Collateralized Borrowing and Increasing Risk

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Abstract

This paper uses a general equilibrium model with collateralized borrowing to show that increases in risk can have ambiguous effects on leverage, loan margins, loan amounts, and asset prices. Increasing risk about future payoffs and endowments can lead to riskier loans with larger balances and lower spreads even when lenders are risk-averse and borrowers can default. As well, increasing the covariance of either agents' endowments with the asset payoff can have ambiguous consequences for equilibrium. Though the effects are ambiguous, key determinants of how increased risk translate into changes in prices and allocations are the correlation of agents' endowments with the asset payoff, agents' risk aversion, and the location of increased risk in the distribution of future states. Some restricted changes in the borrower's or lender's endowments can have unambiguous but asymmetric effects on equilibrium.

Keywords: Leverage, risk, collateral constraints, asset prices

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1 Introduction

Economists have observed that leverage, at the security level and at the institutional level, is pro-cyclical and that the *VIX*, a common measure of market volatility is counter-cyclical.[2] Counter-cyclical volatility is commonly understood as the explanation for counter-cyclical margins for collateralized loans: agents can borrow less against an asset when economic outcomes become more risky. In fact, it is almost taken as axiomatic that asset prices, loan levels, and collateral levels fall when risk increases. This paper asks, how tight is the theoretical link between increased risk and margins, asset prices, and leverage?

This paper provides a simple framework with two agents and collateralized borrowing to demonstrate that risk ambiguously affects equilibrium margins, leverage, loan levels, and asset prices. I consider two types or risk: aggregate risk and endowment risk. Aggregate risk refers to changes in the distribution of future states, which affects the distribution of the asset payoff as well as agents' endowments. Endowment risk refers to changing the the distribution of agents' endowments while fixing the asset payoff and distribution. Increasing endowment risk has the effect of increasing the covariance between endowments and the asset payoff. I characterize how equilibrium margins, leverage, loan levels, and asset prices change in response to these increases in risk.

The main result of this paper is that increases in risk of either type can increase or decrease margins, prices, and loan amounts. However, some precise predictions can be made conditional on knowing the risk aversion of agents, the correlation of future endowments with the asset, and the location of risk in the distribution. In some instances results depend on the expected default rate for collateralized loans. These results imply that we may observe very different margin responses across markets (for example, comparing investment grade versus subprime mortgages).

There are two intuitions for these results. First, when an asset is used for collateral, the asset's payoff is "split" between the borrower and lender: the borrower receives the asset when there is no default, and the lender receives the asset when the borrower defaults. Thus, each

[2] see, e.g., Adrian and Shin (2010, 2011)

agent "prices" the asset but in different states. Second, effects will differ depending on the concavity of agents' pricing kernels in the states in which they price the risky asset and debt contracts. Agents' pricing kernels are determined by the covariance of their endowments with the asset and their degrees of risk aversion. Furthermore, because agents' pricing kernels are endogenous, increases in risk of either type can endogenously affect how agents price assets and contracts in equilibrium. Thus, the results stem from incorporating information about agents' stochastic discount factors, and not simply from exploiting the "threshold property" of equilibria with debt contracts.

One of the key results is that increasing the covariance of endowments with the asset payoff can have ambiguous effects on equilibrium loan levels, margins, and asset prices. This is perhaps surprising because the insight from the C-CAPM literature is that a higher covariance should decrease asset prices. The insight from this model is that with collateralized borrowing, different agents price assets in different states, and so increasing covariance need not necessarily affect asset prices in the usual way. In other words, increase aggregate risk or increased covariance, while having unambiguous implications in standard models, have ambiguous implications in models with collateralized borrowing.

Many papers have tried to endogenize this link in general equilibrium models, and some restrictive models deliver counter-cyclical leverage. Because it is generally difficult to prove results in general equilibrium models with incomplete markets and collateral, the literature has typically considered two types of simplifications: binomial economies, or risk-neutral agents with belief disagreements. With binomial economies, all equilibrium traded contracts are essentially default-free, which makes it easy to characterize equilibrium. In this direction, Fostel and Geanakoplos (2012, 2013) study how equilibrium leverage and margins depend on risk about the future. These papers build off of Geanakoplos (1997, 2003, 2010), whose work developed the general equilibrium analysis of collateralized lending and asset prices. These papers provide a theoretical framework in which leverage falls when risk about asset payoffs increases.[3] When "scary-bad news" hits the economy, leverage falls as tomor-

[3]Fostel and Geanakoplos (2012) prove the link between risk and leverage in a special class of binomial economies; however, Fostel and Geanakoplos (2013) show that this link is not general, and in binomial

row's bad payoff, which is the amount borrowers promise to repay, is lower. These models produce countercyclical margins by restricting payoffs be binary. However, this restriction prohibits analyzing what happens to default rates and equilibrium contracts because loans are always default free. While we do not need default to understand how risk ambiguously effects margins and leverage, once default arises, precise (and interesting) predications can be established. One may wish to relax the no-default restriction for other reasons as well.

Other papers, such as Simsek (2013) and Geerolf (2014) consider many states with risk-neutral traders. In Simsek (2013), trade arises because agents disagree about the distribution of the asset's payoff, and the key result is that the nature of belief disagreements matter more than just the level of disagreements. What he calls "upside" optimism leads to higher leverage and asset prices, while "downside" optimism is disciplined by tight margin requirements and therefore lower leverage and prices. He calls this result the "asymmetric disciplining of optimism." Geerolf (2014) uses a continuum of risk-neutral traders with heterogeneous "point-mass" beliefs to characterize the distribution of margins and contracts traded in equilibrium.

In my model both agents hold common, objective beliefs about the future, but agents may differ in their preference toward risk or in their distribution of future endowments; hence, agents trade because future marginal utilities differ. My paper shows how increased objective aggregate risk, affecting payoffs and endowments for both agents, affects equilibrium.[4] I also am able to consider the equilibrium consequences of changing the covariance between endowments and the asset, something that would be irrelevant in a model with risk-neutral agents.

My paper also relates to a large theoretical literature that concerns the effect of borrowing constraints on asset prices, such as Kiyotaki and Moore (1997) and Fostel and Geanakoplos (2008). Many papers with financial frictions directly assume the link between risk and margins in their models (see, e.g., Brunnermeier and Pedersen (2009) and Adrian and Bo-

economies what seems to matter for leverage is tail risk and not volatility.

[4]An implication of this analysis is that some of the asset-pricing results in Simsek (2013) are not robust to changing the lenders' risk aversion, taking as given the effects on debt riskiness which *do* follow from his analysis.

yarchenko (2012)). In Gorton and Ordoñez (2014), a deleveraging looks like a reduction in the number of available collateral in the economy, which induces an increase in cross sectional dispersion. The model produces a correlation between risk and deleveraging, but the causality is different from what I propose in this paper. Toda (2014) considers a general equilibrium model with collateralized borrowing and shows that in his setup agents always use maximum leverage. Araujo et al. (2012) study the effect of default and collateral on risk sharing. Other papers that study repo, collateral, and borrow constraints include Gorton and Metrick (2012) and Gârleanu and Pedersen (2011).

2 The Model

The economy has two periods, $t = \{0, 1\}$. There is a continuum of possible states of nature in period 1, denoted by $s \in S = [0, \bar{s}]$. Denote the probability distribution function by $f(s)$, which is positive and continuous over S, and denote the cumulative distribution by $F(s)$.

There is a single consumption good X, which is perishable, and a risky asset Y, which produces dividends of the consumption good at time 1 and is in unit supply. In state s the asset pays s, so that the state corresponds to the asset payoff.[5] The risky asset Y has price p in period 0.

There are two types of agents, A and B, and there is a measure 1 of each type of agent. In period 0 agents are endowed with e_0^A and e_0^B units of the consumption good X. As well, agent A is endowed with one unit of the risky asset Y and B has no endowment of Y, denoted by $y_0^A = 1, y_0^B = 0$. Allocating endowments in this way simplifies the analysis, but it is not critical. What is important is that agent B does not have the full asset. In period 1 agents are endowed with e_s^A and e_s^B units of X in state s.[6]

The von-Neumann-Morgenstern expected utility to agent i over consumption of X is given by

$$U^i(x_0, x_s) = x_0 + \mathbb{E}[u_i(x_s)]. \tag{1}$$

[5] The risky asset is a Lucas tree.

[6] Equivalently, we can let e_s^i for $i = A, B$ be the certainty-equivalent of the distribution of endowments in state s.

Both agents have linear utility over period-0 consumption. Over period-1 consumption, agents have C^2 utility functions that are increasing and (weakly) risk-averse; $u_i'(x) > 0$, and $u_i''(x) \leq 0$.

Financial Contracts and Collateral

The financial contracts available for agents to trade at time 0 are collateralized debt contracts. Without loss of generality, the set of contracts is restricted to a set containing contracts using exactly one unit of Y as collateral. The set of contracts is denoted by J and a contract is defined by the promised face value $j \in J$ to be repaid in period 1.

Because collateral is the only enforcement mechanism, agents default (repay less than j) whenever the value of the collateral is less than the value of the promise. Therefore, because the payoff of Y is s in state s, the actual payoff to contract j in state s is $\min\{j, s\}$.

