Financial Intermediation, Leverage, and Macroeconomic Instability

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Draft: Monday 4th January, 2016

This paper investigates how financial-sector leverage affects macroeconomic instability and welfare. In the model, banks borrow (use leverage) to allocate resources to productive projects and provide liquidity. When banks do not actively issue new equity, aggregate outcomes depend on the level of equity in the financial sector. Equilibrium is inefficient because agents do not internalize how their decisions affect volatility, aggregate leverage, and the returns on assets. Leverage creates systemic risk, which increases the frequency and duration of crises. Limiting leverage decreases asset-price volatility and increases expected returns, which decrease the likelihood that the financial sector is undercapitalized.

JEL: E44,G01,G20

Keywords: Leverage, Macroeconomic Instability, Borrowing Constraints, Banks, Macroprudential Regulation, Financial Crises

Many economists believe excessive financial-sector leverage played a critical role during the recent financial crisis, yet economists do not agree on what appropriate leverage regulation should be. The debate weighs the benefits of intermediation against the costs of more frequent crises arising from higher leverage. The costs and benefits of leverage and intermediation, however, are dynamic and

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time-varying, reflecting the ways the financial sector responds to economic fundamentals and affects economic outcomes. Changing leverage during good times can have qualitatively different consequences from changing leverage during "crisis" times. The dynamic nature of leverage and intermediation raises two fundamental questions. First, when is leverage too high or too low? And second, how should leverage be regulated?

To answer these questions, I use a continuous-time stochastic general equilibrium model in which banks allocate resources to productive projects, and bank deposits provide liquidity services. Banks can invest in certain projects more efficiently than households can directly, but banks can issue only risk-free debt and not equity. As a result, banks invest more when they have more equity, and the economy's resources are allocated more efficiently when financial-sector net worth is high. Equity acts as a buffer against adverse shocks, but banks continue to use debt to finance investments because deposits require a low interest rate as a result of their liquidity services. The model builds on Brunnermeier and Sannikov (2014), which demonstrates the inherent instability of economies with financial sectors and the pecuniary externalities caused by equity constraints. My modifications show how household welfare is affected by the tradeoff, created by financial-sector leverage, between stability and higher output.

I show that financial-sector leverage increases social efficiency in the short run, but in the long run it increases the frequency and duration of states with bad economic outcomes. In other words, volatile financial-sector net worth increases the likelihood that the financial sector is undercapitalized. Banks' leverage choices lead to inefficient levels of macroeconomic instability because every bank's action affects the possibility that other banks have low net worth, but limited equity issuance creates a distortion between the private and social values of bank equity. Leverage constraints can alleviate this market failure. The banking system would be less volatile if banks collectively used less leverage, which is better for bank profits and for households.

Regulating leverage can improve welfare by changing the frequency and duration of good and bad outcomes. I solve for the global dynamics of the economy to demonstrate the welfare consequences of regulating leverage across the state space. Limiting leverage in general improves stability, but when banks are subject to endogenous borrowing constraints (for example, Value-at-Risk borrowing constraints, which I show can produce procyclical leverage), increasing leverage during crises ("countercyclical regulation") can improve welfare.

Additionally, the model presents a related result that I call an "Intermediation Paradox." The paradox is that household welfare may worsen as households become better at investing directly in the activities banks finance. This is because the economy is more stable when banks earn higher returns, which occurs when banks have a larger advantage.

A. Related Literature

My paper is most closely related to Brunnermeier and Sannikov (2014), who use continuous-time methods to demonstrate that the economy exhibits instabilities that cannot be adequately studied by steady-state analysis. Near the steady state the economy is stable, with high output and growth, but away from the steady state the economy features high asset price volatility and nonlinear amplifications.

