agents are risk neutral, every agent will choose exactly one of the above positions based on how optimistic they are. The following result characterizes equilibrium in this economy.

Lemma 3. *In this regime, no agent chooses to hold safe assets without selling financial contracts. That is, no agent chooses to hold simply X or the cash-synthetic asset made of a portfolio of Y and CDS_Y. In fact, any agent who holds X will also sell the maximum allowable amount of CDS_Y.*

The intuition is straightforward. Any agent who does not want to buy X and sell the CDS must value consumption in state D. This is because selling the CDS means that the agent loses

[12] Agents have no desire to use X to issue debt contracts since leveraging a completely safe asset provides no benefits

consumption if the down state occurs. Thus, these agents are relatively pessimistic (compared to agents who do choose to sell the CDS) and must therefore be willing to sacrifice consumption in state U for the chance to have even more consumption in state M or D. Since CDS_Y pays $(0, 1-M, 1-D)$, in equilibrium prices must be such an agent will want to invest in CDS_Y rather than hold X. The basis must be negative in this economy (Proposition 6).

In this equilibrium regime, agents choose to hold X rather than the cash-synthetic asset even though the two have equivalent payoffs and the latter is cheaper. While this outcome may seem illogical, the result occurs in equilibrium because neither Y nor CDS_Y can be used as collateral: neither have collateral value. Thus, agents hold X precisely because it allows them to sell the CDS, and therefore isolate payoffs in states U and M. Any agent who chooses to hold the portfolio of Y and CDS_Y cannot isolate payoffs in any states but accepts equal payoffs in every state. It is worth contrasting this result with traditional theories that ignore collateral. Traditional theory predicts that the CDS spread should be equal to the bond spread, due to the arbitrage opportunity that would arise otherwise. Even when agents cannot short-sell assets, the spreads should still be equal because agents can always choose buy the cheaper option—either the safe asset or a combination of the risky asset and its CDS. It is the ability of X to issue financial contracts that gives X a higher price. Combining these results, we obtain the following lemma, which describes equilibrium in this regime.

Lemma 4. *In this economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) X/CDS_Y, (2) Y, and (3) CDS_Y.*

There are two marginal buyers h_1 and h_2. The most optimistic agents in the economy $h > h_1$ will sell their endowment of Y to buy X and issue the maximum allowable number of CDS_Y. Moderate agents $h \in (h_1, h_2)$ will sell their endowment of X to buy all the units of the risky asset Y. Pessimists $h < h_2$ will sell their endowment of X and Y to buy the financial contract CDS_Y sold by optimists. Figure 6 illustrates the equilibrium regime. Arrows point from lender to borrower and we see pessimists (those holding CDS_Y) lending to optimists (those holding X/CDS_Y) in this economy.

Marginal investors are indifferent between two different options. Agent h_1 is indifferent

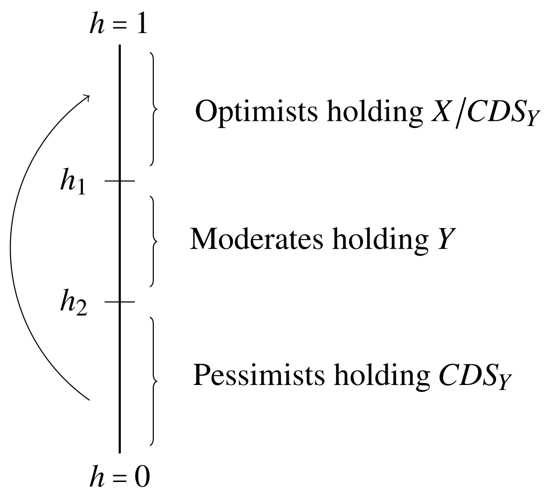


Figure 6: Equilibrium with CDS_Y, no leverage. Holders of CDS_Y fund optimists.

between selling the CDS_Y collateralized by X and buying the risky asset Y

$$\frac{\gamma_U(h_1)(1-D)+\gamma_M(h_1)(M-D)}{1-D-\pi_C^Y}=\frac{\gamma_U(h_1)+\gamma_M(h_1)M+\gamma_D(h)D}{p}. \tag{7}$$

Agent h_2 is indifferent between buying Y and buying the financial contract CDS_Y

$$\frac{\gamma_U(h_2)+\gamma_M(h_2)M+\gamma_D(h)D}{p}=\frac{\gamma_M(h_2)(1-M)+\gamma_D(h_2)(1-D)}{\pi_C^Y}. \tag{8}$$

Market clearing for X requires

$$\frac{(1-h_1)(1+p)}{1-\frac{\pi_C^Y}{1-D}}=1, \tag{9}$$

and market clearing for Y requires

$$\frac{(h_1-h_2)(1+p)}{p}=1. \tag{10}$$

Equation (9) states that agents buying X, $h\in(h_1,1)$ will spend all of their endowment, $(1+p)$ to purchase X, which has price 1. With each unit of X they buy, they will also sell $\frac{1}{1-D}$ units of CDS_Y, which has price π_C^Y. The revenue from these sales is used to buy more X. The demand for X is equal to the supply, which is 1. Equation (10) states that agents buying the risky asset Y, $h\in(h_2,h_1)$ will spend all of their endowment on Y, which has price p, and that the amount demanded by these agents must be equal to the unit supply in the economy.

A.4 Leverage Economy: $C^j \in \{X, Y\}$

Consider when the risky asset Y can be used as collateral to issue debt contracts and CDS_Y. In particular, one unit of Y can back a non-contingent debt promise (ℓ, ℓ, ℓ), or $\frac{1-D}{D}$ units of Y can back one (fully collateralized) CDS contract. This is due to the fact that the CDS pays $1-D$ in the same state when Y pays D.

The results of Fostel and Geanakoplos (2012a,b) characterize which contracts will be traded in equilibrium in an economy with only debt contracts, and these results allow us to characterize equilibrium with CDS. In an economy with debt contracts and without leverage limits, two debt contracts are traded in equilibrium: $j_D = D$ and $j_M = M$, with prices π^D and π^M respectively. The contract j_D delivers (D, D, D), while j_M delivers (M, M, D). Unlike the safe promise j_D, the delivery of j_M depends on the realization of the state at time 1. Therefore, j_M is risky and has price $\pi^M < M$. The interest rate for j_M is strictly positive and is given by $i_M = \frac{M}{\pi^M} - 1$, and is endogenously determined in equilibrium.

First, note that holding $1-D$ units of Y and selling D units of CDS contracts yields $(1-D, M-D, 0)$, which is the same payoff as holding one unit of Y and selling the promise j_D. Second, holding $(1-D)$ of X and selling one unit of CDS_Y also yields the same payoff as holding one unit of Y and selling the promise j_D. We denote buying Y and selling CDS by Y/CDS_Y, buying Y and selling j_D by Y/j_D, and buying X and selling CDS by X/CDS_Y, where all positions are appropriately scaled to be fully collateralized: Y/CDS_Y costs $(1-D)p - D\pi_C^Y$; Y/j_D costs $p - \pi^D$; X/CDS_Y costs $1 - D - \pi_C^Y$. Since all positions yield the same cash flows, investors will choose the positions which are cheapest. An immediate implication is that the equilibrium basis is non-negative.

If the basis were negative, then agents would prefer to use Y as collateral to issue CDS over using X, and so no agent would hold X. In fact, we can say more: if the basis is zero, then X/CDS_Y is equivalent to Y/CDS_Y and both will be traded in equilibrium; when the basis is strictly positive then X/CDS_Y is cheaper and no agent will trade Y/CDS_Y in equilibrium. Accordingly, equilibrium in the leverage economy can be described by three marginal investors h_1, h_2, h_3. Investors $h > h_1$

buy the risky asset Y and issue risky debt. Investors with $h \in (h_2, h_1)$ issue CDS contracts, using either X or Y as collateral. Investors with $h \in (h_3, h_2)$ buy risky debt, and the remaining investors buy CDS.

Lemma 5. *In the leverage economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) Y/j_M, (2) $X/CDS_Y \equiv Y/CDS_Y$, (3) j_M, (4) and CDS_Y. When the basis is zero, then a fraction of Y is used for Y/CDS_Y, but no agents trade Y/CDS_Y when the basis is positive.*

That the four positions exist in equilibrium is immediate. Figure 7 shows the equilibrium regime. Arrows point from lender to borrower. In this economy, pessimists lend to optimists.

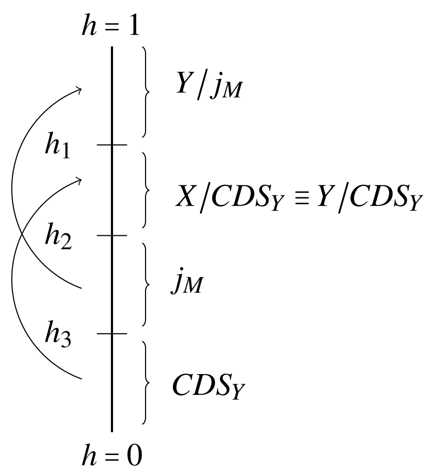


Figure 7: Equilibrium with leverage and CDS_Y backed by X. Buyers of CDS_Y fund moderates holding X/CDS_Y. Agents purchasing j_M lend to optimists.