The sale of a contract corresponds to borrowing the sale price, and the purchase of a promise is tantamount to lending the price in return for the promise. I denote the sale of promise j by $\varphi_j > 0$ (borrowing), and the purchase of the same contract by $\theta_j > 0$ (lending). The significance of the collateral is that the borrower must own the collateral at time 0 in order to make the promise j. The sale of $\varphi_j > 0$ units of contract type $j \in J$ requires the ownership of φ_j units of Y, whereas the purchase of θ_j number of contracts does not require any ownership of Y.

Each contract $j \in J$ trades for a price $q(j)$. An investor can borrow $q(j)$ today by selling contract j in exchange for a promise of j tomorrow, provided the investor owns the collateral.

Budget Constraint

Given asset and contract prices at time 0, $(p, (q(j))_{j \in J})$, each agent i chooses consumption of X in each state, denoted by $x_0 \geq 0$ and $x_s \geq 0$, positions in the risky asset Y, denoted by $y \geq 0$, and contract trades $\theta_j \geq 0, \varphi_j \geq 0$ in state 0, in order to maximize utility (1) subject to the budget set defined by

$$B^i(p,(q(j))_{j\in J}) = \{(x_0,y,\theta,\varphi,x_s)\} \in R_+ \times R_+ \times R_+^J \times R_+^J \times R_+ :$$

$$py^i + x_0^i + \int_{j\in J} \theta_j^i q(j)dj \leq e_0^i + py_0^i + \int_{j\in J} \varphi_j^i q(j)dj. \tag{2}$$

$$x_s^i + \int_{j\in J} \varphi_j^i \min\{j,s\}dj \leq e_s^i + y^i s + \int_{j\in J} \theta_j^i \min\{j,s\}dj, \tag{3}$$

$$\int_{j\in J} \varphi_j^i dj \leq y^i \}. \tag{4}$$

The budget set $B^i(p,(q(j))_{j\in J}) = \{(x_0,y,\theta,\varphi,x_s)\}$, is the set of consumption decisions and portfolios that satisfy budget constraints in period 0 and 1 and the collateral constraint in period 0. Equation (2) is the budget constraint for agent i in period 0, equation (3) is the state-s budget constraint, and equation (4) is the collateral constraint, which applies only to borrowing.

At time 0 expenditures on the assets purchased (or sold) can be at most equal to the money borrowed selling contracts using the assets as collateral. The assets put up as collateral must indeed be owned. In the final states, consumption must equal dividends of the assets held minus debt repayment. Notice that short selling of assets is not possible.

Agents choose portfolios in $B^i(p,(q(j))_{j\in J})$ to maximize expected utility. In other words, the problem for each agent is to solve

$$\begin{aligned} \max \quad & x_0 + \mathbb{E}[u_i(x_s)] \\ \text{s.t. } & (x_0,y,\theta,\varphi,x_s) \in B^i(p,(q(j))_{j\in J}) \end{aligned}$$

Equilibrium

A *Competitive Collateral Equilibrium* in this economy is a price of asset Y, contract prices, asset purchases, contract trade and consumption decisions by all the agents such that markets for the consumption good in all states clear, assets and promises clear in equilibrium at time 0, and agents optimize their utility given their budget sets.

Definition 1 (**Competitive Collateral Equilibrium (CE)**). *A Competitive Collateral Equilibrium is a collection of prices* $(p, (q(j))_{j\in J})$ *and allocations* $(x_0^i, y^i, \theta^i, \varphi^i, x_s^i)_{i\in\{A,B\}}$ *such that agents' positions solve their problems, and markets clear:*

$$y^A + y^B = 1,$$
$$\varphi_j^A + \varphi_j^B = \theta_j^A + \theta_j^B \quad \forall j \in J,$$
$$x_0^A + x_0^B = e_0^A + e_0^B,$$
$$x_s^A + x_s^B = e_s^A + e_s^B + s \quad \forall s \in S.$$
$$(x_0^i, y^i, \theta^i, \varphi^i, x_s^i) \in B^i(p, (q(j))_{j\in J}), \forall i$$
$$(x_0, y, \theta, \varphi, x_s) \in B^i(p, (q(j))_{j\in J}) \Rightarrow U^i(x) \leq U^i(x^i), \forall i$$

3 Characterizing Equilibrium

Because the purpose of the model is to study collateralized borrowing, I restrict preferences and endowments so that agent B leverages her endowment to buy the risky asset on margin using risky debt (default must be possible). The following conditions are sufficient for equilibrium to take this form. The first ensures that agent B values consumption in high-payoff states more than agent A does, and the second condition ensures that agent B is not very wealthy. For expositional ease, the first condition is stronger than necessary (but also less technical than the weaker condition required, which is stated in the Appendix).

Condition 1 (Marginal Utilities): Agents' marginal utilities satisfy the following:

$$u_A'(e_s^A) \quad \text{is stricly decreasing for } s \in S, \tag{C1a}$$
$$u_B'(e_s^B + s) \quad \text{is weakly increasing for } s \in S, \tag{C1b}$$
$$\exists s^* \in (0, \bar{s}) \text{ s.t. } \frac{u_B'(e_{s^*}^B + s^*)}{u_A'(e_{s^*}^A)} = 1. \tag{C1c}$$

This set of conditions captures two key features: differences in risk aversion, and/or differences in covariances between endowments and the asset.[7] Condition (C1c) implies that B is the natural buyer of the asset because she values consumption in high-payoff states comparatively more than A does.[8] Notice that conditions (C1a) and (C1b) imply

$$u''_A(e^A_s)\frac{de^A_s}{ds} < 0, \text{ and } \quad u''_B(e^B_s + s)\left(\frac{de^B_s}{ds} + 1\right) \geq 0. \tag{5}$$

Agent A is risk-averse and her endowments increase with the asset payoff. Agent B can either be risk-averse with endowments that decrease with the asset payoff, or she can be risk-neutral without any condition on her endowments. As stated earlier, these conditions are stronger than necessary. What is required is some combination of either (1) for agent A to be sufficiently more risk-averse than agent B or (2) for agents to have the same preferences but for their endowments to covary differently. This insight, of course, follows directly from the asset pricing literature, which plays a central role in the Capital Asset Pricing Model as well as its more modern variants. Naturally, combinations of differences in risk aversion and endowment covariances will satisfy Condition 1.

Condition 2 (Endowments): Given s^* defined above,

$$e^B_0 < \frac{\mathbb{E}\left[u'_A(e^A_s + s)|s \geq s^*\right]}{\mathbb{E}\left[u'_B(e^B_s)|s \geq s^*\right]}\mathbb{E}[u'_B(e^B_s + s)\max(s - s^*, 0)]. \tag{C2}$$

This condition ensures that agent B is not so wealthy that she can buy the asset outright, but she must use all of her wealth with risky debt to buy the asset. As a result, the collateral constraint binds for B.[9]

Given these conditions, in equilibrium agents trade a single contract $j = \gamma$. Agent A sells the asset and buys contract γ, while agent B uses all her wealth (does not consume in period

[7]Notice that because agents are weakly risk-averse, condition (C1a) implies that $u'_A(e^A_s + s)$ is also strictly decreasing, and condition (C1b) implies that $u'_B(e^B_s)$ is weakly increasing.

[8]By conditions (C1a) and (C1b), this ratio is strictly greater than 1 for any $s > s^*$, particularly if agent B consumes less than $e^B_s + s$ and if A consumes more than e^A_s, which will be true in equilibrium.

[9]If (C2) did not hold, then agent B would be sufficiently wealthy so that her leveraged return would equal one and the collateral constraint would not bind. In this case, equilibrium traded contract and asset price would be determined by setting the leveraged return in equation (8) equal to one, which, given the quasi-linear preferences, is the marginal utility of consumption at time 0 when consumption is non-negative.

0) to buy the asset on margin by selling contract γ. A single contract is traded in equilibrium because marginal utilities differ uniformly across states (Condition 1): if any contract $j \neq \gamma$ were priced so that one agent would be willing to trade it, the other agent would not be willing to. Since agents have linear utility over x_0, the marginal utility of wealth for agent A is 1, and the marginal utility of wealth for agent B will be greater than 1 (equal to her leveraged return).

Two conditions determine the asset price p and the equilibrium contract γ: market clearing for the asset, and an optimality condition for the contract. The rest of this section presents the key equilibrium equations. The Appendix contains the full derivation of equilibrium conditions.[10] Given an equilibrium contract γ and the equilibrium form described above, agents' consumptions are given by $x_s^A = e_s^A + \min(s, \gamma), x_s^B = e_s^B + \max(s - \gamma, 0)$. For notational ease, define the stochastic discount factors by

$$\mu_s^A = u'_A(x_s^A), \quad \mu_s^B = u'_B(x_s^B).$$

Naturally, these marginal utilities are endogenous and depend on γ.

Optimality Conditions

Since agent A consumes at time 0 and has linear utility over x_0, agent A has marginal utility of wealth equal to one. Therefore, if agent A buys contract j

$$q(j) = \int_0^j s\mu_s^A dF(s) + j \int_j^{\bar{s}} \mu_s^A dF(s) = \mathbb{E}[\mu_s^A \min(s, j)], \tag{6}$$

which is a standard asset-pricing equation using first-order conditions.[11]

Next, let $R_B(j)$ be the leveraged return agent B gets from purchasing the risky asset

[10]Because agents marginal utilities in each state are endogenous—they depend on the endogenous portfolio allocations—solving for the competitive equilibrium allocation is not equivalent to solving a bargaining problem or a principal-agent economy in which one agent prices contracts and the other chooses which contract to trade. This result was formally proved in earlier versions of the paper.