Brunnermeier and Sannikov (2014) is a critical contribution, but the model is not designed to study leverage regulation: limiting leverage can improve stability, but it is very difficult to improve the welfare of either households or banks ("experts"). In order to study effective leverage regulation, I modify their model in two principal ways. First, I consider a model with two goods that are combined into a consumable good: good 1 is more effectively produced by intermediaries and good 2 is more effectively produced by households.[1] Regulating leverage

[1]With only one good, there is little room for leverage regulation whenever experts have plenty of equity. Robustly, when experts have lots of wealth, the socially optimal allocation is for experts to control all of the capital because the wedge between the marginal efficiency of experts and households is discrete. Thus, leverage regulation in good states is a question of when experts consume/pay dividends rather than how to allocate resources. There is, however, room to regulate allocations when experts are

modifies the allocation of resources in the economy, affecting the marginal productivity for each good and the goods prices (i.e., the returns to the intermediary sector).[2] Second, I model banks as firms owned by households, rather than as competing agents, so that the flow utility of households is monotonic in and completely determined by the condition of the banking sector.[3] Modeling banks in this way creates a tradeoff between stability and flow utility; adding the second good makes this tradeoff robustly exploitable using leverage regulation.

The key assumption that equity is "sticky" or "slow-moving" is closely related to He and Krishnamurthy (2012, 2013).[4] These papers use continuous-time models to study the effect of financial intermediation on asset prices and risk-premia when banks have a "participation constraint" that limits how much equity they can raise after bad shocks. Relaxing this constraint decreases risk-premia and shortens crises. As a complementary exercise, I study how to regulate leverage when banks cannot—or would choose not to—increase equity. Because issuing equity can be costly and time-consuming, regulating leverage can be a desirable tool to use in conjunction with other policies.[5]

Finally, this paper is related to the "Financial Accelerator" literature associ-

poor and households manage capital, but it is still the case that there is a discrete difference between agents' marginal productivities, greatly diminishing the ability of a regulator to improve welfare by reallocating resources to improve stability.

[2]This effect is related to Brunnermeier and Sannikov (2015), who show that, in an international context, capital controls can improve stability and welfare by improving the terms of trade countries face. Including two sectors also allows us to match the empirical evidence of Adrian, Colla and Shin (2012), which documents how direct and intermediated financing—and their costs—change during financial disruptions.

[3]A third modification is that, instead of assuming that intermediaries are less patient, I assumes that households get a utility flow from holding deposits, i.e., bank debt. As a result, banks pay out dividends to reduce equity, and so the equity constraint always binds. There are not misaligned incentives between banks and households.

[4]The assumption that bank equity is "sticky" is empirically supported by Acharya et al. (2011), which shows that the capital raised by banks during the crisis was almost entirely in the form of debt and preferred stock and not in the form of common equity. Adrian and Shin (2010, 2011) provide evidence that the predetermined balance sheet variable for banks and other financial banks is equity, not assets. The "pecking order" theories of corporate finance shed light on why equity may be so "sticky." In Myers and Majluf (1984), equity issuance is costly because the firm has better information on the value of its assets, and any attempt to raise new equity financing will encounter a lemon problem. Similarly, Jensen and Meckling (1976) argue that there are agency costs associated with the entrenched "inside" equity holders, which entail a discount when issuing new equity to "outside" equity holders.

[5]My results generalize so long as banks do not issue equity too frequently. In Section II of the Online Appendix I show that equilibrium is similar with costly equity issuance, though the issuance policy differs from that in He and Krishnamurthy (2013) where experts adjust their equity continuously. Countercyclical leverage regulation is even more attractive when there is costly equity issuance.

ated with Bernanke and Gertler (1989), Bernanke, Gertler and Gilchrist (1999), and Kiyotaki and Moore (1997), and to the literature on the effects of credit constraints. Danielsson, Shin and Zigrand (2011) and Adrian and Boyarchenko (2012) investigate how VaR constraints affect endogenous volatility in continuous-time models. In Geanakoplos (2003) and Brunnermeier and Pedersen (2009), borrowing capacity is limited by possible adverse price movement in the next period. Using a model similar to mine, Brunnermeier and Sannikov (2015) show that capital controls can improve global welfare by improving global stability.

I. The Baseline Model

In this section I develop a baseline model populated by households and banks, which are owned by households. There is a single factor of production that can be used to produce two intermediate goods. Banks have an advantage for producing one intermediate good and households for the other. As a result, output and growth depend endogenously on land ownership. With financial frictions, outcomes will depend on the level of equity in the banking sector.

A. The Model Setup

Technology. — Time is continuous and infinite, and there are aggregate productivity shocks which follow a Wiener process.