With leverage, equilibrium consists of three marginal investors, h_1, h_2, and h_3 and the following equations defining the marginal investors. Agent h_1 is indifferent between holding the risky asset with leverage promising M and buying the risky asset with leverage promising D,

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D)+\gamma_M(h_1)(M-D)}{p-D}. \tag{11}$$

Agent h_2 is indifferent between buying the safe asset to sell CDS_Y and holding the risky debt promising M

$$\frac{\gamma_U(h_2)(1-D)+\gamma_M(h_2)(M-D)}{1-D-\pi_C^Y} = \frac{(\gamma_U(h_2)+\gamma_M(h_2))(M)+\gamma_D(h_2)D}{\pi^M}. \tag{12}$$

Agent h_3 is indifferent between holding the risky debt j_M and buying the CDS_Y contract.

$$\frac{(\gamma_U(h_3)+\gamma_M(h_3))M+\gamma_D(h_3)D}{\pi^M} = \frac{\gamma_M(h_3)(1-M)+\gamma_D(h_3)(1-D)}{\pi_C^Y}. \tag{13}$$

Denote by η the fraction of Y used to back CDS_Y. Market clearing for the risky asset Y requires

$$\frac{(1-h_1)(1+p)}{p-\pi^M} = 1-\eta. \tag{14}$$

Market clearing for risky debt j_M requires

$$\frac{(h_2-h_3)(1+p)}{\pi^M} = \frac{(1-h_1)(1+p)}{(p-\pi^M)}. \tag{15}$$

The market clearing condition for CDS_Y is

$$\frac{h_3(1+p)}{\pi_C^Y} = (1+\eta D)\left(\frac{1}{1-D}\right). \tag{16}$$

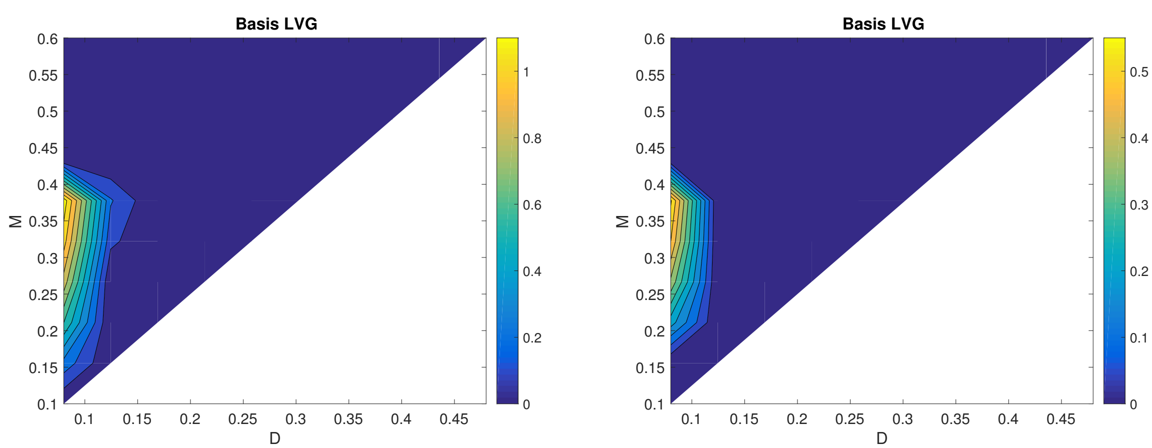
Equation (14) states that the amount of risky asset Y demanded by agents $h \in (h_1, 1)$ is equal to the amount of risky assets not backing CDS_Y. Equation (15) states that agents $h \in (h_3, h_2)$ will sell their endowment which has value $1+p$ and buy the risky debt, costing π^M for each unit; this demand must equal the amount supplied, which is created by the agents $h \in (h_1, 1)$ who sell one unit of j_M for every unit of Y they hold. Finally, Equation (16) states that agents $h \in (0, h_3)$ will sell their endowment to buy CDS_Y, which has price π_C^Y and that this demand is equal to the amount supplied in the economy—a total of $\frac{1}{(1-D)}$ units of CDS_Y are created from the one unit of X and $\frac{D}{1-D}$ units are created from the equilibrium amount η backed by Y.

Notice that we could implement this equilibrium if we let any safe asset—specifically, j_D in addition to X—be used as collateral to back CDS_Y. Whether or not Y can back CDS_Y, equilibrium would be unchanged. In equilibrium, if the basis is zero, then agents will trade Y/j_D, and every agent that buys j_D will use it as collateral to sell CDS_Y (just as they do with X). Thus, $\pi^D = D$, and the following positions will be equivalent: X/CDS_Y, Y/j_D, j_D/CDS_Y. The risky asset Y would implicitly back CDS_Y because it would be used to back safe debt which was used to back CDS_Y.

A.4.1 Leverage Constraints and Negative Bases

Before investigating how leverage limits affect the basis, we document that for almost all parameters, the basis is zero with full leverage. (Our theoretical result is simply that the basis is non-zero.) Figure 8 plots the basis in leverage economies, with beliefs parametrized by the form $\gamma_U(h) = h^\zeta$ and $\gamma_M(h) = h^\zeta(1-h^\zeta)$, when beliefs are given by $\zeta = 0.5$ and $\zeta = 1$. The parameter ζ determines the relative frequency of optimists and pessimists in the economy; equivalently, the frequency of pessimists can be interpreted as the relative demand for assets that pay in bad states (negative-beta assets), perhaps from hedging needs or risk aversion. High ζ corresponds to relatively more pessimists and low ζ to more optimists (with $\zeta > 1$, γ's are convex; $\zeta < 1$, concave).

In general the basis is zero, but as noted earlier the basis can be positive. In these cases, the risky asset Y is not used to issue CDS but is exclusively used to issue risky debt. There is a small range with a positive basis around $M = 0.3, D = 0.08$. This region grows slightly as ζ decreases, but for ζ sufficiently high (for example, $\zeta = 1.5$) the basis is always zero for all payoffs.



(a) Leverage: $\zeta = 0.5$

(b) Leverage: $\zeta = 1$

Figure 8: Comparative Statics: basis (times 100) varying payoffs M, D with leverage.

The zero-basis result emerges when Y and X have equal abilities to serve as collateral, albeit to make different promises. However, if Y is imperfect collateral, perhaps because of regulations or because financial markets have concerns arising from informational issues, then the basis will be negative. This follows because if the collateral value of Y decreases, then a negative basis emerges. Suppose that Y can be used to issue debt contracts, but the maximum promise is $\bar{\ell} < M$. That is, one unit of Y can at most back a non-contingent promise $(\bar{\ell}, \bar{\ell}, \bar{\ell})$. Furthermore, Y cannot be used to issue *CDS*.

When $\bar{\ell} \le D$, the only debt contract traded is $j_{\bar{\ell}} = \bar{\ell}$ which delivers the promised amount in every state of the world. However, because this safe debt cannot be used to issue *CDS*, it trades at a discount to X (there is a basis on the safe debt), and so $\pi^{\bar{\ell}} < \bar{\ell}$. Equilibrium in this case is ordered as follows (starting with the most optimistic): agents holding X to issue *CDS*; agents holding Y and issuing safe debt (the leverage constraint); agents holding safe debt; agents holding *CDS*. Furthermore, the basis is negative. While we have not been able to prove so, numerical examples suggest that the basis is monotonic in $\bar{\ell}$ for $\bar{\ell} < D$, with the basis more negative the tighter is the leverage constraint (lower $\bar{\ell}$).

When $D < \bar{\ell} < M$, two debt contracts are potentially traded: the safe contract $j_D = D$ and a risky contract $j_{\bar{\ell}} = \bar{\ell}$. The j_D contract delivers (D, D, D) while the $j_{\bar{\ell}}$ contract delivers $(\bar{\ell}, \bar{\ell}, D)$ because agents default in the down state. Depending on parameters, in equilibrium agents may trade the risky contract only. While we have not been able to prove so in this case, numerical results (below) suggest that in either case the leverage constraint decreases the basis.

Figure 9 plots the basis with beliefs parametrized by the form $\gamma_U(h) = h^{\zeta}$ and $\gamma_M(h) = h^{\zeta}(1 - h^{\zeta})$, with $D = 0.1$ and $M = 0.3$, solving for the basis as a function of $\bar{\ell}$ and varying the parameter ζ.

The numerical examples provide two results in addition to our propositions. First, for low ζ (corresponding to high levels of optimism or high marginal utilities in good states), the basis with leverage limits and when Y cannot be used to issue *CDS* converges to the basis without leverage limits and when Y *can* be used to issue *CDS*. In particular, in these cases the restriction that Y cannot issue *CDS* is not binding when leverage limits are relaxed (note that the basis would actually be positive in this case). In these economies, when $\bar{\ell} > D$ agents trade only risky debt in equilibrium.

However, when ζ is high (corresponding to low levels of optimism or high marginal utilities in bad states), the basis with leverage limits does not converge to the basis when Y can be used to issue *CDS*. In these cases, in equilibrium agents use Y to issue safe debt, and the basis on the asset

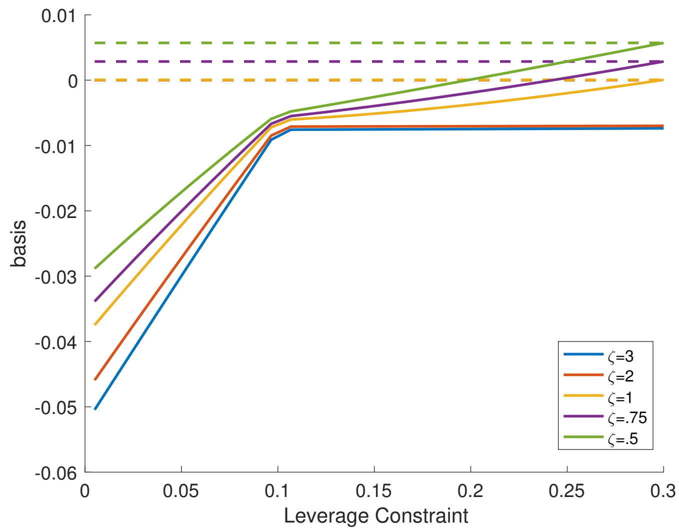
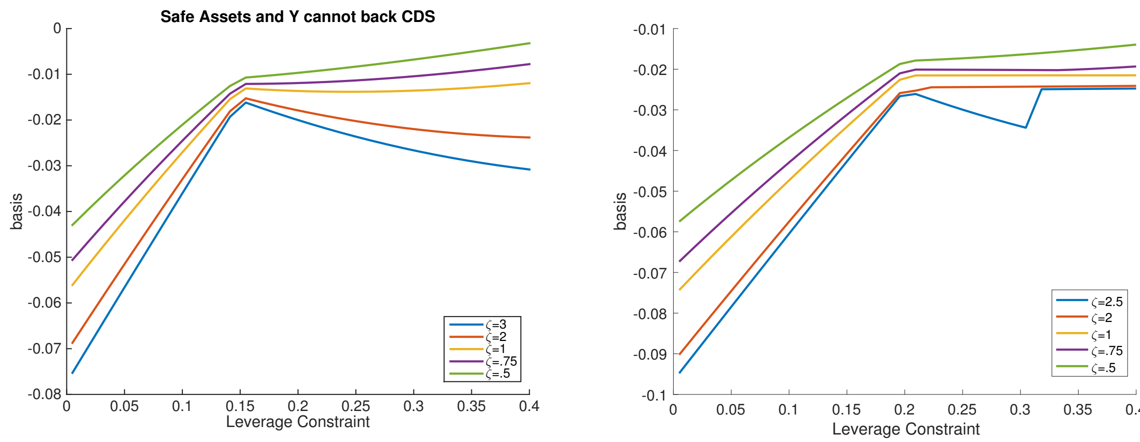


Figure 9: Leverage Constraints and the Basis. Dashed lines are the basis in an economy without leverage constraints and in which Y can be used to issue CDS.

exactly equals the basis on the safe debt .