[11]Notice that the assumption of linear preferences of x_0 is not critical but simplifies the results. By assuming linearity we essentially rule out any income effects for asset pricing. One could relax this restriction and replace it with bounds on e_0 to achieve the same results throughout the paper.

and selling contract j. The expected marginal return delivered from an asset bought against contract j is $\int_j^{\bar{s}} \mu_s^B (s-j) dF(s)$. (The asset bought on margin delivers $s-j$, and only in states $s \geq j$, and these payments are valued using agent B's marginal utility.) Because the downpayment, or margin, associated with buying the asset against contract j is $p-q(j)$, the leveraged return is thus

$$R_B(j) = \frac{\int_j^{\bar{s}} \mu_s^B (s-j) dF(s)}{p-q(j)} = \frac{\mathbb{E}[\mu_s^B \max(s-j,0)]}{p-q(j)}. \tag{7}$$

Each contract delivers a different leveraged return. Making a larger promise (using a larger j) allows agent B to borrow more against each unit of the asset, since $q(j)$ is increasing in j, and thus B can get more leverage and buy more assets. However, a larger promise means that agent B gets a smaller payment whenever the asset pays off $s > j$—with a higher j, agent B receives a net dividend in fewer states and receives a smaller payment in those states.

There is a unique contract $j = \gamma$ that maximizes the leveraged return for agent B. The intuition is that the contract prices $q(j)$ are priced using agent A's marginal utilities, while agent B calculates the expected marginal return using her own marginal utilities. By Condition 1, the ratio of marginal utilities is increasing, which means that increasing j beyond a certain point does not decrease the downpayment fast enough to make up for the foregone return. The contract γ exactly balances making the largest promise possible with receiving the largest expected value of payments from the asset.

The equilibrium traded contract satisfies

$$R_B(\gamma)\mathbb{E}\left[\mu_s^A | s \geq \gamma\right] = \mathbb{E}\left[\mu_s^B | s \geq \gamma\right] \implies R_B(\gamma) = \frac{\int_\gamma^{\bar{s}} \mu_s^B dF(s)}{\int_\gamma^{\bar{s}} \mu_s^A dF(s)}. \tag{8}$$

The intuition for equation (8) is as follows. If agent B promises an additional ϵ, agent A's utility improves by μ_s^A in state $s \geq \gamma$ for a total increase of $\int_\gamma^{\bar{s}} \mu_s^A dF(s)$. As a result, agent B can increase her borrowing by this amount, since agent A is willing to give an additional $\int_\gamma^{\bar{s}} \mu_s^A dF(s)$ in period-0 for the larger promise. Agent B can leverage that amount to receive a marginal return of $R_B(\gamma)$. Thus, the total benefit to agent B is $R_B(\gamma) \int_\gamma^{\bar{s}} \mu_s^A dF(s) =$

$R_B(\gamma)\mathbb{E}\left[\mu_s^A|s \geq \gamma\right](1 - F(\gamma))$. The marginal cost to agent B of promising an additional ϵ is the lost utility in states $s \geq \gamma$, i.e., $\int_\gamma^{\bar{s}} \mu_s^B dF(s) = \int_\gamma^{\bar{s}} \mu_s^B dF(s) = \mathbb{E}\left[\mu_s^B|s \geq \gamma\right](1 - F(\gamma))$. Combining these conditions so that marginal cost equals marginal benefit yields the result. Given Conditions 1 and 2, $\gamma > s^*$ and therefore the leveraged return $R_B(\gamma) > 1$, and thus the collateral constraint binds.[12]

An alternative interpretation is that agents are effectively trading upper and lower "tranches" corresponding to the asset payoffs above and below γ, where the collateral constraint implicitly defines the positions agents can take in the upper tranche. Thus, the intuition is that the numerator is the valuation of the lower tranche sold by the borrower to the lender, and the denominator is the valuation of the upper tranche sold by the lender to the borrower. The previous intuition goes through: the return B gets from buying an up tranche by selling a down tranche satisfies equation (8) for the optimal tranches.

We can implicitly solve for the asset price that satisfies equation (8) for any given contract j. Using the definition of $R_b(j)$ and solving for p defines:

$$p^{opt}(j) - q(j) = \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\int_j^{\bar{s}} \mu_s^B dF(s)} \int_j^{\bar{s}} \mu_s^B (s - j) dF(s) + q(j). \tag{9}$$

Equation (9) defines the asset price p that makes contract j the optimal contract. This is the first equilibrium condition.

Market Clearing

The second equilibrium condition is that the asset market clears. Since agent B does not consume in period 0,[13] we can use her budget constraint together with the collateral constraint to define her asset holdings as a function of the debt price and the asset price. Using the equation for the debt price, market clearing requires

$$p^{mc}(j) = e_0^B + \mathbb{E}[\mu_s^A \min(s, j)]. \tag{10}$$

[12]This optimality condition has an analog in Simsek (2013), but the difference is that the integrals are over *endogenous* values.

[13]Because her leveraged return is greater than the marginal utility of consumption in period 0.

Agent B will purchase the one risky asset, which costs p, using her wealth e_0^B and proceeds from borrowing. Since there is one asset to use as collateral, and since the collateral constraint binds, she will borrow one unit of contract j, yielding $q(j)$ to spend in period 0.

In fact, the interpretation that agents trade upside and downside tranches is precisely why B can buy the entire asset even though she is constrained and agents' marginal utilities are endogenous. By Condition 1, B always values consumption in high-payoff states relatively more than A does. By buying the asset and making a very large promise (trading a high γ debt contract), B effectively buys the asset *only in upside states*, while A buys the asset in downside states (via default). Thus, B's low wealth and collateral constraint affects the states in which she buys the asset rather than the quantity of assets she buys.

The equilibrium price p clears the asset market for the optimally chosen debt contract j, which occurs when $p^{mc}(j) = p^{opt}(j)$ (satisfying (9) and (10)). Thus we can define $z(j) = p^{opt}(j) - p^{mc}(j)$, which is analogous to excess demand:

$$z(j) = \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\int_j^{\bar{s}} \mu_s^B dF(s)} \int_j^{\bar{s}} \mu_s^B (s-j) dF(s) - e_0^B. \tag{11}$$

By Conditions 1 and 2, $z(j)$ is strictly decreasing and has a unique zero in $(s^*, \bar{s})$.

Equilibrium Margins and Leverage

The margin is the fraction of the asset price that must be paid for using wealth—it is the downpayment. Leverage is the inverse of the margin—it is how many units of an asset each unit of wealth buys. Denote the margin associated with debt contract γ as $m_\gamma = \frac{p-q(\gamma)}{p}$. Leverage for contract γ is $L_\gamma = 1/m_\gamma$. Since in equilibrium $p = q(\gamma) + e_0^B$, we know that $m_\gamma = \frac{e_0^B}{p}$ and $L_\gamma = \frac{p}{e_0^B}$. That is leverage increases with asset prices, and margins decrease. Since $p = q(\gamma) + e_0^B$, the asset price, margin, and leverage are entirely determined by the debt price $q(\gamma)$.

4 Increased Risk

This section considers two sources of risk: aggregate risk, and covariance between endowments and the asset. The first considers changes in the distribution of aggregate states $f(s)$, which affects the distribution of both the asset payoff and of agents' endowments. Second, I consider changes in the distribution of an agent's endowments in period 1, which determines the covariance between an agent's endowments and the asset payoff.

I use a mean-preserving spread (MPS) to denote increased risk. As is well-known, if Z is a random variable and $\widetilde{Z}$ is an MPS of Z, then $\widetilde{Z}$ can be defined as Z plus a mean-zero noise term, and this is a stronger condition than an increase in variance. Thus a mean-preserving spread is a desirable criterion for measuring risk.[14] I model an increase in aggregate risk as an MPS of $f(s)$, and I model an increase in endowment risk as an MPS of the distribution of future endowments over s. An MPS of endowments implies an increase in the covariance between endowments and the asset payoff.

There are three classes of results that all imply ambiguous effects of increases in risk, either aggregate or endowment. First, even when changes in risk do not change the equilibrium debt contract γ (or holding γ fixed as a partial-equilibrium exercise), the effect on asset prices depends on the concavity of agents' pricing kernels and the location of the MPS. The concavity (or convexity) of each agent's pricing kernel depends on the interaction between risk aversion and the distribution of endowments. However, because agents' preferences and endowments can in general take any shape (subject to Condition 1), there is no unconditional prediction about how risk affects prices and margins in equilibrium.

Second, changes in risk can ambiguously affect the equilibrium contract γ: the face value of the equilibrium debt contract γ can either increase or decrease as a result of increases in risk. This result is reminiscent of the result in Simsek (2013) that margins change in

[14] Formally, a distribution $\widetilde{F}$ differs from F by a mean-preserving spread if

1. $\int z d\widetilde{F}(z) = \int z dF(z)$
2. $\int_0^y \widetilde{F}(z)dz \geq \int_0^y F(z)dz$

That is, $\varepsilon = \widetilde{f} - f$ is a mean-preserving spread.

response to upside or downside belief disagreements (that the type of disagreement matters more than the level)—and there is a close correspondence with my results concerning changes in agents' endowment risk. Nonetheless, the result that increases in aggregate risk, which is a fundamental affecting both agents' valuations, is not an immediate implication of the result about changing types of belief disagreements.

Third, given how the equilibrium debt contract changes, the effect on how pricing kernels change in equilibrium depends fundamentally on agent's degree of risk aversion. Thus, even for a precise prediction in how the equilibrium contract γ changes, the effects on prices and margins will depend on the lender's risk aversion in particular.