There is one factor of production, land. An "effective unit" of land can be used to produce either good 1 or 2 at unit rate. Land quantity y_t evolves according to equation (1),

$$\frac{dy_t}{y_t} = g_y dt + \sigma dW_t, \tag{1}$$

where dW_t is an exogenous Brownian aggregate (common) shock, and where g_y depends on who manages land and what it is used to produce. The values of g_y are

Table 1—: Expected Productivity Growth Rates

g_y	Good 1	Good 2
Households	$g-m$	g
Banks	g_B	g_B

given in Table 1, which imply that banks are comparatively better at managing good 1 and households are better at managing good 2. I define the parameter restriction more clearly later in this section.

Denote by Y_t the stock of effective land at time t, which is also the flow production of goods at time t. At any point a plot of land can be used to create an instantaneous flow of either good regardless of how the land was used in the past.

The consumption good is produced using goods 1 and 2 according to

$$C_t = Y_{1t}^{\alpha} Y_{2t}^{1-\alpha}, \tag{2}$$

where C_t is the quantity of the consumption good, Y_{jt} is the quantity of good j (equivalently the quantity of land used to produce good j), and α is the parameter determining the relative importance the two goods for production. Letting the consumption good C_t be the numeraire, the equilibrium prices of goods 1 and 2 are thus given by

$$p_{1t} = \alpha \left(\frac{Y_{2t}}{Y_{1t}}\right)^{1-\alpha}, \quad p_{2t} = (1-\alpha)\left(\frac{Y_{1t}}{Y_{2t}}\right)^{\alpha}.$$

Let $\lambda_t = \frac{Y_{1t}}{Y_t}$ be the fraction of effective land cultivating good 1. Then

$$p_{1t} = \alpha \left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha}, \quad p_{2t} = (1-\alpha)\left(\frac{\lambda_t}{1-\lambda_t}\right)^{\alpha}. \tag{3}$$

Households . — There is a continuum of risk-neutral households denoted by $h \in [0,1]$ with initial wealths n_0^h. Households have the discount rate r, may consume both positive and negative amounts (though in equilibrium their consumption will always be positive), and have "liquidity-in-the-utility-function" with constant marginal utility over bank deposits. Lifetime utility is given by equation (4)

$$V_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(c_t^h + \phi_L \delta_t^h \right) dt \right], \tag{4}$$

where r is the discount rate, c_t^h is flow consumption, δ_t^h are bank deposits and $\phi_L > 0$ is the liquidity preference.

I model households as risk neutral to emphasize the effects of leverage on stability (they do not directly care about risk and volatility, though in equilibrium households' welfare will depend on volatility). I model the liquidity value of bank deposits directly in the utility function as a modeling convenience, as is common in the New Keynesian literature. Deposits have liquidity values for a variety of reasons outside of the model, which I leave out to simplify exposition.[6]

It follows that the expected return on any asset that households own (whether land employed to produce either good or bank equity) must be r, and the expected the return on deposits must be $r - \phi_L$, owing to the liquidity value.

Banks. — There is a continuum of banks, denoted by $b \in [0, 1]$, with book value ("equity") n_0^b. Banks are owned by households, who choose dividend payouts, the level of deposits, and the portfolio weights on land used to produce goods 1 and 2. The objective is to maximize the present value of dividends discounted at rate r (households' time preference), subject to the constraints that dividends cannot be negative (i.e., banks cannot raise new equity) and the value of banks' assets minus liabilities n_t^b cannot become negative. Banks can borrow using debt at an interest rate $r^L = r - \phi_L < r$ and so banks will never choose a capital structure

[6]See for example Diamond and Dybvig (1983), Gorton and Pennacchi (1990), or Lagos and Wright (2005). Because my objective is not to study liquidity provision or demand, I use constant marginal utility for deposits to simplify the model and so that bank leverage does not affect liquidity value.

that is completely equity.[7]

I restrict parameters so that banks do not have a *net* financing advantage for both goods: $g_B = g - \phi_L$. This condition says that banks have a net advantage at cultivating good 1 but not at cultivating good 2. Part of banks' advantage comes from their ability to issue deposits that provide liquidity services. Thus, even when banks earn a lower *real* rate of growth, they are still useful for allocating resources to good 1.[8]

Market for Land and Returns. — Agents are price takers in the perfectly competitive market for land, with equilibrium price q_t per effective unit. I postulate that its law of motion is of the form

$$\frac{dq_t}{q_t} = \mu_t^q dt + \sigma_t^q dW_t, \tag{5}$$

which will be determined endogenously in equilibrium. A plot of y_t effective units of land has price $q_t y_t$, regardless of how land will be used. I will refer to q_t as the "asset price" since land is the only factor of production.