Second, when neither safe assets nor Y can back CDS contracts, the basis need not be monotonic in $\bar{\ell}$ when $D < \bar{\ell} < M$. In particular, when the economy features a relatively high demand for risk (ζ is low, marginal utilities are high for higher states), the basis is monotonic. However, when the economy features a substantially high demand for negative-beta assets (ζ is high, marginal utilities are high for low states), the basis can decrease as $\bar{\ell}$ increases from D to M. Varying the asset payoffs emphasizes these non-monotonicity results. Figure 10 plots the effects of leverage constraints on the basis, varying ζ, for two different sets of payoffs. When in equilibrium agents do not use Y to issue safe debt, the basis decreases significantly when $\bar{\ell}$ increases beyond D. In panel (a) to the left, for $\zeta = 2,3$ agents use Y to exclusively issue risky debt. In this case, increasing the leverage limit actually decreases the basis. However, when agents use Y to issue safe debt, there is a basis on safe debt (because it cannot be used to issue CDS while X can), and the basis on the asset exactly equals the basis on the safe debt. Panel (b) to the right shows this for $\zeta = 0.75, 1, 2$, and for $\bar{\ell} > .3$ for $\zeta = 2.5$. For $\zeta = 2.5$ the equilibrium regime shifts as leverage constraints rise. For the loosest constraints, agents use Y to issue safe debt, but this is not the case for tighter constraints.



(a) $M = 0.4, D = 0.15$

(b) $M = 0.4, D = 0.2$

Figure 10: Leverage Constraints and the Basis.

A.5 Structured Finance (Pyramiding, Tranching) Economy: $C^j \in \{X, Y, j_M\}$

Given our earlier results, we can proceed to characterize equilibrium.

Corollary 5. *In the economy with debt collateralization and CDS_Y backed by X, it is cheaper to hold X/CDS_Y than Y/j_D. Thus, no agent will hold Y/j_D. That is, $(1-D) - \pi_C^Y < p - D$.*

Lemma 6. *In this economy, equilibrium consists of the following portfolio positions, ordered by investors: (1) Y/j_M, (2) $X/CDS_Y \equiv j_D^1/CDS_Y$, (3) j_M/j_D^1, and (4) CDS_Y. This characterization of equilibrium is not dependent on which assets can be used to issue CDS_Y. In fact, the equilibrium regime does not change even if we allow agents to use Y and j_M to back the CDS,*

It is clear from earlier results that the above four positions must exist in equilibrium. Figure 11 depicts the equilibrium regime. There are three marginal buyers. Arrows demonstrate the lender-borrower relationship in this economy, pointing from lenders to borrowers. Compared to the leverage economy, there is no longer a clean lending relationship, with pessimistic investors always lending to more optimistic agents. In addition to the usual lending flows, in this equilibrium we also see relatively optimistic agents (those holding the safe asset and selling CDS) lending to more pessimistic agents (those holding the risky debt contract) by buying the safe debt contract issued by the pessimists. This occurs because the safe debt issued by these pessimists can be leveraged to make an even more optimistic trade. (This is a form of financial entanglement.)

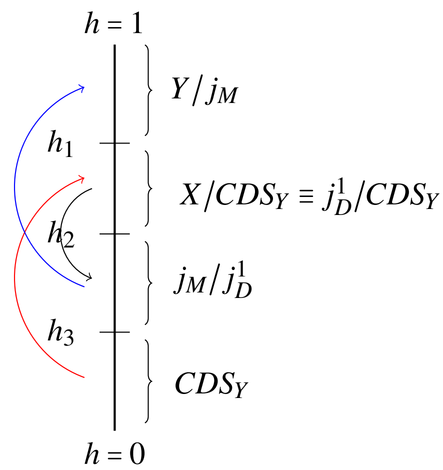


Figure 11: Equilibrium with debt collateralization and CDS_Y backed by X. Regime features financial entanglement.

The following equations define marginal investors (given by equalizing expected returns on two investment options) in the debt collateralization economy. Agent h_1 is indifferent between buying Y with leverage promising M and holding X while selling CDS_Y

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D)+\gamma_M(h_1)(M-D)}{1-D-\pi_C^Y}. \tag{17}$$

Agent h_2 is indifferent between buying X to sell CDS_Y and buying j_M with leverage D

$$\frac{\gamma_U(h_2)(1-D)+\gamma_M(h_2)(M-D)}{1-D-\pi_C^Y} = \frac{(\gamma_U(h_2)+\gamma_M(h_2))(M-D)}{\pi^M-D}. \tag{18}$$

Agent h_3 is indifferent between buying the risky debt with leverage promising D and buying the CDS

$$\frac{(\gamma_U(h_3)+\gamma_M(h_3))(M-D)}{\pi^M-D} = \frac{\gamma_M(h_3)(1-M)+\gamma_D(h_3)(1-D)}{\pi_C^Y}. \tag{19}$$

Market clearing for the safe asset X requires

$$\frac{(h_1-h_2)(1+p)}{1-\frac{\pi_C^Y}{1-D}} = 1. \tag{20}$$

Market clearing for the risky debt j_M implies that

$$\frac{(h_2 - h_3)(1+p)}{\pi^M - D} = 1. \tag{21}$$

Finally, market clearing for CDS_Y requires

$$\frac{h_3(1+p)}{\pi_C^Y} = \frac{1}{1-D}. \tag{22}$$

A.5.1 Numerical Example

While our results hold across parameters and are not quantitative, a numerical example is helpful to fix ideas. We let beliefs be $\gamma_U(h) = h$, $\gamma_M(h) = h(1-h)$, and let payoffs be $d_M^Y = 0.3$ and $d_D^Y = 0.1$. Table 2 compares equilibrium with no leverage, leverage, and debt collateralization. When debt backed by Y can be used to back further debt contracts, the basis is positive since Y now has two levels of collateralization. Our results explicitly demonstrate that the basis does not only depend on whether Y can be used as collateral—it is also intrinsically linked to the collateral value of "downstream" promises backed by Y.

Table 2: Equilibrium with No Leverage, Leverage, and Debt Collateralization

	No Leverage	Leverage	Debt Collateralization
p	0.447	0.508 ↑	0.529 ↑
π_C^Y	0.513	0.492 ↓	0.491 ↓
π^M	–	0.204	0.224↑
Basis_Y	-0.040	0 ↑	0.020 ↑

An agent in the no-leverage regime could choose to buy the cash-synthetic asset consisting of a portfolio of Y and CDS_Y—at a lower price than X while earning the same return—but this portfolio is less valuable to agents because it cannot be used as collateral to back financial contracts. Thus, the cash-synthetic asset does not provide agents the ability to isolate payoffs in a state of the world. Similarly, every investor in the debt collateralization economy could sell Y and CDS_Y to buy X at a price lower than the cash-synthetic asset. However, in equilibrium, no agent chooses to do so because the value of "downstream" contracts backed by X is lower than those backed by Y, and it is also cheaper for the agent to buy X while selling the CDS_Y contract.

In fact, a *positive* basis could emerge in a leverage economy when there is a strong demand to use Y to issue risky debt, rather than to use Y to issue CDS, which is the equivalent leveraging with safe promises. To see this, consider the following comparative static for the economy above. Redistribute wealth from agents $h < h_3$ to agents $h > h_1$. For small redistribution, the only equilibrium variable affected would be η, the fraction of Y used to back CDS_Y, and thus the supply of CDS. Taking wealth from agents $h < h_3$ would decrease demand for CDS, and increasing wealth for agents $h > h_1$ would increase demand for Y/j_M. A large enough redistribution would require $\eta = 0$, at which point marginal agents and prices would change and the basis could be positive so that agents trading X/CDS_Y would not trade Y/CDS_Y.

However, if agents could sell partially collateralized CDS, then a zero-basis would re-emerge because a issuing a partially collateralized CDS is equivalent to Y/j_M. Thus, the positive basis emerges with the restriction that CDS be fully collateralized because X is "constrained" in the set of promises it can make while Y is not. See Figure 8 for comparative statics regarding positive bases with leverage.

A.5.2 Comparative Statics and Tail Risk

We now consider how variations in the payoffs M and D affect the size of the basis in the economy with debt collateralization. Figure 12 plots the basis (multiplied by 100) with debt collateralization varying the payoffs M and D. We parameterize beliefs as before (results are qualitatively the same for other belief structures). The comparative statics provides the following main qualitative results, which are interesting testable implications for our model. With debt collateralization the basis is more positive when tail risk is larger (when D is small and M is large). Debt collateralization endogenously shifts equilibrium so that investors purchase the asset only with the riskiest contract. When M and D are very different, leveraging the asset with a safe promise is not very valuable. Since debt collateralization endogenously increases the fraction of investors issuing expensive promises to buy the asset, with substantial tail risk, the collateral value of Y substantially. Thus, variations in tail risk ought to correspond to variations in the size of the CDS basis.