4.1 Aggregate Risk

I first state the results for aggregate risk, modifying $f(s)$, and then revisit endowment risk/covariance later this section.

Expectations and Pricing Kernels

The first result is that even when changes in risk do not change the equilibrium debt contract γ, the effect on asset prices depends on the concavity of agents' pricing kernels (their marginal utilities in each state, which are influenced by risk aversion and endowments). A change in the distribution $f(s)$ will affect debt prices for all contracts. Define $Q_\gamma(s) = \mu_s^A \min(s, \gamma)$. Then the price of debt contract γ can be written as $q(\gamma) = \mathbb{E}[Q_\gamma(s)]$. That is, $Q_\gamma(s)$ is the kernel used to price the debt payments, agent A's marginal utility times the debt payment. If $Q_\gamma(s)$ is concave, then the expectation will decrease after applying an MPS; it if is convex it will increase after an MPS. Generally, we cannot say anything about this effect because $Q_\gamma(s)$ can take any shape, and the function may be concave or convex over different regions.

However, to fix ideas, consider when agent A has constant relative risk aversion (CRRA) with $u_A(x) = \frac{x^{1-\sigma_A}}{1-\sigma_A}$, and let agent A's endowment be given by $e_s^A = \kappa s$ with $\kappa > 0$ so that the endowment and the asset is positively correlated. Then, we have

$$s\mu_s^A = \kappa^{-\sigma_A} s^{1-\sigma_A}, \quad \gamma\mu_s^A = \kappa^{-\sigma_A} s^{-\sigma_A}\gamma$$

The concavity/convexity of $Q_\gamma(s)$ is determined by σ_A. For $s < \gamma$, $Q_\gamma(s)$ is constant for $\sigma_A = 1$; $Q_\gamma(s)$ is increasing and concave for $\sigma_A < 1$; and $Q_\gamma(s)$ is decreasing and convex for $\sigma_A > 1$. For $s > \gamma$, $Q_\gamma(s)$ is decreasing and convex always; there is a concave kink at γ.

As a result, the location of aggregate risk—whether it affects states $s < \gamma$ ("downside risk") or affects states $s > \gamma$ ("upside risk")—matters for how the price of debt, and thus the asset price, changes. From equation (11), any change in $f(s)$ isolated to states $s < \gamma$ does not change which contract is traded in equilibrium. Thus we can state clearly downside risk does not change γ, and the change in the debt and asset price depend systematically on the risk aversion of agent A, the lender (higher debt prices, i.e., more lending, when $\sigma_A > 1$, and lower debt prices, i.e., less lending, when $\sigma_A < 1$). This is stated formally in Proposition 6 in the Appendix.

This may be surprising. When the risk only concerns states when default occurs, and in which the lender is effectively holding the asset, more risk aversion leads to more lending and a higher asset price. However, this follows immediately from the standard asset-pricing equation using marginal utility and probabilities to discount asset payments. This effect on asset prices, which I'll call the *expectation effect*, arises from taking expectations of the pricing kernel using a different distribution.

Consider instead an MPS that affects states $s > \gamma$, or "upside risk." This change has potentially two effects. First, since $Q_\gamma(s)$ is convex right of γ (for any degree of risk aversion) the expectation effect will tend to increase the price of debt—holding the face value of the equilibrium contract fixed. Second, the MPS will generally affect the equilibrium contract. From equation (11), the equilibrium traded contract depends on $\mathbb{E}\left[\mu_s^A | s \geq \gamma\right]$ and also on $\frac{\mathbb{E}[\mu_s^B(s-\gamma)|s\geq\gamma]}{\mathbb{E}[\mu_s^B|s\geq\gamma]}\left[1 - F(\gamma)\right]$.

One possibility is that B is risk-neutral, implying μ_s^B is constant. In this case $z(j)$ would increase after an MPS because $\mathbb{E}\left[\mu_s^A | s \geq \gamma\right]$, which is convex, would increase. Thus, increased upside risk in the good tail would increase γ and tend to increase $q(\gamma)$ for two reasons: $q(j)$ is increasing in j and γ increases, and $Q_\gamma(s)$ is convex in the region of the MPS (there is a third effect, which is that μ_s^A will endogenously change with γ, which we discuss later in this section). Only the states with full debt repayment are affected—but in these states, the asset ends up owned by the risk-neutral agent. The asset price increases because B can borrow more against the asset. Agent A is willing to lend more no matter the degree of risk aversion.

More generally, though, agent B's pricing kernel will have important consequences for equilibrium. When, for example, agent B's endowment is *negatively* correlated with the asset payoff so that B's marginal utilities are increasing, $\mu_s^B(s-\gamma)$ is likely to be convex. (Furthermore, differentiating shows that $\mu_s^B(s-\gamma)$ is more convex than μ_s^B.) Thus, there would be an additional kick increasing borrowing rates because agent B would have a higher expected marginal return after an MPS. However, the first-order effect is the expectation effect coming from the lender's marginal utility.

Changing the Equilibrium Contract

The second result is that γ, the face value of the equilibrium traded contract, can either increase or decrease when the distribution is modified by an MPS. It is convenient to parametrize the function $z(j;f)$ by the payoff distribution $f(s)$. Let the distribution $\widetilde{f}(s)$ differ from $f(s)$ by an MPS. Denote the equilibrium traded contracts in each case respectively by $\widetilde{\gamma}$ and γ. In other words, $z(\gamma;f)=0$ and $z(\widetilde{\gamma};\widetilde{f})=0$.

Proposition 1. *There exists a distribution $\widetilde{f}(s)$ which is an MPS of $f(s)$ such that $\widetilde{\gamma}>\gamma$, and there exists $\widetilde{f}(s)$ such that $\widetilde{\gamma}<\gamma$. In other words, one can find an MPS to either increase or decrease the equilibrium contract.*

The proof is in Appendix B. The intuition is that an MPS can shift mass to either side of γ and can do so in way that increases or decreases the value of the "upper tranche" associated with buying the asset on margin. Thus, the default level of the equilibrium traded contract can either increase or decrease when the asset's risk increases. Notice that when γ increases, agents are making bigger promises.[15]

Changing the Pricing Kernel

Knowing what happens to the equilibrium γ is only one part of knowing what happens to equilibrium levels of borrowing and asset prices. Pricing kernels change when the face value of the equilibrium traded contract changes, and the effect depends fundamentally on agents' degrees of risk aversion. When γ changes in equilibrium, there is an additional effect on asset prices arising from what I call the *pricing kernel effect* as marginal utilities endogenously change because agents hold different portfolios. When risk changes the equilibrium traded contract, agents' future consumption changes, and thus how they value payments in each state changes. In particular, this implies that in equilibrium, the same contract may have a different value because the marginal utility of consumption in each state will potentially change.

To see the pricing kernel effect, consider

$$\begin{aligned}\frac{dq(\gamma)}{d\gamma} &= \int_{\gamma}^{\bar{s}} u_A'(x_s)dF(s) + \int_{\gamma}^{\bar{s}} \gamma u_A''(x_s)\frac{dx_s}{d\gamma}dF(s) \\ &= \int_{\gamma}^{\bar{s}} u_A'(x_s)\left(1 - \gamma ARA(s)\right)dF(s)\end{aligned}$$

where $ARA(s) = -\frac{u''(x_s)}{u'(x_s)}$ is absolute risk aversion at s. The pricing kernel effect states that lending increases with γ if the lender's risk aversion is not too large. When the lender's risk aversion is high, the pricing kernel decreases more than the increase in the payment so that lending falls even though the promise is larger.

When endowments are negligible compared to the asset payoff s, we can say even more. Let $e_s^i \to 0$, then $x_s^A = \min\{s, \gamma\}, x_s^B = \max\{s - \gamma, 0\}$, implying that agent A's marginal

[15] One might be tempted to compare this result to the result in Simsek (2013) regarding the type-versus-degree of belief disagreement. However, this result is about changing risk that affects both agents so that both agents simultaneously value the asset and debt contracts differently; whereas the belief disagreement result is about changing how agents *differentially* value assets and debt contracts.

utility is constant above γ because consumption is constant above γ. The optimality condition for the equilibrium contract simplifies to $u'_A(\gamma)R_B(\gamma) = \mathbb{E}\left[\mu_s^B | s \geq \gamma\right]$, and the pricing kernel effect is given by

$$\frac{dq(\gamma)}{d\gamma} = u'_A(\gamma)\left(1 - RRA(\gamma)\right)\left(1 - F(\gamma)\right),$$

where $RRA(\gamma)$ is the relative risk aversion. When the lender's risk aversion is greater than 1, an equilibrium with riskier loans (higher γ) has more lending in equilibrium. Given the pricing kernel effect, increases in γ will tend to raise debt prices (i.e. more lending) when $RRA < 1$, and to lower debt prices (less lending) when $RRA > 1$.

Define the spread on the loan by $R_\gamma = \frac{\gamma}{q(\gamma)}$. When the lender's risk aversion is high, equilibria with higher γ also have higher spreads, since $q(\gamma)$ falls. That is, loan amounts will be higher when spreads and riskiness are lower. The net effect on prices of an increase in risk depends on the combination of the pricing kernel and expectations effects, which can either reinforce and counteract each other depending on the degree of risk aversion. Similarly, when γ increases, agent B receives lower consumption in each state, which increases her marginal utility in each state.