When an agent buys and holds y_t units of land to produce good j, by Ito's

[7] While equity provides a buffer against bankruptcy, debt provides attractive financing because it earns a liquidity yield. In general, banks use debt because debt provides a tax shield or earns a liquidity yield, debt alleviates certain agency issues, or agency problems create incentives for agents to use debt. Harris and Raviv (1990) argue that debt incentivizes good management because it is a hard claim that can force managers into bankruptcy. Similarly, Calomiris and Kahn (1991) argue that demandable deposits can force liquidation and thus induce good management. Jensen (1986) argues that debt removes agency costs associated with free cash flows. Thus, if equity leads to wasteful costs spent disciplining managers, agency costs increase as the firm becomes less leveraged. Compensation structures could cause managers to take on excess risk or behave impatiently. For example, Admati and Hellwig (2013) and Admati et al. (2010) argue that the preference for leverage does not arise from agency costs that are mitigated by debt, but because banks enjoy an implicit backstop from the government that they will be bailed out in case of bankruptcy.

[8] Many of the qualitative results in the paper go through so long as $g_B \geq g - m - \phi_L$, and the model can handle when banks are better at both, i.e., $g_B > g - \phi_L$. The differences between banks' and households' growth rates are meant to capture the various advantages that banks may have for particular types of investments. First, banks can undertake certain investments owing to lower financing costs. Second: banks may be better able to monitor or control managers, or to identify and select good investment projects; they may have advantages arising from scale allowing them to minimize coordination costs and fixed costs, to overcome indivisibility problems, and to achieve better diversification. Third, banks typically tolerate more risk than households do. That said, banks are not better at financing every investment in the economy, and their activities are costly.

Lemma the value of this land evolves according to

$$\frac{d(y_t q_t)}{y_t q_t} = (g_y + \mu_t^q + \sigma\sigma_t^q)\, dt + (\sigma + \sigma_t^q) dW_t,$$

where g_y is appropriately defined; this is the capital gains rate of the land investment. The return to owning land includes the value of the output produced and the capital gains on the value of the land. If y_t effective units of land cultivate good j, the rate of return is given by

$$dr_t^j = \left(\frac{p_{jt}}{q_t} + g_y + \mu_t^q + \sigma\sigma_t^q\right) dt + (\sigma + \sigma_t^q) dW_t.$$

The volatility of returns on investments is $\sigma + \sigma_t^q$. Returns include fundamental risk, σ, and price risk, or endogenous risk, σ_t^q. Agents get different returns because their productivity growth rates differ, and the realized returns depend on the realization of the aggregate shock dW_t. Denote by dr_t^{jb} and dr_t^{jh} the returns respectively to banks and households from owning land to grow good j. To simplify notation, I denote the expected returns as: $\mathbb{E}[dr_t^{jb}] = \bar{r}_t^{jb} dt$, $\mathbb{E}[dr_t^{jh}] = \bar{r}_t^{jh} dt$.

Banks' Portfolio Choice. — Banks choose a dividend policy and portfolio weights to maximize the expected discounted value of dividends subject to a dynamic budget constraint:

$$\max_{\{\omega_t^{1b} \geq 0, \omega_t^{2b} \geq 0, d\zeta_t^b \geq 0\}} U_\tau = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} d\zeta_t^b \right]$$

subject to

$$dn_t^b = (\omega_t^{1b} n_t^b) dr_t^{1b} + (\omega_t^{2b} n_t^b) dr_t^{2b} - \delta_t^b r^L dt - d\zeta_t^b \tag{6}$$

$$n_t^b \geq 0, \tag{7}$$

where ω_t^{jb} is the fraction of equity invested in land used to produce good j, $\delta_t^b = (\omega_t^{1b} + \omega_t^{2b} - 1)n_t^b$ is the amount borrowed using deposits, and $d\zeta_t^b \geq 0$ is the rate at which dividends are paid (that is, the cumulative dividends paid to shareholders is given by $\{\zeta_t^b\}$).[9]

Competitive Equilibrium. — Informally, a competitive equilibrium is characterized by market prices for land and goods, together with land allocations and consumption decisions such that given prices, agents optimize and markets clear. The formal definition is given in the Online Appendix.