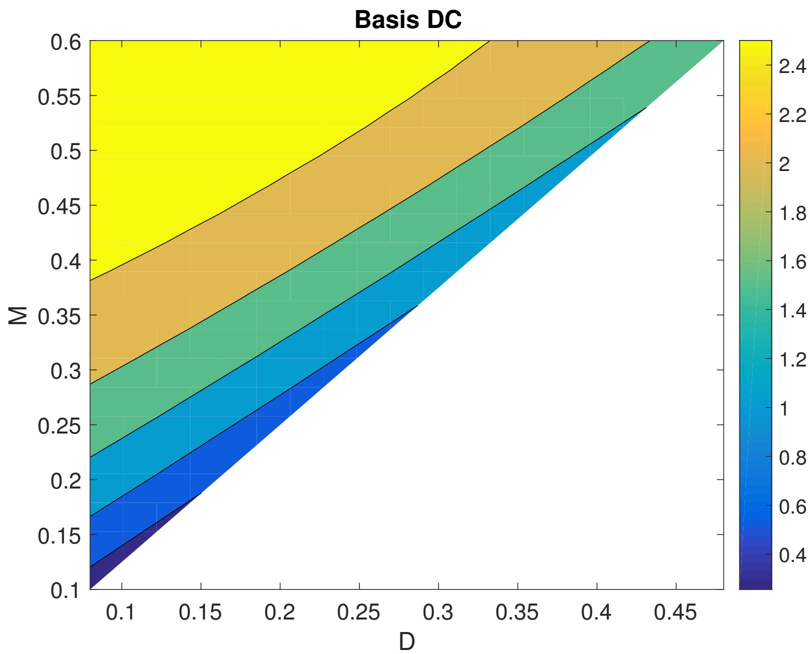


Figure 12: Comparative Statics with debt collateralization: basis (times 100) varying payoffs M, D.

A.6 Equilibrium Conditions with CDS_M and Leverage

Proposition 7. *Consider an economy with CDS contracts CDS_Y and CDS_M, which are backed by safe assets:*

1. *(Leverage) In an economy with level-0 contracts only, the basis on the risky debt is negative and the basis on the risky asset is non-negative. That is, $\pi^M + \pi_C^M < M$ and $p + \pi_C^Y \geq 1$.*

2. *(Pyramiding) In an economy with level-1 contracts, the basis on the risky debt is zero and the basis on the risky asset is positive, $\pi^M + \pi_C^M = M$ and $p + \pi_C^Y > 1$*

Marginal investors are given by equalizing expected return on two investment options. There are five marginal investors in equilibrium and they are as follows: agent h_1 is indifferent between buying Y while making the j_M promise and buying Y while making the j_D promise

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D)+\gamma_M(h_1)(M-D)}{1-D-\pi_C^Y}. \tag{23}$$

Agent h_2 is indifferent between buying X leveraged with the CDS_Y contract and buying X leveraged with the CDS_M contract

$$\frac{\gamma_U(h_2)(1-D)+\gamma_M(h_2)(M-D)}{1-D-\pi_C^Y}=\frac{(\gamma_U(h_2)+\gamma_M(h_2))(M-D)}{M-D-\pi_C^M}. \tag{24}$$

Agent h_3 is indifferent between holding X to sell the CDS_M contract and buying the risky debt j_M

$$\frac{(\gamma_U(h_3)+\gamma_M(h_3))(M-D)}{M-D-\pi_C^M}=\frac{(\gamma_U(h_3)+\gamma_M(h_3))M+\gamma_D(h_3)D}{\pi^M}. \tag{25}$$

Agent h_4 is indifferent between buying j_M debt contract and buying the CDS_Y contract

$$\frac{(\gamma_U(h_4)+\gamma_M(h_4))M+\gamma_D(h_4)D}{\pi^M}=\frac{\gamma_M(h_4)(1-M)+\gamma_D(h_4)(1-D)}{\pi_C^Y}. \tag{26}$$

Agent h_5 is indifferent between buying the CDS on the risky asset and the CDS on the risky debt.

$$\frac{\gamma_M(h_5)(1-M)+\gamma_D(h_5)(1-D)}{\pi_C^Y}=\frac{\gamma_D(h_5)(M-D)}{\pi_C^M}. \tag{27}$$

We obtain market clearing conditions by equating the supply and demand for a given asset. For any asset, agents demanding the asset will spend their endowment $(1+p)$ to buy the asset, at some price either with or without leverage. Market clearing for the safe asset X requires

$$\frac{(h_1-h_2)(1+p)}{1-\frac{\pi_C^Y}{1-D}}-\left(1-\frac{(1-h_1)(1+p)}{p-\pi^M}\right)+\frac{(h_2-h_3)(1+p)(M-D)}{M-D-\pi_C^M}=1. \tag{28}$$

Market clearing for the risky debt implies

$$\frac{(h_3-h_4)(1+p)}{\pi^M}=\frac{(1-h_1)(1+p)}{p-\pi^M}. \tag{29}$$

Market clearing for CDS_Y guarantees

$$\frac{(h_4-h_5)(1+p)}{\pi_C^Y}=\frac{(h_1-h_2)(1+p)}{1-D-\pi_C^Y}-\left(1-\frac{(1-h_1)(1+p)}{p-\pi^M}\right). \tag{30}$$

Finally, market clearing for CDS_M necessitates

$$\frac{h_5(1+p)}{\pi_C^M} = \frac{(h_2 - h_3)(1+p)}{(M - D - \pi_C^M)}. \tag{31}$$

Figure 13 illustrates the equilibrium regime with the direction of the arrow indicating the direction of funding. In general, pessimists lend to optimists in this economy. The most pessimistic agents buy the CDS_M promise from moderates, thereby lending to agents holding X/CDS_M. Agents who are slightly less pessimistic hold CDS_Y, funding those who hold X/CDS_Y. Moderates buying the risky debt contracts lend to the most optimistic agents in the economy, who are buying Y while making the j_M promise. However, financial entanglement occurs between agents who hold X/CDS_Y, Y/j_D or X/CDS_M; the safe debt contracts, j_D are being bought by agents who hold X. Thus, within (h_1, h_2), agents are (potentially) lending to each other, and agents in (h_2, h_3) are also lending to those in (h_1, h_2).

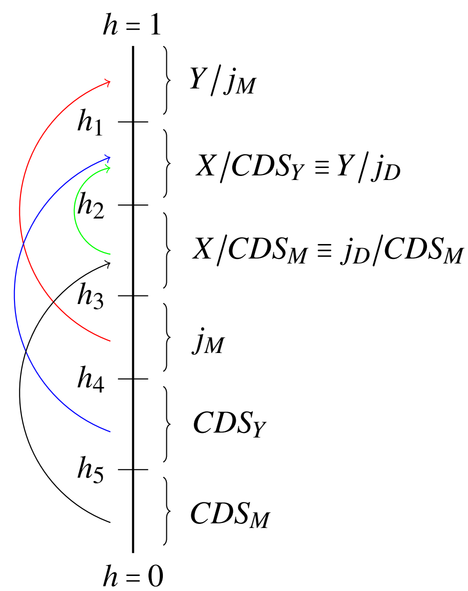


Figure 13: Equilibrium with CDS_Y and CDS_M (backed by X). No debt collateralization.

A.7 Economy with CDS_M and Pyramiding

Figure 14 depicts the equilibrium regime and shows the direction of funding between agents. The borrower-lender relationships are similar to those in the previous regime. However, agents who are buying safe assets and selling the CDS_Y contract are now lending to more pessimistic investors holding the risky debt contract with with leverage. This occurs because the safe debt issued by the moderates can be used as collateral to issue CDS_Y, which is a riskier position.

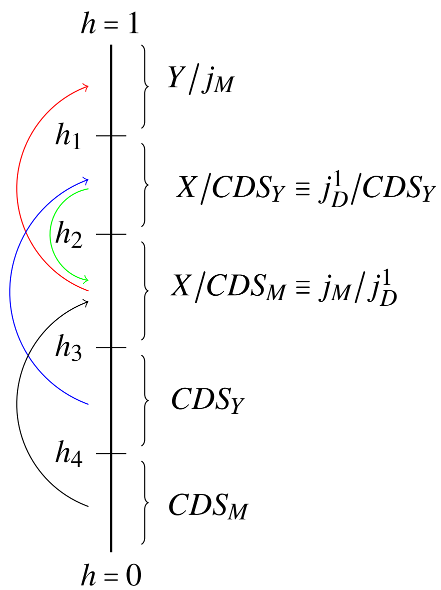


Figure 14: Equilibrium with CDS_Y, CDS_M, and Debt Collateralization.