In practice, we would expect lenders to have a variety of other sources of income. For instance, they might have labor income, or they might have other assets (e.g., stocks, bonds, or houses). Once we account for these other sources of wealth, a model in which lenders' stochastic discount factor is exogenous might very well be the more appropriate model (though the exogenous marginal utilities need not be constant, i.e., need not correspond to risk-neutrality). For instance, if subprime borrowers in practice were to borrow with safer loans, I would expect this to have very little effect on the wealth of the lenders in the subprime market (which tend to be very rich individuals). Nonetheless, to the extent that many assets *can* be systematically correlated, or if these loan sizes are substantial, the equilibrium consequences for the pricing kernel is at least worth noting.

4.2 Endowment Risk

The previous subsection considered increases in risk that affect the endowments of each agent together with the asset payoff. However, we can consider modifications in agents' endowments for a given payoff s, which has the effect of changing the covariance between endowments and the asset payoff. This exercise is meaningful precisely because agents are (weakly) risk-averse and so agents' marginal utilities and asset demands will in part depend on their endowments.

Modifying endowments by a mean-preserving spread has two effects: first, the covariance between endowments and the asset payoff increases; second, agent's marginal utilities, to a first approximation, change by a mean-preserving *contraction*. Suppose for example that endowments are shifted from e_s^i to $\widetilde{e}_s^i$ so that

$$\widetilde{e}_s^i = e_s^i + \varepsilon_s,$$

where ε_s acts as MPS according to $f(s)$. That is, we shift endowments from middle states toward tail states in such a way as to keep the expected endowment constant; the distribution $f(s)$ does not change. This has the effect of increasing the covariance. Let $\mathbb{E}[e^i] = \int_0^{\bar{s}} e_s^i dF(s)$ and $\mathbb{E}[s] = \int_0^{\bar{s}} s dF(s)$. Then

$$cov(\widetilde{e}_s^i, s) = \int_0^{\bar{s}} s\widetilde{e}_s^i dF(s) - \mathbb{E}[\widetilde{e}^i]\mathbb{E}[s] \tag{12}$$

$$= \int_0^{\bar{s}} s\left(e_s^i + \varepsilon_s\right) dF(s) - \mathbb{E}[e^i]\mathbb{E}[s] \tag{13}$$

$$= cov(e_s^i, s) + \int_0^{\bar{s}} s\varepsilon_s dF(s). \tag{14}$$

Since $\int_0^{\bar{s}} \varepsilon_s dF(s) = 0$, $\int_0^{\bar{s}} s\varepsilon_s dF(s) > 0$ and therefore $cov(\widetilde{e}_s^i, s) > cov(e_s^i, s)$.

Modifying endowments naturally means modifying marginal utilities. For simplicity, consider the effect of ε_s on marginal utilities when agents consume their endowments. For a small perturbation,

$$u_i'(\widetilde{e}_s^i) = u_i'(e_s^i) + u_i''(e_s^i)\varepsilon_s. \tag{15}$$

Since $u_i''(x) \leq 0$, a decrease in endowments leads to an increase in marginal utilities and vice versa. In fact, if the MPS is small or if marginal utility is linear (e.g., consider quadratic utility), then a mean-preserving spread of the endowments yields a mean-preserving *contraction* ("MPC") of marginal utilities. To simplify exposition, in the rest of the paper I will suppose that the third-order effect is small enough to ignore.[16]

With this in mind, we can revisit the results we considered for asset risk instead applied to endowment risk. Define

$$g_A(s) = \mu_s^A f(s), \quad g_B(s) = \mu_s^B f(s). \tag{18}$$

We can rewrite $z(j)$ and the debt price $q(j)$ as

$$z(j) = \frac{\int_j^{\bar{s}} g_A(s)ds}{\int_j^{\bar{s}} g_B(s)ds} \left(\mathbb{E}_{g_B}[\max(s-j,0)]\right) - e_0^B, \quad q(j) = \mathbb{E}_{g_A}\left[\min\{s,j\}\right],$$

which are pseudo-expectations taken over $g_A(s)$ and $g_B(s)$.

Generally when agent A's endowments change by an MPS, which is an MPC of agent A's marginal utility, $q(j) = \mathbb{E}_{g_A}[\min\{s,j\}]$ increases since $\min\{s,j\}$ is concave.[17] When the

[16]To the extent that the change in the endowments is large (i.e., ε_s are large) then an MPS of endowments will contract marginal utilities in a way that does *not* necessarily preserve the mean. A second-order expansion makes this clear.

$$u_i'(\tilde{e}_s^i) = u_i'(e_s^i) + u_i''(e_s^i)\varepsilon_s + \frac{u_i'''(e_s^i)}{2}\varepsilon_s^2 \tag{16}$$

Additionally, consider the integral that uses equation (15):

$$\int_0^{\bar{s}} u_i'(\tilde{e}_s^i)dF(s) = \int_0^{\bar{s}} u_i'(e_s^i)dF(s) + \int_0^{\bar{s}} u_i''(e_s^i)\varepsilon_s dF(s) \tag{17}$$

Since ε_s is an MPS, $\int_0^{\bar{s}} \varepsilon_s dF(s) = 0$. The value of the final integral depends on the third-derivative and how the endowment varies with s. For example,

$$\frac{d\left(u_i''(e_s^B)\right)}{ds} = u_i'''(e_s^B)\frac{de_s^B}{ds}$$

Whenever the third-derivative of utility is positive, as is the case for CRRA, the second-derivative is increasing and negative. Thus, if the agent's endowment increases with the state, then $\int_0^{\bar{s}} u_i''(e_s^i)\varepsilon_s dF(s) < 0$, but if the agent's endowment is decreasing with the state, then $\int_0^{\bar{s}} u_i''(e_s^i)\varepsilon_s dF(s) > 0$. In these cases marginal utilities are not modified in a mean-preserving way.

[17]There is an additional force if A's marginal utilities are sufficiently convex. In this case, the expected

endowment MPS is isolated to upside or downside risk, i.e., strictly applying to states above γ or below γ, then there is no change in the debt price because the debt payoff is piecewise-linear. The effect, from equation (17), depends on how agent A's endowment varies with the asset payoff. When A's marginal utility is linear, there is no change in $\int_j^{\bar{s}} g_A(s)ds$. However, when A's utility is CRRA and the endowment is positively correlated with the asset payoff, then lending will decrease, $q(\gamma)$ will decrease.

Changing agent B's endowment below γ has no effect on equilibrium. Thus an increase in downside endowment risk will affect borrowing so long as the lender is affected, but downside endowment risk for the borrower has no effect. However, upside endowment risk for the borrower has an entirely different effect. An MPS of B's endowment affecting states $s > \gamma$ will have a very modest effect on $\int_\gamma^{\bar{s}} g_B(s)ds$, as discussed. However, $\int_\gamma^{\bar{s}}(s-\gamma)g_B(s)ds$ will certainly change because $s - \gamma$ is an increasing function.

As well, agents' endowments can change to ambiguously effect the equilibrium traded contract γ. Changing endowments for agent A or B can ambiguously affect γ.

Proposition 2. *There exists an MPS of agent A's endowments* $\widetilde{e}_s^A$ *such that* $\widetilde{\gamma} > \gamma$*, and there exists* $\widetilde{e}_s^A$ *such that* $\widetilde{\gamma} < \gamma$*.*

Proposition 3. *Suppose* $\bar{s} - \gamma > 1$*. Then there exists an MPS of agent B's endowments* $\widetilde{e}_s^B$ *such that* $\widetilde{\gamma} > \gamma$*, and there exists* $\widetilde{e}_s^B$ *such that* $\widetilde{\gamma} < \gamma$*.*

The proofs are in the Appendix. The intuition is that endowments can change so that states above and below the default threshold γ can be valued relatively more or less by either agent as marginal utilities change. This is perhaps surprising because the insight from the C-CAPM literature is that a higher covariance should decrease asset prices. The insight from this model is that with collateralized borrowing, different agents price assets in different states, and so increasing covariance need not necessarily affect asset prices in the usual way.

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marginal utility decreases because the second-derivative is increasing, as in equation (17).

Endowment Tail Risks

These existence results suggests that it is worth limiting the way that risk can increase. I will consider two types of modifications to agents' endowments that move mass strictly into the tails, though in different ways. The important result of this section is that equilibrium responds differently to changes in endowments for agents A and B, and that the result can depend on whether default is likely or unlikely for collateralized debt contracts.

The first result considers modifying agent A's endowments in a way that strictly moves mass away from the mean and toward the tails. The concept is flexible enough to encompass nonparametric changes in distributions, but captures the idea of what happens when the variance increases for a parametric distribution. I use the term *strict* for this type of MPS.

Definition 2 (Strict Mean-Preserving Spread). *Let ε be an MPS. Then ε is strict if the following conditions hold.*

1. *Consider any $s_1 < \mathbb{E}(s)$. If $\varepsilon(s_1) > 0$, then for all $s < s_1$, $\varepsilon(s) \geq 0$*
2. *Consider any $s_2 > \mathbb{E}(s)$. If $\varepsilon(s_2) > 0$, then for all $s > s_2$, $\varepsilon(s) \geq 0$*

Since ε is an MPS (and must integrate to zero), clearly $\varepsilon(s) < 0$ for some $s \in S$; in particular, $\varepsilon(s) \leq 0$ in a neighborhood (perhaps a large one) around $\mathbb{E}(s)$. In other words, if there are states "in the tails" with greater mass as a result of the MPS, then states farther into the tails have weakly greater mass, too; mass moves to the tails from the mean, and not from farther in the tails.