Marginal investors

- h_1: indifferent between Y/j_M and X/CDS_Y

$$\frac{\gamma_U(h_1)(1-M)}{p-\pi^M} = \frac{\gamma_U(h_1)(1-D)+\gamma_M(h_1)(M-D)}{1-D-\pi_C^Y}$$

- h_2: indifferent between X/CDS_Y and X/CDS_M

$$\frac{\gamma_U(h_2)(1-D)+\gamma_M(h_2)(M-D)}{1-D-\pi_C^Y} = \frac{(\gamma_U(h_2)+\gamma_M(h_2))(M-D)}{M-D-\pi_C^M}$$

- h_3: indifferent between X/CDS_M and CDS_Y

$$\frac{(\gamma_U(h_3)+\gamma_M(h_3))(M-D)}{M-D-\pi_C^M} = \frac{\gamma_M(h_3)(1-M)+\gamma_D(h_3)(1-D)}{\pi_C^Y}$$

- h_4: indifferent between CDS_Y and CDS_M

$$\frac{\gamma_M(h_4)(1-M)+\gamma_D(h_4)(1-D)}{\pi_C^Y} = \frac{\gamma_D(h_4)(M-D)}{\pi_C^M}$$

Market Clearing

- Market for Y

$$\frac{(1-h_1)(1+p)}{p-\pi^M} = 1$$

- Market for CDS_Y

$$\frac{(h_3-h_4)(1+p)}{\pi_C^Y} = \frac{(h_1-h_2)(1+p)}{1-D-\pi_C^Y}$$

- Market for CDS_M

$$\frac{h_4(1+p)}{\pi_C^M} = \left(1+D-\frac{(h_1-h_2)(1+p)(1-D)}{1-D-\pi_C^Y}\right)\left(\frac{1}{M-D}\right)$$

- Market for X and j_M

$$\frac{(h_1-h_2)(1+p)(1-D)}{1-D-\pi_C^Y} + \frac{(h_2-h_3)(1+p)(M-D)}{M-D-\pi_C^M} = 1+M$$

A.7.1 Numerical Example

Table 3 compares the prices and bases in the CDS_M regime with leverage and the CDS_M regime with equilibrium. The price of the risky asset increases because debt backed by Y can now serve as collateral. The price of risky debt increases because agents can now buy the debt with leverage, increasing demand for risky debt. Furthermore, allowing j_M to serve as collateral for non-contingent debt contracts increases the supply of safe assets in the economy. Since safe assets are used to issue both CDS_Y and CDS_M, the supply of both these CDS contracts increase, resulting in a lower π_C^Y and π_C^M.

A.7.2 Double Basis in Four-State Economy

While in the 3-state economy Basis$_M$ can never be positive because j_M can be collateralized at most once, we can obtain a positive basis on both the risky debt in a four-state model in which

Table 3: Double-Basis Equilibrium with Leverage and Debt Collateralization

	Leverage	Collateralization
p	0.502	0.527 ↑
π^M	0.196	0.223 ↑
π_C^Y	0.498	0.491 ↓
π_C^M	0.090	0.077 ↓
Basis_Y	0	0.018 ↑
Basis_M	-0.014	0 ↑

downstream debt contracts can be used to back multiple layers of debt. See Gong and Phelan (2019) for a theoretical characterization of debt collateralization with $N > 3$ states.

The setup is as before, but now the set of states is given by by $S = (0, S_1, S_2, S_3, S_4)$, where $s = 0$ is the initial state of the world at time $t = 0$. Let the payout of the risky asset Y be $(1, s_2, s_3, s_4)$ in states (S_1, S_2, S_3, S_4), where $1 > s_2 > s_3 > s_4$. Let j_i be the debt contract promising s_i, and let the price of j_i be π^i. We set $s_2 = 0.5,\ s_3 = 0.3,\ s_4 = 0.1$, and we let beliefs be given by $\gamma_4(h) = (1-h)^3$, $\gamma_3(h) = h(1-h)^2$, $\gamma_2(h) = h^2(1-h)$, $\gamma_1(h) = 1 - \gamma_4(h) - \gamma_3(h) - \gamma_2(h)$, which preserves the properties in the three-state model.

Let there be full debt collateralization in the economy, and let there be a CDS on Y (with price π_C^Y) and a CDS on j_2 (with price π_C^2). We let Basis_α denote the basis on the asset α. In equilibrium, $p = 0.585$, $\pi^2 = 0.339$, $\pi^3 = 0.228$, $\pi_C^Y = 0.431$, $\pi_C^2 = 0.169$, $\text{Basis}_Y = 0.016$, $\text{Basis}_{j_2} = 0.009$, and we see a positive basis on both the risky asset and the risky debt.

A.8 Equilibrium when Y and j_D cannot serve as collateral for CDS

Let the set of financial contracts in the economy be given by $J = J^X \cup J^Y$, where J^X consists of CDS_Y backed by X and J^Y consists of non-contingent debt contracts. Note that we no longer allow j_D to back CDS_Y. By Proposition 3, it must be the case that $\pi^D < D$ or no one will want to buy the safe debt. We define the basis on j_D, denoted Basis_D, to be $D - \pi^D = \text{Basis}_D$. Equilibrium features four marginal buyers, $h_1 > h_2 > h_3 > h_4$. All agent $h > h_1$ will hold Y/j_M. Agents $h \in (h_2, h_1)$ will hold a combination of X/CDS_Y and Y/j_D (or just X/CDS_Y if it is cheaper). $h \in (h_3, h_2)$ will sell their endowments to buy j_M and $h \in (h_4, h_3)$ will buy j_D instead. Finally, $h < h_4$ will hold only CDS_Y. Furthermore, we see a double basis in this case—one on the risky asset and one on the *safe* *debt*. Additionally, when j_D is traded, the basis for Y must be the same as the basis on j_D because

$$p - \pi^D = 1 - D - \pi_C^Y \implies 1 - p - \pi_C^Y = D - \pi^D \implies \text{Basis}_Y = \text{Basis}_D.$$

Note that j_D is not always traded in this equilibrium. Specifically, for low enough values of M, no agent strictly prefers to buy the safe debt. The intuition here is that a lower M raises increases the payout of CDS_Y in the M state, making the CDS a more attractive option for moderate agents who wish to isolate payoffs in state M.

A.9 Economies with Short Selling

Thus far we have been silent about the possibility of short sales. One could understandably worry that, given the literature on limits to arbitrage, ignoring short selling would be a central driver of our results. We now show that this is not the case. In this section we provide agents the ability to sell short Y and we show that in general agents will *not* choose to do so. The intuition for our result is that to bet against Y, a collateral-efficient strategy is to buy CDS (requiring no collateral) rather than to sell short the asset.

In addition to letting agents trade debt and CDS, now let agents also be allowed to issue a contract promising $(1, M, D)$, which we call a Y-promise. This Y-promise is collateralized by 1 unit of X and costs π^Y_{short}. Note that buying X and issuing a Y-promise is a collateralized short position in Y, which costs $1 - \pi^Y_{short}$ and delivers $(0, 1 - M, 1 - D)$, which is exactly the payoff to a CDS. Thus, agents can bet against Y by either buying CDS or by shorting Y. However, a unit of X can issue more CDS than Y-promise: one CDS is backed by $1 - D$ units of X as collateral while selling Y-promise requires one unit of X. This is precisely what we mean when we say that buying the CDS to bet against Y is collateral efficient.[13]

We now reinforce our previous results by showing that our results hold even when short sales are allowed.

[13] An alternative modeling strategy follows Bottazzi et al. (2012) by explicitly requiring agents to borrow the asset Y at a funding cost in order to sell it short in the market. This "box constraint" is how short sales are done in reality. They show that a binding box constraint leads to a liquidity premium (bonds are special in repo), increasing the cost of shorting. Our setup will deliver a similar result—the Y-promise may trade at a discount to Y, implying that shorting Y entails a funding cost.

Proposition 8. *In an economy with short sales, suppose that agents can use X to issue Y-promises, but these promises cannot be used as collateral.*

1. *(Shorting with no leverage) If Y cannot be be used as collateral, then in equilibrium, agents do not issue Y-promises and the basis is negative.*

2. *(Shorting with leverage) If y can be used as collateral to issue debt contracts (but these debt contracts cannot serve as collateral), then in equilibrium, the basis on Y is non-negative, as it was without short sales.*

3. *(Shorting with debt collateralization) If Y can be used as collateral to issue debt, and these debt contracts can also be used as collateral, then in equilibrium the basis on Y is strictly positive.*

In all of these cases, it is important to note that more optimistic agents will always be willing to use X as collateral for CDS because this position isolates payoffs in the U and M states. So the CDS on Y is always traded.

In case 1 with no leverage, since neither Y nor Y-promises can be used as collateral, investors are indifferent between buying Y or the Y-promise. If the Y-promise is traded in equilibrium it must be that $\pi^Y_{short} = p$. Since buying X and issuing a Y-promise delivers the same payoffs as buying a CDS, a Y-promise will be issued in equilibrium only if $\pi^Y_C = 1 - \pi^Y_{short}$, implying that $p + \pi^Y_C = 1$—that is, the basis is zero. But, we have already shown that the basis is strictly negative when X can issue CDS and Y cannot be leveraged since X has higher collateral value (the proof of 6 still holds with short-selling). This contradiction implies that in equilibrium, no agent will trade the Y-promise. The intuition for the result is immediate: when Y cannot be used as collateral, the basis is negative (Y is cheap) and so investors do not want to sell short the already-cheap asset, but those who wish to bet against it do so by buying CDS.

In case 2 with leverage, because Y can be used as collateral while the Y-promise cannot, it must be that $\pi^Y_{short} \leq p$ if the Y-promise is traded. Suppose that short sales do occur in equilibrium. As we just argued, agents are only willing to issue Y-promises (to short Y) if the basis is non-negative since a negative basis implies it is cheaper to buy CDS. Thus, the presence of short-sales imply a non-negative basis. In particular, the equilibrium regime would feature a set of agents buying Y promises with these agents lying between those using X to issue CDS and those buying the risky debt. Even if short sales do not occur, then the equilibrium regime is exactly as discussed in the previous section so the basis is non-negative. The result in case 3 with debt collateralization follows from the same argument.

The restriction that Y-promises cannot be completely collateralized as the underlying asset can reflect either (i) direct limitations in borrowing underlying assets to short or (ii) the fact that assets that are used in CDOs or other structured securities cannot be replicated frictionlessly to be used in these same structures. (Technically, the result holds when the Y-promise can be collateral but the debt backed by the Y-promise cannot be, implying that a risky promise backed by the Y-promise would be different from the risky promise backed by Y.) These restrictions are empirically relevant given the assets we have in mind (corporate bonds, mortgage- and asset-backed securities, etc.).

B Additional Proofs

Proof of Proposition 6. We prove for each case.

Case 1, No leverage: From Lemma 3, the position X/CDS_Y must be traded in equilibrium, otherwise no agent will hold X. Thus X/CDS_Y cannot be more expensive than Y/CDS_Y. Hence, it must be that $1-D-\pi_C^Y \leq (1-D)p - D\pi_C^Y$, which simplifies to $1 \leq \pi_C^Y + p$. In order for any agent to hold j_D, which offers the same payoff as X but which cannot be used as collateral, it must be that $\pi^D < D$ in equilibrium. But since $\pi_C^Y + p \geq 1$, then $p - \pi^D > (1-D)p - D\pi_C^Y$, which means that agents would strictly prefer to use Y to issue CDS rather than to issue debt.

Case 2, Leverage and limits on Y: Consider the agent h who is indifferent between holding X/CDS_Y and holding Y. For h, $\mathbb{E}_h[X/CDSY] = \mathbb{E}_h[Y]$, thus

$$\frac{\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D}{p} = \frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1-D-\pi_C^Y}. \tag{32}$$

Furthermore, this agent is relatively optimistic and strictly prefers both of these two options to holding the safe asset, X. It follows that

$$\frac{\gamma_U(h)(1-D)+\gamma_M(h)(M-D)}{1-D-\pi}>1. \tag{33}$$

Rearranging and simplifying Equation 32, we have that

$$p+\pi_C^Y=(1-D)+\frac{D(1-D-\pi)}{\gamma_U(h)(1-D)+\gamma_M(h)(M-D)}.$$

Combining this with 33, $p+\pi_C^Y<(1-D)+D=1 \implies p+\pi_C^Y<1$.

Case 3, Leverage, no limits on Y: First, suppose $\bar{\ell}=D$. Then in equilibrium investors must be indifferent between X/CDS and Y/j_D (the alternative is investors will hold Y without leverage, in which case the basis is negative per earlier results). Since the payoffs to these positions are the same, the costs of these positions are the same, $1-D-\pi_C^Y=p-\pi_D$, implying the basis is π_D-D, which is negative since the safe debt cannot be used as collateral while X can. Note that if $D<\bar{\ell}<M$ and safe debt is issued in equilibrium, then the same argument implies the basis is negative.

If $\bar{\ell}<D$ then investors are ordered X/CDS, $Y/j_{\bar{\ell}}$,$j_{\bar{\ell}}$, CDS. The position $Y/j_{\bar{\ell}}$ pays $(1-D+D-\bar{\ell}, M-D+D-\bar{\ell}, D-\bar{\ell})$. This position can be replicated using X/CDS and buying $\frac{D-\bar{\ell}}{\bar{\ell}}$ units of $j_{\bar{\ell}}$. Note that the investor indifferent between $Y/j_{\bar{\ell}}$ and $j_{\bar{\ell}}$ is indifferent between buying and selling $j_{\bar{\ell}}$, but strictly prefers $Y/j_{\bar{\ell}}$ over X/CDS. Thus, the position must be cheaper:

$$1-D+\pi_C^Y+\frac{D-\bar{\ell}}{\bar{\ell}}\pi^{\bar{\ell}}>p-\pi^{\bar{\ell}}.$$

Since $\bar{\ell}<D$ and $\pi^{\bar{\ell}}<\bar{\ell}$, $D\left(\frac{\pi^{\bar{\ell}}}{\bar{\ell}}-1\right)<0$, and the basis satisfies

$$p+\pi_C^Y<1-D+D\left(\frac{\pi^{\bar{\ell}}}{\bar{\ell}}-1\right)<1.$$

□

Proof of Lemma 3. Suppose some agent strictly prefers to hold only the safe asset X without selling any financial contracts. Let $\mathbb{E}_h[a]$ denote the expected return on holding the position a.

Then there exists some agent h such that $\mathbb{E}_h[X] > \mathbb{E}_h[X/CDS]$. This implies that:

$$1 > \frac{\gamma_U(h) + \gamma_M(h)\left(\frac{M-D}{1-D}\right)}{1 - \frac{\pi_C^Y}{1-D}} \implies (1-D) - \pi_C^Y \; > \; \gamma_U(h)(1-D) + \gamma_M(h)(M-D). \tag{34}$$

Additionally, since h strictly prefers to hold X, it must be the case that $\mathbb{E}_h[X] > \mathbb{E}_h[CDS_Y]$, implying

$$1 > \frac{\gamma_M(h)(1-M) + \gamma_D(h)(1-D)}{\pi_C^Y} \implies \pi_C^Y > \gamma_M(h)(1-M) + \gamma_D(h)(1-D). \tag{35}$$

Note that adding together equations 34 and 35 implies the following contradiction:

$$(1-D) > (1-D)(\gamma_U(h) + \gamma_M(h) + \gamma_D(h)) \implies (1-D) > (1-D).$$

Thus, no agent ever prefers to hold X. By risk neutrality, it is also follows that any agent who choses to sell CDS will sell as many units of CDS as they can. To see that no agent is willing to hold the cash-synthetic asset, suppose for contradiction that some agent h, strictly prefers the cash-synthetic asset. That is, $\mathbb{E}_h[Y + CDS_Y] > \mathbb{E}_h[Y]$ Then,

$$\frac{1}{p + \pi_C^Y} > \frac{\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D}{p} \implies p > (p + \pi_C^Y)(\gamma_U(h) + \gamma_M(h)M + \gamma_D(h)D). \tag{36}$$

Additionally, we must also have , $\mathbb{E}_h[Y + CDS_Y] > \mathbb{E}_h[CDS_Y]$, which means

$$\frac{1}{p + \pi_C^Y} > \frac{\gamma_M(h)(1-M) + \gamma_D(h)(1-D)}{\pi_C^Y} \implies p > (p + \pi_C^Y)(\gamma_M(h)(1-M) + \gamma_D(h)(1-D)). \tag{37}$$

Combining the above two inequalities yields the following contradiction:

$$p + \pi_C^Y > (p + \pi_C^Y)(\gamma_U(h) + \gamma_M(h) + \gamma_D(h)) \implies p + \pi_C^Y > p + \pi_C^Y.$$

□

Proof of Lemma 5. First note that because the minimum payout of Y is D and the maximum payout of CDS_Y is $1-D$, each unit of Y can back $\frac{D}{1-D}$ units of CDS_Y. The payoff of buying one unit of Y and selling $\frac{D}{1-D}$ units of CDS_Y (holding Y/CDS_Y) is $\left(1, \frac{M-D}{1-D}, 0\right)$ in states (U, M, D). However, this return is equivalent to holding Y and selling j_D, so the choice-set of agents has not been increased by this financial innovation.

Now consider when j_D could also be used to back CDS_Y. Without letting agents use Y to issue CDS_Y, agents holding Y were still able to do this indirectly by selling the promising j_D. One unit of j_D can back $\frac{D}{1-D}$ units of CDS_Y, which is the exactly the amount issued when agents holding Y issue CDS_Y directly. In short, the leverage equilibrium, as we have characterized, does not depend on which assets can back CDS_Y. □

Proof of Corollary 5. From previous theorem, we have that $p + \pi_C^Y > 1 \implies p > 1 - \pi_C^Y \implies p - D > (1-D) - \pi_C^Y$. Note that $1 - D - \pi_C^Y$ is the cost of holding X/CDS_Y while $p - D$ is the cost of holding Y/j_D. Thus, all agents will choose the cheaper option and hold X while selling the CDS_Y contract. □

Proof of Lemma 6. There are two parts to this proof. First we will show that no one holds Y/CDS_Y. Second, we will show that no agent strictly prefers to hold j_M/CDS_Y. Note that we know that $1 - D - \pi_C^Y < p - D$ and hence $p + \pi_C^Y > 1$. Then,

$$\implies (p + \pi_C^Y)(1-D) > 1 - D \implies p - \frac{D}{1-D}\pi_C^Y > 1 - \frac{1}{D}\pi_C^Y.$$

So, the cost of holding Y/CDS_Y is higher than the cost of holding X/CDS_Y even though these two positions have equivalent returns. Thus, no agent will choose to hold Y/CDS_Y.

Now, suppose for contradiction that there is an agent, h who strictly prefers to hold j_M/CDS_Y. This means that for investor h, the expected return of j_M/CDS_Y must be greater than the return of X/CDS_Y. Thus,

$$\frac{\gamma_U(h)M(1-D) + \gamma_M(h)(M-D)}{(1-D)\pi^M - D\pi_C^Y} > \frac{\gamma_U(h)(1-D) + \gamma_M(h)(M-D)}{1 - D - \pi_C^Y}. \tag{38}$$

Rearranging this equation and simplifying, we obtain

$$\gamma_U(h)M(1-D) + \gamma_M(h)(M-D) > \tag{39}$$

$$\pi^M\big(\gamma_U(h)(1-D) + \gamma_M(h)(M-D)\big) + \pi_C^Y(M-D)\big(\gamma_U(h) + \gamma_M(h)\big). \tag{40}$$

Since h strictly prefers j_M/CDS_Y, the expected payout of this position must also be higher than the expected payout of holding j_M/j_D^1. So,

$$\frac{\gamma_U(h)M(1-D)+\gamma_M(h)(M-D)}{(1-D)\pi^M - D\pi_C^Y} > \frac{\gamma_U(h)(1-D)+\gamma_M(h)(M-D)}{1-D-\pi_C^Y}. \tag{41}$$

Rearranging and simplifying the above, we obtain

$$-\big(\gamma_U(h)M(1-D)+\gamma_M(h)(M-D)\big) > \tag{42}$$

$$-\pi^M\big(\gamma_U(h)(1-D)+\gamma_M(h)(M-D)\big) - \pi_C^Y(M-D)\big(\gamma_U(h)+\gamma_M(h)\big). \tag{43}$$

Combining Equations (40) and (43) yields $0 > 0$, a contradiction. So, there does not exist a set of agents with positive measure who strictly prefer to sell CDS_Y backed by j_M. □

Proof of Proposition 7. We prove the result in the three-state economy for each case.