When lenders endowments become riskier in this "strict sense," the equilibrium outcome depends on the riskiness of the equilibrium traded contract before the change. What is important is that the effect depends on the types of the loan. When repayment is likely, as is the case with prime mortgages and investment grade bonds, increased endowment risk, relative to the asset payoff, decreases margins. The asset has become relatively less risky for the lender and so margins decrease. But for loans in which repayment is not likely, as might be the case for subprime loans or for lower tranches of CDO's, risk works in a different way. In this case, increasing endowment risk relative to the asset payoff leads to higher margins.

The asset is less risky relative to endowments, but margins increase, because in this case repayment is a tail event and so decreased risk (relative to endowments) makes repayment less likely (in a weighted marginal utility sense).

Proposition 4 (**Asymmetric Effect of Strict Lender Endowment Risk**). *Let $\widetilde{e}_s^A$ differ from e_s^A by a strict MPS while still satisfying Conditions 1 and 2. The following are true:*

1. *If $\gamma \leq \mathbb{E}(s)$, then $\widetilde{\gamma} \geq \gamma$*

2. *If $\gamma \geq \mathbb{E}(s)$, then $\widetilde{\gamma} \leq \gamma$*

Proof. Because $\widetilde{e}_s^A$ is an MPS of e_s^A, then $\widetilde{g}_A(s)$ is an MPC of $g_A(s)$. When $\gamma \leq \mathbb{E}(s)$, then $\int_\gamma^{\bar{s}} g_A(s)ds \leq \int_\gamma^{\bar{s}} \widetilde{g}_A(s)ds$. More endowment risk decreases the marginal utility weighting in states above γ. From this we have that $z(j;e) \leq z(j;\widetilde{e})$. Since the $z(j)$ is increasing, the face value of the equilibrium contract increases, $\widetilde{\gamma} \geq \gamma$. The reverse is true when $\gamma \geq \mathbb{E}(s)$. □

The second type of endowment change considers upside and downside states, or the skewness of endowments, for borrowers. Above a threshold, s^R, endowments become increasingly larger, and below s^R endowments become increasingly smaller. While the expected endowment is the same, the conditional expected endowment is better for states above s^R. Using this definition, we can unambiguously describe what happens to the equilibrium borrowing contract, borrowing amounts, and the asset price when the borrower's endowments become more skewed to the upside relative to the lender's endowments.

Definition 3 (**Upside Skew of Endowments**). *Endowments $\widetilde{e}_s$ is skewed more to the upside than e_s if the expected endowment is the same, $\mathbb{E}\big[\widetilde{e}_s\big] = \mathbb{E}\big[e_s\big]$, and the ratio of endowments satisfy the following (weak) single crossing condition. For some $s^R \in S$*

$$\frac{\widetilde{e}_s}{e_s} \text{ is increasing for } s > s^R, \quad \frac{\widetilde{e}_s}{e_s} \text{ is decreasing for } s < s^R.$$

Proposition 5. *If agent B's endowments become more skewed to the upside, i.e. e_s^B is changed to $\widetilde{e}_s^B$, while still satisfying Conditions 1 and 2, then $\widetilde{\gamma} \leq \gamma$, i.e., the face value of* *the equilibrium contract will weakly decrease. As a result, borrowing will decrease and the asset price p will decrease. Margins and leverage respond accordingly.*

The proof is in Appendix B. Thus, relative upside and downside endowment/asset risk have entirely different implications for equilibrium margins and asset prices. The intuition for this result comes from the CAPM together with the fact that the borrower is buying an "upper tranche." When endowments change in this way, marginal utilities are lower precisely when the asset pays the most, and so agent B values the asset payoffs less. If endowments change in the opposite way, then marginal utilities will be higher precisely when the asset pays the most and so the asset will be more valuable to B, increasing B's willingness to borrow to buy the asset.

4.3 Numerical Examples

In light of these results, what conditions would produce countercyclical leverage? How should we expect γ to change and which pricing effect—the expectations effect or the pricing kernel effect—dominates when γ increases? Should we expect γ to increase or decrease? To answer these questions, I numerically investigated the comparative statics for three different parametric distributions when borrowers are risk-neutral, lenders have CRRA utility with relative risk aversion σ_A, and endowments are negligible.[18] Because the distribution functions do not have closed-form expectations, analytical results are hard to come by. I increase risk by increasing the standard deviations of the distribution of asset payoffs, corresponding to an increase in the asset risk.

My results suggest that margins are countercyclical when $\sigma_A < 1$. When volatility (the standard deviation) increases, the equilibrium loan γ increases and loan amounts $q(\gamma)$ decrease. This result is robust to whether γ is above or below the expected asset payoff. With CRRA utility for these parametric distributions, the expectation effect dominates the pricing kernel effect. When $\sigma_A < 1$, increased risk leads to lower loan amounts, lower asset

[18] The three distributions solved are: Symmetric Truncated Normal ($S = [0, 10]$ and $\mu = 5$); Symmetric Logit-Normal ($S = [0, 10]$ and $\mu = 0$); Symmetric Beta Distributions ($S = [0, 10]$ and $\alpha = \beta$).

prices, lower leverage, and higher margins, which is what we expect given empirical research. When $\sigma_A > 1$, γ increases and $q(\gamma)$ increases because the expectation effect dominates, which means that increased risk leads to higher asset prices and higher leverage. Thus, for standard examples widely used in the literature, risk does not lead to lower prices and higher margins, but the reverse.

What drives the results in each case is the shape of marginal utilities below γ. In these examples, the pricing kernel is concave whenever $\sigma_A < 1$. What this suggests is that for models of this type to produce countercyclical margins, we will likely need concave pricing kernels for lenders.

5 Conclusion

I have considered a general equilibrium model with heterogeneous agents and collateralized borrowing to show that changes in aggregate risk or in endowment risk can have ambiguous effects on leverage, loan margins, loan amounts, and asset prices. As is well known, agents' endogenous stochastic discount factors matter for asset pricing, though this effect has not been extensively studied in the collateral equilibrium literature. While the set of states in which default occurs determines margins, leverage, and asset prices, there is no systematic relationship between increased risk and when default occurs. However, the correlation of agents' endowments with the asset payoff and agents' risk aversion are key determinants of how increased risk affects equilibrium outcomes.

Future work should seek to address under what conditions changes in risk lead to countercyclical margins, as well as what changes in the economic environment lead to countercyclical margins with endogenous loan riskiness. Numerical results suggest that in this model, for a variety of distribution functions, margins are countercyclical and leverage is pro-cyclical when lenders have relative risk aversion less than one (vice versa). This may strike many as an unsatisfactory result. It may be worthwhile to consider how leverage and risk are related in a model with multiple agents and assets—and therefore with multiple leverage levels.

Appendices

A Solving for Collateral Equilibrium

In this section I first state the weaker, but more technical, condition required for equilibrium to have the form already outlined. I then show that given this condition agents trade a single contract, that the contract satisfies the optimality conditions already listed, and that $z(j)$ is strictly decreasing with an interior root.

The following weaker condition can replace conditions (C1a) and (C1b).

Condition 1-Weak: The following ratio of agents' marginal utilities is strictly increasing in s:

$$\text{for any } s' > s^*, \ \frac{u_B'(e_s^B + s - s')}{u_A'(e_s^A + s')} \quad \text{is strictly increasing for } s \in S. \tag{C1weak}$$

This condition ensures that in equilibrium $\frac{\mu_s^B}{\mu_s^A}$ is strictly increasing for $s \geq \gamma$. This condition holds if

$$\text{for any } s' > s^*, \ -\frac{u_A''(e_s^A + s')}{u_A'(e_s^A + s')} \left(\frac{de_s^A}{ds}\right) > -\frac{u_B''(e_s^B + s - s')}{u_B'(e_s^B + s - s')} \left(\frac{de_s^B}{ds} + 1\right). \tag{19}$$

Letting $ARA_i = -\frac{u_i''}{u_i'}$, this condition holds if

$$\inf_{s' > s^*} ARA_A(e_s^A + s') \left(\frac{de_s^A}{ds}\right) > \sup_{s' > s^*} ARA_B(e_s^B + s - s') \left(\frac{de_s^B}{ds} + 1\right). \tag{20}$$

The intuition for this equation is that the ratio of marginal utilities is increasing, where we require stronger conditions on bounds for absolute risk aversion since marginal utilities are endogenous.

Let the equilibrium contract traded be γ with price $q(\gamma)$. I first derive bounds on contract prices so that given the optimality condition in equation (8), no other contract is traded in equilibrium. Then, given these bounds, I show that the traded contract indeed satisfies equation (8).