Case 1, Leverage. Consider the agent who is indifferent between holding X/CDS_M and j_M. Since this agent is relatively optimistic, the expected return of both of these two options must be greater than 1. Then, we have that $\mathbb{E}_h[X/CDS_M] = \mathbb{E}_h[j_M] > 1$.

$$\frac{(\gamma_U(h)+\gamma_M(h))(M-D)}{M-D-\pi_C^M} = \frac{(\gamma_U(h)+\gamma_M(h))M + \gamma_D(h)D}{\pi^M} > 1. \tag{44}$$

Rearranging and simplifying 44, we obtain

$$(\pi^M+\pi_C^M)(\gamma_U(h)+\gamma_M(h))(M-D) = D(M-D-\pi_C^M) + (M-D)^2(\gamma_U(h)+\gamma_M(h))$$

$$\implies \pi^M+\pi_C^M = \frac{D(M-D-\pi_C^M)}{(\gamma_U(h)+\gamma_M(h))(M-D)} + M - D. \tag{45}$$

Combining the above with Equation 44, it follows that $\pi^M + \pi_C^M < M$.

Case 2, Debt Collateralization. Because X/CDS_M is equivalent to j_M/j_D^1, any equilibrium in this economy must feature a zero basis on j_M ($\text{Basis}_M = 0$). A positive basis, $\text{Basis}_M > 0$ would imply that j_M/j_D^1 is expensive relative to X/CDS_M and no agent would want to buy j_M. This is not an equilibrium because optimists who want to isolate payoffs in state U would be willing to sell j_M at a lower price, driving the basis toward 0. A negative basis, $\text{Basis}_M < 0$ is not an equilibrium because this implies j_M/j_D^1 is cheap relative to X/CDS_M and CDS_M is never issued as a result. However, extreme pessimists who want to isolate payoffs in state D and would therefore be willing to buy CDS_M even at a higher price, driving the basis toward 0.

□

C Empirical Test Using Markit CDX Inclusion/Exclusion

This section describes in greater detail the empirical test of our theory using inclusion and exclusion in Markit CDX indices.

C.1 The logistics of index inclusion/exclusion and index tranching

The HY (IG) index is composed of 100 (125) liquid North American entities with high yield (investment grade) credit ratings that trade in the CDS market. There are two roll dates every year for both the HY and IG indices, once in March and once in September. The IG index rolls out on September 20 (March 20) and the HY index rolls out on September 27 (March 27). When the 20th or the 27th falls on a non-trading day, the IG and HY indices are rolled out on the trading day closest to the 20th and 27th, respectively. Prior to a new index being rolled out, Markit releases information about which CDS contracts are added to or removed from the CDX index and Markit keeps publicly available records of these announcements from 2013-2017, as well as a finalized list of the CDS basket for each roll.

All CDS contracts in the basket are equally weighted, though Markit does have target sector-specific weights for the composition of each index. Markit publishes the list of entities removed or added to the indices around a week before the roll date. Entities are removed from the index if any of the following conditions are satisfied

1. There is a corporate event (i.e. merger or acquisition).
2. There is a credit event–the bond matures, is called, or is defaulted upon.
3. For the HY index, the debt outstanding of the entity falls below a certain level; for the IG index, the debt outstanding rises above a certain level.
4. The credit default swap no longer meets the liquidity requirement.
5. The target sector weights in the index are not met; or

6. There is a change in the relevant credit rating. For example, a formerly investment-grade corporation that gets demoted to high-yield would be removed from the IG index to the HY index and vice versa.

Entities are added to the index if

1. the CDS satisfies the liquidity requirement
2. The referenced corporation meets the required amount of debt outstanding.

Entities are also added or removed from an index based on the results of a dealer poll conducted amongst institutions that frequently trade these indices. Furthermore, the HY and the IG index are tranched separately and the tranches are shown in figure 15.

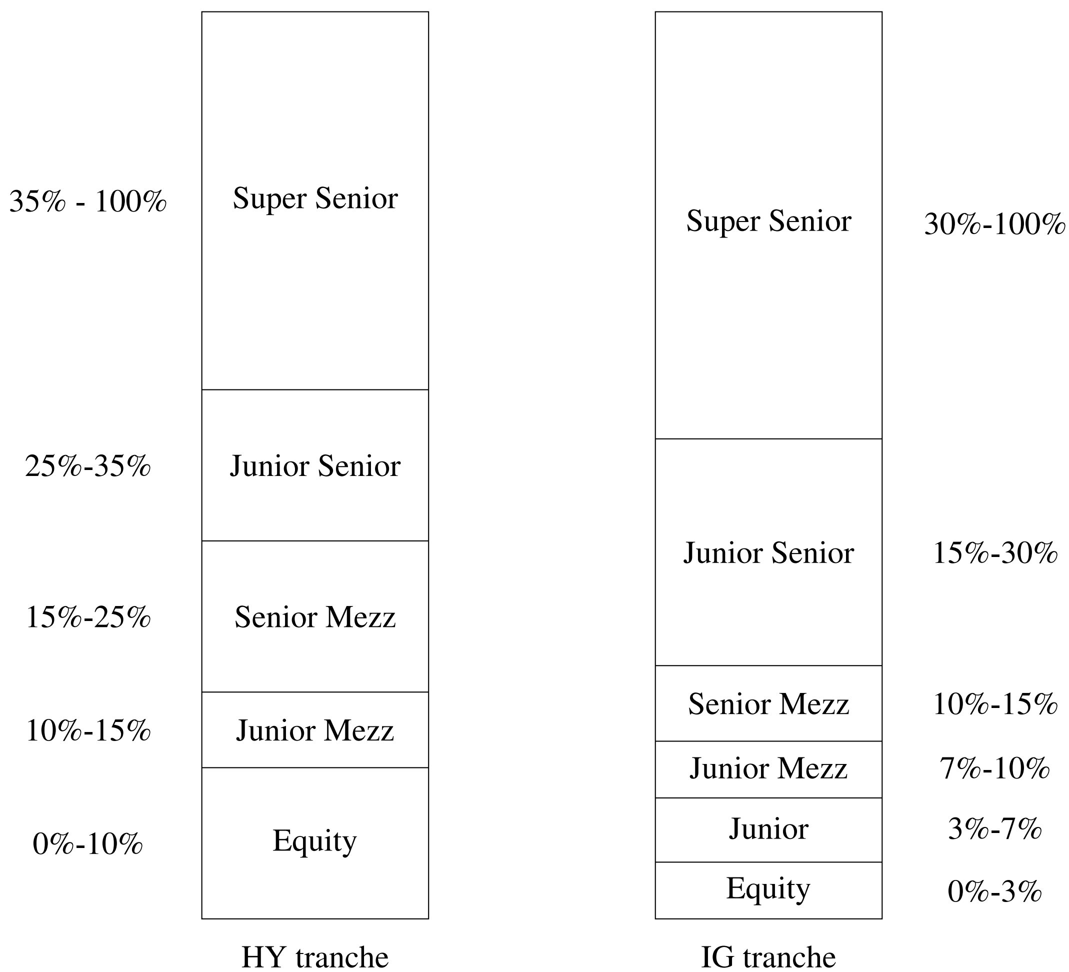


Figure 15: Tranches for HY and IG indices

C.2 Data

Based on Markit's publicly available record of changes to the CDX.NA.HY index and CDX.NA.IG index from March 2013 to September 2017, we compiled a list of CDS contracts that were included, excluded, or stayed on the index. We exclude contracts that were removed from an index due to a credit event. We obtained end-of-day mid CDS bases from Bloomberg, where the mid CDS basis is defined as the CDS spread minus the Z-spread (zero-volatility spread) for a fixed rate cash bond of the same issuer and maturity. The z-spread takes into account the full term structure of the benchmark swap curve and is defined as the spread that must be added to a give benchmark zero swap curve so that the sum of the bond's discounted cash flows equals its price, with each cash flow discounted at its own rate. As such, the Z-spread is a reasonably realistic valuation of a bond's cash flows. We obtained CDS data for all entities that were excluded or included from a roll and a random subsample of entities that were unaffected by the roll.

We group the CDS data into 5 different status categories: included, excluded, updated (removed), updated (added), and remain. Included and excluded observations are self-explanatory, though the sample sizes for these two groups are rather small (approximately 50 observations in each group) because most rolls do not involve substantial changes to the index. A referenced bond has a status of "updated" if the firm has issued another a newer bond with maturity closer to five years. Markit prefers to select newer bonds with a maturity of around five years because five-year CDS contracts are the most commonly traded and are the most liquid on the market.

C.3 Estimation Method

We use a difference-in-difference approach to estimate the change in the CDS-bond basis for credit default swaps that are added to or removed from either index over a two-day window, both around the time of announcement and around the time of index roll. We use the end-of-day CDS bond-basis the day before the announcement and the day of the announcement (as well as the day before roll and the day of roll). The baseline regression examines percentage changes for the CDS-bond basis of CDS contracts that were added, removed, or unaffected by the announcement/roll and does not include observations that were updated.

We use percent changes because different bonds exhibit a great degree of heterogeneity in the magnitude of the CDS-bond basis. In our sample, the largest bases (in absolute value) was over 1000 basis points while the smallest observed basis was 0.5 basis points. CDS contracts with large bases typically were much more volatile in levels, changing by over 200 basis points between trading days. Because of this, running a regression on levels introduces a lot of noise into the estimation. The normalization procedure is as follows: Letting basis_0 and basis_1 be the observed basis before and after an announcement (or roll). We define

$$\widehat{\text{basis}_0} = 1, \qquad \widehat{\text{basis}_1} = \frac{\text{basis}_1 - \text{basis}_0}{\frac{|\text{basis}_1| + |\text{basis}_0|}{2}}$$

so that the normalized basis before treatment is equal to 1, and the the after-treatment observation is the percentage change in the basis. To calculate the percentage change in the basis, we divide by the mean of the absolute values of the pre- and post-announcement basis for several reasons

1. Dividing by the absolute value of the pre-announcement (roll) basis or post-announcement (roll) basis introduces systematic bias into the regression. Consider the case when added entities tend to have both positive bases and and experience a increase in basis while removed entities tend to have negative bases and experience a decrease in the basis. Dividing by the pre-announcement basis would upward bias the estimated coefficients while dividing by the post-announcement basis biases the estimates toward zero.