Since A lends in equilibrium, it follows from the first-order-conditions for j that for all $j \in J$

$$q(j) \geq \int_0^j \mu_s^A s dF(s) + j \int_j^{\bar{s}} \mu_s^A dF(s). \tag{21}$$

with strict equality for $j = \gamma$. Remember that A consumes in period 0 and has linear utility over x_0; therefore A's marginal utility of wealth is 1. Thus, agent A will not trade in $j > \gamma$ so long as

$$q(j) - q(\gamma) \geq \int_\gamma^j \mu_s^A (s-\gamma) dF(s) + \int_j^{\bar{s}} \mu_s^A (j-\gamma) dF(s). \tag{22}$$

Next, define $V_j = \int_j^{\bar{s}} \mu_s^B (s-j) dF(s)$. Agent B will not trade in a contract j so long as $R_B(j) \leq R_B(\gamma)$:

$$\frac{V_\gamma}{p - q(\gamma)} \geq \frac{V_j}{p - q(j)}. \tag{23}$$

Rearranging terms, this requires that

$$\begin{aligned} p - q(j) \geq & \frac{V_j}{R_B(\gamma)} \\ \geq & \frac{V_j - V_\gamma + V_\gamma}{R_B(\gamma)} \\ \geq & \frac{V_\gamma}{R_B(\gamma)} - \frac{V_\gamma - V_j}{R_B(\gamma)} \\ \geq & p - q(\gamma) - \frac{V_\gamma - V_j}{R_B(\gamma)}. \end{aligned}$$

This implies that

$$q(j) - q(\gamma) \leq \int_\gamma^j \frac{\mu_s^B}{R_B(\gamma)} (s-\gamma) dF(s) + \int_j^{\bar{s}} \frac{\mu_s^B}{R_B(\gamma)} (j-\gamma) dF(s). \tag{24}$$

Putting together equations (22) and (24), there exists a price $q(j)$ such that neither agent trades in contract j so long as

$$\int_\gamma^j \left(\frac{\mu_s^B}{R_B(\gamma)} - \mu_s^A \right) (s-\gamma) dF(s) + \int_j^{\bar{s}} \left(\frac{\mu_s^B}{R_B(\gamma)} - \mu_s^A \right) (j-\gamma) dF(s) > 0. \tag{25}$$

Since the optimal contract satisfies equation (8), and since the ratio of marginal utilities $\frac{\mu_s^B}{\mu_s^A}$ is strictly increasing in s (by Condition 1-Weak), $\frac{\mu_s^B}{R_B(\gamma)} - \mu_s^A > 0$ and so this equation holds; therefore agents will trade in a single contract. By a symmetric argument, contract prices exist for $j < \gamma$ so that agents will not trade in j.

Because the bounds from equations (22) and (24) are sharp at γ, we can use the equation for $q(j)$ to determine the slope of the contract prices at $j = \gamma$. Agent B will choose the contract j to maximize $R_B(j)$. Differentiating $R_B(j)$ with respect to j at $j = \gamma$ yields

$$\frac{dR_B(j)}{dj} = \frac{d}{dj}\left[\frac{V_j}{p - q(j)}\right] = \frac{(p - q(j))\frac{dV_j}{dj} + V_j\frac{dq(j)}{dj}}{(p - q(j))^2}. \tag{26}$$

Using $q(\gamma) = \int_0^\gamma \mu_s^A s dF(s) + \gamma \int_\gamma^{\bar{s}} \mu_s^A dF(s) = \mathbb{E}[\mu_s^A \min(s, \gamma)]$, the derivative at $j = \gamma$ is

$$q'(j) = \int_j^{\bar{s}} \mu_s^A dF(s).$$

By the definition of V_j, the derivative is

$$V_j'(j) = -\int_j^{\bar{s}} \mu_s^B dF(s).$$

Setting $\frac{dR_B(j)}{dj} = 0$ yields

$$\frac{V_j}{p - q(j)} = -\frac{V_j'(j)}{q'(j)},$$

which yields the optimality condition

$$R_B(\gamma) = \frac{\mathbb{E}\left[\mu_s^B | s \geq \gamma\right]}{\mathbb{E}\left[\mu_s^A | s \geq \gamma\right]}.$$

To show that equilibrium occurs for some $x \in (0, \bar{s})$, I first show that $z(j) = p^{opt}(j) -$

$p^{mc}(j)$ is strictly decreasing, which follows from Condition 1. Differentiating $z(j)$ yields

$$\begin{aligned}\frac{dz(j)}{dj} =& -\mu_j^A f(j) \frac{\int_j^{\bar{s}} \mu_s^B (s-j) dF(s)}{\int_j^{\bar{s}} \mu_s^B dF(s)} \\ & + \mu_j^B f(j) \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\left(\int_j^{\bar{s}} \mu_s^B dF(s)\right)^2} \int_j^{\bar{s}} \mu_s^B (s-j) dF(s) \\ & - \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\int_j^{\bar{s}} \mu_s^B dF(s)} \int_j^{\bar{s}} \mu_s^B dF(s).\end{aligned}$$

Thus,

$$\frac{dz(j)}{dj} = z(j) \left(\frac{\mu_j^B f(j)}{\int_j^{\bar{s}} \mu_s^B dF(s)} - \frac{\mu_j^A f(j)}{\int_j^{\bar{s}} \mu_s^A dF(s)} - 1 \right)$$

Since $\frac{\mu_s^B}{\mu_s^A}$ is strictly increasing, this implies that

$$\frac{\mu_\gamma^B}{\mu_\gamma^A} < \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\int_j^{\bar{s}} \mu_s^A dF(s)},$$

and therefore $\frac{dz(j)}{dj} < 0$.

Finally, evaluating $z(j)$ at the boundaries shows that equilibrium must be interior. Evaluating p^{opt} and p^{mc} at $s = 0$ and $s = \bar{s}$, we have

$$p^{opt}(\bar{s}) = q(\bar{s}) < p^{mc}(\bar{s}) = q(\bar{s}) + e_0^B,$$

and

$$p^{mc}(0) = q(0) + e_0^B = e_0^B.$$

And $p^{opt}(0) = \frac{\mathbb{E}[\mu_s^A]}{\mathbb{E}[\mu_s^B]} \mathbb{E}\left[s\mu_s^B\right]$, which by Conditions 2 is greater than e_0^B. Thus, $z(0) > 0$ and $z(\bar{s}) < 0$. As a result, it follows that A consumes in period 0 but B does not.

B Proofs and Propositions

Proof of Proposition 1

Proof. I will show that there are MPS such that $z(j; f)$ increases or decreases when evaluated using the new probabilities. Define the simpler function

$$h(j; f) = \int_j^{\bar{s}} (s - j) dF(s)$$

The behavior of $z(j; f)$ closely follows the behavior of $h(j; f)$.

First, consider a mean-preserving spread that takes mass from the left of γ and places some of that mass to the right of γ, as shown in Figure 1. Risk about future states has increased under $\widetilde{f}(s)$, and in the new equilibrium γ will increase.

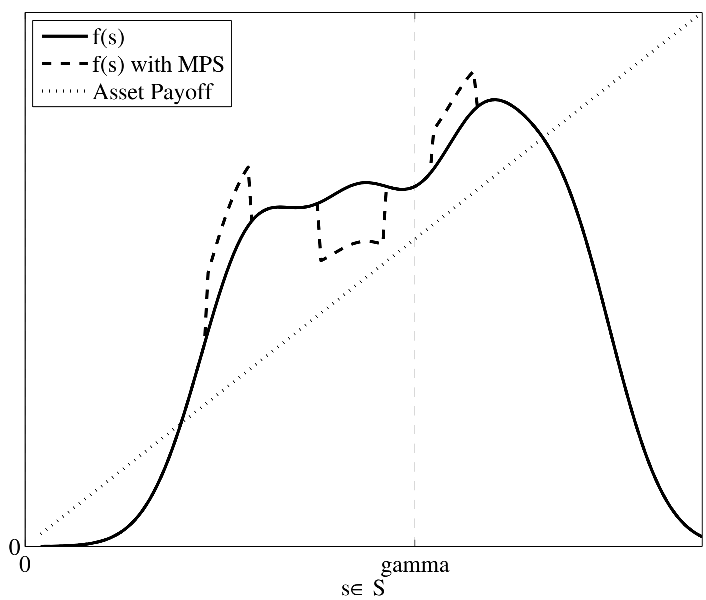

Figure 1: Increased Risk Increases Equilibrium Loan γ.

Because under the new distribution $\widetilde{f}(s) \geq f(s)$ for $s \geq \gamma$

$$\int_{\gamma}^{\bar{s}} (s - \gamma) d\widetilde{F}(s) > \int_{\gamma}^{\bar{s}} (s - \gamma) dF(s)$$

Thus $h(j;\widetilde{f}) > h(j;f)$.

It is little more work to show that $z(j;\widetilde{f}) > z(j;f)$. Define

$$z_1(j) = \frac{\mathbb{E}\left[\mu_s^A | s \geq j\right]}{\mathbb{E}\left[\mu_s^B | s \geq j\right]}, \quad z_2(j) = \mathbb{E}[\mu_s^B \max(s-j,0)]$$

so that

$$z(j) = z_1(j) z_2(j) - e_0^B$$

First, $h(j;\widetilde{f}) > h(j;f)$ implies $z_2(j;\widetilde{f}) > z_2(j;f)$. For small changes we can write

$$\Delta z = \Delta z_1 z_2 + z_1 \Delta z_2$$

Next, if the change in the probability distribution is local to γ, then the change in $z_1(j)$ is on the order of

$$\frac{\mu_\gamma^A}{\int_\gamma^{\bar{s}} \mu_s^A dF(s)} - \frac{\mu_\gamma^B}{\int_\gamma^{\bar{s}} \mu_s^B dF(s)}$$

Since $\frac{\mu_s^B}{\mu_s^A}$ is strictly increasing by Condition 1, this implies that

$$\frac{\mu_j^B}{\mu_j^A} < \frac{\int_j^{\bar{s}} \mu_s^A dF(s)}{\int_j^{\bar{s}} \mu_s^A dF(s)}$$

As a result, $\Delta z_1 > 0$ and hence $z(j;\widetilde{f}) > z(j;f)$. (Intuitively, since the ratio of marginal utilities is increasing, increasing the mass of the distribution close to γ gives more weight to marginal utilities that are not so different.) Finally, since $z(j)$ is decreasing in j, $\widetilde{\gamma} > \gamma$.