2. We use the mean of the absolute values because there are several observations for which the basis switches signs over the observation period. This is problematic if the mean of the basis is close to zero.[14]

The regression equation is given by

$$\text{basis}_{ist} = \beta_0 + \beta_1 \lambda_t + \beta_2 \gamma_a + \beta_3 \gamma_r + \beta_4 \lambda_t \cdot \gamma_a + \beta_5 \lambda_t \cdot \gamma_r + \varepsilon_{ist} \tag{46}$$

Here, basis_{ist} is the basis for CDS i with status s at time t. λ_t is an indicator variable that takes a value of 0 before the announcement (or roll) and a value of 1 after the announcement/roll. The regression includes dummy variables γ_a and γ_r that takes values of 1 if the CDS is in the added or

[14]An example: a CDS basis switches from -5 to 5.25 during the window of observation. Taking a simple mean mean would inflate the actual change in basis.

removed group respectively. In this equation, β_1 is interpreted as the time fixed effect and β_2 and β_3 are the status-group fixed effects for entities that are added or removed from an index, respectively. β_4 is the difference-in-difference estimator that provides the level change in the CDS-bond basis around the time of the announcement (or roll) for entities added to an index *relative* to entities that were unaffected. β_5 is the analogous estimator for entities that were removed from an index relative to swaps that are unaffected. Margin-based asset pricing predicts that $\beta_4 > 0$ and $\beta_5 < 0$. In order for β_4 and β_5 to be interpreted as the *causal* effect of the implied margin change on the CDS-bond basis, it must be the case that nothing else changed in the included or excluded group at the time of the announcement (or roll) that did not change for the unaffected group.

We rerun the baseline regression including controls for index switching and year-month fixed effects.

$$\text{basis}_{ist} = \beta_0 + \beta_1\lambda_t + \beta_2\gamma_a + \beta_3\gamma_r + \beta_4\lambda_t \cdot \gamma_a + \beta_5\lambda_t \cdot \gamma_r + \beta_6\delta_{HY} + \beta_7\delta IG + \alpha T + \varepsilon_{ist} \tag{47}$$

δ_{HY} is an indicator variable that takes a value of 1 if the entity switched from the IG index to the HY index. β_7 is the analogous variable indicating whether an entity switched from HY to IG. T is a matrix of dummy variables to estimate month-year effects. The results of the two regressions are reported in the next section.

Here, the biggest threat to identification is that the announcement of inclusion or exclusion from an index conveys some information about the bond/CDS or for some other reason causes market participants to treat the bond/CDS contracts differently. Theoretically, index inclusion does not reveal *new* information about the future prospects of the newly included firm, since the requirements of index inclusion or exclusion are published by Markit and publicly available to all market participants. However, a number of papers have found that inclusion in an equity index is associated with improved stock prices (see Harris and Gurel (1986), Shleifer (1986), Denis et al. (2003), Chen et al. (2004), and Wurgler and Zhuravskaya (2002)). For equities, this change in price can be explained by, among other hypotheses, increased monitoring by investors after inclusion which leads to more effort by the firm's management; increased demand for the equity pushing up prices from funds that track the index; or greater reputation costs if management performs poorly. However, because trading CDS contracts is fundamentally different from trading equities, these hypotheses do not apply to the CDX index. Investors in CDS buy protection against a firm credit default and there is no monitoring incentive involved and funds do not track CDX indices.

This does not exclude the possibility of a behavioral response by market participants when an entity is added to or removed from the index. To eliminate differential changes in behavioral reactions for the included, excluded, and unaffected groups, we run a difference-in-difference-in-difference regression on CDS entities that were added to or removed rom the HY and IG indices.

$$\text{basis}_{ist} = \beta_0 + \beta_1\lambda_t + \beta_2\gamma_a + \beta_3\kappa_I + \beta_4\lambda_t\gamma_a + \beta_5\lambda_t\kappa_I + \beta_6\gamma_a\kappa_I + \beta_7\lambda_t\gamma_a\kappa_I \tag{48}$$

As before, λ_t takes a value of 1 if the observation is post announcement (or roll). γ_a is 1 when the observation was added to an index and 0 if it is removed. κ_I indicates which index the entity was added to or removed from and takes a value of 1 when the affected index is the HY index. β_7 in the above equation is the triple difference estimator–it is the difference in relative changes (inclusion relative to removal) in the CDS basis between entities that were added to the HY index versus the IG index. While there might be differences in behavioral response for entities that are added rather than removed, it is harder to come up with plausible explanations for there being significant difference for swaps added to the HY index rather than the IG since the only fundamental difference between the two indices is the credit rating of the firm. As credit rating is known publicly before the announcement, the difference in credit rating cannot explain why $\beta_7 \neq 0$. Margin-based asset pricing, however, does predict that $\beta_7 \neq 0$ precisely because the cash flows and tranching of the two indices are different. The result is reported in Table 5.

C.4 Empirical Results

As shown in table 1, we find that inclusion into an index is associated with an 18% increase in the CDS bond basis at the time of announcement while removal has no significant effects on the CDS bond basis around the announcement. There are no significant changes for either group around the time of the roll, which suggests that the market has already adjusted by the time of the roll. The lack of a significant coefficient on removal could be due to the fact that the "removed" sample is much smaller because many entities removed after experiencing a credit event.

To better compare included observations from excluded ones, we run a difference-in-difference regression comparing only included entities to excluded entities with results reported in Table 4.[15] we find that the difference in the change of bases between these two groups is significant–addition relative to removal is associated with a 26% increase in the CDS-bond basis. This estimate matches the differences in the point estimates in Table 4.

Table 4: Difference in difference estimations comparing inclusion to exclusion. The last two specifications include controls for the month and year, as well as indicators for whether the entity switched indices. The month and year controls are not shown in the table.

	Dependent variable: Normalized CDS basis (percent changes)			
	announcement	roll	announcement	roll
	(1)	(2)	(3)	(4)
time	−0.046	0.091	−0.046	0.091
	(0.066)	(0.060)	(0.066)	(0.060)
added	−0.000	−0.000	0.004	−0.030
	(0.066)	(0.054)	(0.068)	(0.055)
switch to HY			0.123*	0.038
			(0.070)	(0.075)
switch to IG			−0.021	−0.112
			(0.089)	(0.087)
time*added	0.259***	0.007	0.259***	0.007
	(0.094)	(0.077)	(0.093)	(0.077)
Observations	214	486	214	486
R^2	0.081	0.013	0.132	0.038
Adjusted R^2	0.068	0.007	0.071	0.009
Note:			*p<0.1; **p<0.05; ***p<0.01	

C.5 Alternative hypotheses

It is possible that CDS contracts that are added to an index become more liquid as a result of inclusion, and the increase in the liquidity premium increases only the CDS spread and not the bond spread. Figure 3 and 4 show the trading volume of HY and IG CDS contracts with the roll

[15] To increase sample size, we also include "updated" entities. The entities included in the new roll are in the inclusion group while the entities that are phased out are in the exclusion group.

dates highlighted. Trade volumes spike on the roll date of the index, but this increase in trade volume is temporary. There is no significant increase in trade volume around the time of the announcement. The result from the previous section finds that there is no significant change in the CDS-bond basis around the roll date, and this suggests that liquidity is not the driving force behind changes in bases.

To eliminate confounding variables that arise from behavioral responses by market participants, we look at a triple difference estimation following Equation (48). The only difference between addition to the HY index and addition to the IG index is the credit rating of the firm, which is publicly known well before the announcement or roll dates. As such, if different market reactions to addition versus removal resulted in changes to the CDS-bond bases, such reactions should not be present in the triple-difference regression. Table 5 shows that the change in the CDS basis for added entities relative to removed entities is 41 percent higher when the entity was added to the HY index rather than the IG index. This result implies that the implicit margin for a CDS contract in the HY index is much lower than the margin for contracts in the IG index, likely because the HY index is composed of fewer entities, so that each individual entity accounts for a greater percentage of the index's cash flows.

Table 5: Triple-difference estimations comparing the effect of inclusion versus exclusion for the HY index relative to the IG index. The first two specifications looks at the percent change in the CDS-bond basis around the time of the announcement and roll. The last two specifications include controls for the month and year, as well as indicators for whether the entity switched indices. The month and year controls are not shown in the table.

	Dependent variable: Normalized CDS basis (percent changes)			
	announcement	roll	announcement	roll
	(1)	(2)	(3)	(4)
time	−0.046	0.221***	−0.046	0.221***
	(0.091)	(0.076)	(0.091)	(0.076)
added	0.000	−0.000	0.028	−0.011
	(0.092)	(0.070)	(0.096)	(0.071)
HY	0.000	0.000	0.022	0.027
	(0.090)	(0.084)	(0.099)	(0.086)
Switch to HY			0.084	0.048
			(0.077)	(0.076)
Switch to IG			−0.011	−0.099
			(0.086)	(0.085)
announced×HY	0.0002	−0.317***	0.0002	−0.317***
	(0.128)	(0.118)	(0.128)	(0.118)
announced×added	0.038	0.009	0.038	0.009
	(0.130)	(0.099)	(0.130)	(0.099)
added×HY	−0.000	0.000	−0.028	−0.046
	(0.127)	(0.107)	(0.141)	(0.111)
announced×added×HY	0.412**	0.029	0.412**	0.029
	(0.180)	(0.151)	(0.180)	(0.151)
Observations	214	486	214	486
R^2	0.166	0.078	0.209	0.102
Adjusted R^2	0.138	0.064	0.136	0.067
Note:			*p<0.1; **p<0.05; ***p<0.01	