Second, consider the following MPS that moves mass to the left of γ. Pick a small $d, c > 0$ and $s_1 < \gamma - c$. For $s \in (\gamma, \gamma + d)$ set $\widetilde{f}(s) = f(s) - a_1$. For $s \in (\gamma + d, \gamma + 2d)$ set $\widetilde{f}(s) = f(s) + a_2$. And for $s \in (s_1, s_1 + c)$ set $\widetilde{f}(s) = f(s) + a_3$. This is an MPS so long as

$$a_2 d + a_3 c = a_1 d$$

$$a_2(3d^2 + 2\gamma d) - a_1(2\gamma d + d^2) + a_3(2 s_1 c + c^2)$$

The first line says that the distribution integrates to 1 and the second says that the mean is unchanged. If $a_1 = Na_2$, then $s_1 = \gamma - \frac{2c}{N-1}$. As long as $N > 3$, $h(j; \widetilde{f}) < h(j; f)$. By the same argument, local to γ a sufficient choice of parameters ensure that $z(j; \widetilde{f}) < z(j; f)$, and therefore $\widetilde{\gamma} < \gamma$. □

Proof of Proposition 2

Proof. Define $z(j; \widetilde{e}_s^A)$ as parametrized by A's endowment. Consider $\int_j^{\bar{s}} g_A(s)ds$, which is the only component of $z(j)$ that depends on A's endowment. One can shift endowments to the right of γ, just as we shifted probability mass to the right of γ in the proof for Proposition 1, which decreases $g_A(s)$ for $s \geq \gamma$. As a result

$$\int_{\gamma}^{\bar{s}} \widetilde{g}_A(s)ds < \int_{\gamma}^{\bar{s}} g_A(s)ds$$

Thus $z(j; \widetilde{e}_s^A) < z(j; e_s^A)$. Similarly, we can shift endowments as in the second part of the proof for Proposition 1 so that

$$\int_{\gamma}^{\bar{s}} \widetilde{g}_A(s)ds > \int_{\gamma}^{\bar{s}} g_A(s)ds.$$

□

Proof of Proposition 3

Proof. The proof is very similar to the previous one, this time shifting mass either close to γ or close to $\bar{s}$. When mass is shifted close to γ, then $s - \gamma$ is small over the region where $g_B(s)$ changes, and so $\int_j^{\bar{s}} g_B(s)ds$ changes by more than $\int_j^{\bar{s}}(s - j)g_B(s)ds$. Conversely, shifting mass close to $\bar{s}$ implies that $s - \gamma > 1$ over the region where $g_B(s)$ changes, and therefore $\int_j^{\bar{s}} g_B(s)ds$ changes by less than $\int_j^{\bar{s}}(s - j)g_B(s)ds$. □

Proof of Proposition 5

Proof. Since marginal utilities move inversely with endowments, when the endowment becomes more skewed to the upside, then marginal utilities become more skewed to the downside. As a result,

$$\frac{\widetilde{g}_B(s)}{g_B(s)} \text{ is decreasing for } s > s^R, \quad \frac{\widetilde{g}_B(s)}{g_B(s)} \text{ is increasing for } s < s^R$$

Define

$$G_B(j) = \int_j^{\bar{s}} g_b(s)ds, \quad \widetilde{G}_B(j) = \int_j^{\bar{s}} \widetilde{g}_B(s)ds$$

downside skewness implies that $\widetilde{g}_B(s)$ and $g_B(s)$ satisfy the following "hazard rate" property

$$\begin{cases} \dfrac{\widetilde{g}_B(s)}{\widetilde{G}_B(\bar{s}) - \widetilde{G}_B(s)} \leq \dfrac{g_B(s)}{G_B(\bar{s}) - G_B(s)} & \text{if } s < s^R, \\ \dfrac{\widetilde{g}_B(s)}{\widetilde{G}_B(\bar{s}) - \widetilde{G}_B(s)} \geq \dfrac{g_B(s)}{G_B(\bar{s}) - G_B(s)} & \text{if } s > s^R, \end{cases}$$

Additionally, we can write equation (11) as

$$z(j) = \left(\int_j^{\bar{s}} g_A(s)ds\right)\left(\int_j^{\bar{s}} (s-j)\frac{g_B(s)}{G_B(\bar{s}) - G_B(j)}(s)ds\right) - e_0^B$$

The term $\int_j^{\bar{s}}(s-j)\frac{g_B(s)}{G_B(\bar{s})-G_B(j)}(s)ds$ is the analog of the conditional expectation above s^R, taken with respect to $g_B(s)$, which is a pseudo-distribution. Since the hazard rate is decreasing above s^R, the "conditional expectation" is lower after the endowments are skewed upside.

Lemma: Let $\widetilde{f}(s)$ be upside skewed relative to $f(s)$. Then $\mathbb{E}[s|s > x; \widetilde{f}(s)] \leq \mathbb{E}[s|s > x; f(s)]$ for each $x \in (0, \bar{s})$

Proof. Define $t(x) = E[s|s > x; \widetilde{f}(s)] - \mathbb{E}[s|s > x; f(s)]$. I need to show that $t(x) \geq 0$ for all

$x \in (0, \overline{s})$. Note that

$$\begin{aligned} t'(x) =& \frac{\widetilde{f}(x)}{1-\widetilde{F}(x)} \left(\mathbb{E}_B[s|s > x; \widetilde{f}(s)] - x\right) - \frac{f(x)}{1-F(x)} \left(\mathbb{E}_B[s|s > x; f(s)] - x\right) \\ =& \left(\frac{\widetilde{f}(x)}{1-\widetilde{F}(x)} - \frac{f(x)}{1-F(x)}\right) \left(\mathbb{E}_B[s|s > x; \widetilde{f}(s)] - x\right) + \frac{f(x)}{1-F(x)} t(x) \end{aligned}$$

Over the range $(0, s^R)$, $\frac{\widetilde{f}(s)}{1-\widetilde{F}(s)} \geq \frac{f(s)}{1-F(s)}$. Thus, I can write our expression as the differential equation $t'(x) = A(x) + B(x)t(x)$ where $A(x) \geq 0$ and $B(x) > 0$ with initial condition $t(0) = 0$. This implies that $t(x) \geq 0$ over this range.

Over the range $(s^R, \overline{s})$, the hazard-rate ordering is reversed. Using that $\mathbb{E}[s|s > x; \widetilde{f}(s)] - x > t(x)$ we have that

$$t'(x) \leq \frac{\widetilde{f}(x)}{1-\widetilde{F}(x)} t(x) \text{ for each } x \in (s^R, \overline{s})$$

Suppose then that there exists $s_1 < \overline{s}$ such that $t(s_1) < 0$. Choose $\hat{s} = \sup\{s \in [s_1, \overline{s}] | t(s) \leq t(s_1)\}$. Note that $t(\overline{s}) = 0$ and $t(\hat{s}) = t(s_1) < 0$. We have that $t'(\hat{s}) \leq \frac{\widetilde{f}(\hat{s})}{1-\widetilde{F}(\hat{s})} t(\hat{s}) < 0$. But if $t(\hat{s})$ is decreasing, that means that $\hat{s}$ can't be the largest s as defined above. Thus $t(x) \geq 0$ over this range as well. □

A direct result is that, since the skewness is reversed,

$$\int_j^{\bar{s}} (s-j) \frac{\widetilde{g}_B(s)}{\widetilde{G}_B(\overline{s}) - \widetilde{G}_B(j)}(s) ds \leq \int_j^{\bar{s}} (s-j) \frac{g_B(s)}{G_B(\overline{s}) - G_B(j)}(s) ds,$$

and therefore $z(j; \widetilde{e}) \leq z(j; e)$. Since $z(j)$ (weakly) decreases, $\widetilde{\gamma} \leq \gamma$, i.e., the face value of the equilibrium contract (weakly) decreases in equilibrium. Since agent A's marginal utilities are unaffected, we know that $q(\widetilde{\gamma}) \leq q(\gamma)$ and therefore $\widetilde{p} \leq p$. □

Downside Aggregate Risk

Proposition 6. *Let $\widetilde{f}(s)$ differ from $f(s)$ by an MPS such that $f(s) = \widetilde{f}(s)$ for $s \geq \gamma$. Then $\widetilde{\gamma} = \gamma$. The change in the price of debt depends on A's risk aversion: when $\sigma_A < 1$, debt has* *a lower price,* $q(\gamma; \widetilde{f}(s)) < q(\gamma; f)$*; when* $\sigma_A > 1$ *debt has a higher price* $q(\gamma; \widetilde{f}(s)) > q(\gamma; f)$*; where* $\sigma_A = 1$*, the debt price does not change,* $q(\gamma; \widetilde{f}(s)) = q(\gamma; f)$*.*

Proof. States above γ are unchanged, which means trivially that $z(j; f) = z(j; \widetilde{f})$ is unchanged. Therefore $\widetilde{\gamma} = \gamma$ and the equilibrium contract is the same. However, $Q_\gamma(s)$ is not linear below γ, and therefore the expectation will be different, i.e. $q(j)$ will be different. The expectation is higher under $\widetilde{f}(s)$ when $Q_\gamma(s)$ is convex and it is lower when $Q_\gamma(s)$ is concave, which is determined by σ_A, as discussed. □

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