B. Equilibria in Stationary Economics

I first solve for equilibrium in two cases when stationary equilibria exist: without banks, and when banks can freely issue equity. In both cases prices and allocations will be constant, though the economy will still be subject to fundamental shocks.

Equilibrium Without Banks. — Without banks we simply consider households' decisions. Because the expected return to land used for either good is r, we can find a stationary equilibrium with constant prices and allocations. Land is allocated so that the flow production of each good is constant.[10]

PROPOSITION I.1: *With no banks, equilibrium prices and allocations are* $p_{1t} = \underline{p}_1$, $p_{2t} = \underline{p}_2$, $q_t = \underline{q}$, $\lambda_t = \underline{\lambda}$, *where* $\underline{p}_2 < \underline{p}_1$, $\underline{\lambda} < \alpha$, *and* $\underline{q} = \frac{\underline{p}_1}{r+m-g} = \frac{\underline{p}_2}{r-g}$.

With constant prices, returns are given by $dr_t^{1h} = \left(\frac{\underline{p}_1}{\underline{q}} + g - m\right) dt + \sigma dW_t$, and $dr_t^{2h} = \left(\frac{\underline{p}_2}{\underline{q}} + g\right) dt + \sigma dW_t$. Setting expected returns equal to r and solving for $\underline{q}$ yields the land price, which implies the goods prices and the land allocation using equation (3). The supply of good 2 is relatively higher because households are more productive using land for good 2.

[9]This is a basic equity issuance constraint. In Online Appendix II banks are instead subject to a constant marginal cost of issuing equity. I suspect that a variety of issuance constraints would produce similar qualitative results, but it would be important to get the constraint right for quantitative analyses.

[10]Land productivity and output varies due to the aggregate shock dW_t, but land is continuously reallocated across goods so that prices, output, and expected returns are constant.

Equilibrium With Banks and No Equity Issuance Constraint. — Consider when banks can issue equity without constraint (they can choose $d\zeta_t^b < 0$ at no cost). Since banks can finance their investments more cheaply with debt, the required return for banks investments are $r - \phi_L$, which is the deposit rate. Given the productivity growth rates, banks will buy land to produce good 1 and households will buy land to produce good 2. Banks will finance themselves entirely with debt and issue new shares after adverse shocks so that equity is always zero, never negative. Even though banks earn a low growth rate ($g_B < g$), banks buy land to allocate to good 1 because they can borrow at a low rate using deposits.

PROPOSITION I.2: *With no equity issuance constraints, equilibrium prices and allocations are* $p_{1t} = \bar{p}_1$, $p_{2t} = \bar{p}_2$, $q_t = \bar{q}$, $\lambda_t = \bar{\lambda}$, *where* $\bar{p}_1 = \bar{p}_2 = \alpha^\alpha(1-\alpha)^{1-\alpha}$, $\bar{\lambda} = \alpha$, *and* $\bar{q} = \frac{\bar{p}_1}{r-g}$.

Expected returns satisfy $\bar{r}_t^{1b} = \frac{\bar{p}_1}{\bar{q}} + g - \phi_L = r^L$ and $\bar{r}_t^{2h} = \frac{\bar{p}_2}{\bar{q}} + g = r$, and these equations imply $\bar{q} = \frac{\bar{p}_1}{r-g} = \frac{\bar{p}_2}{r-g}$. Hence $\bar{p}_1 = \bar{p}_2$, and the prices and allocations follow using equation (3). Note also that $\underline{q} < \bar{q}$.

The goods prices, land price, and allocation of land to each type is the same as in the economy with no banks and with $m = 0$. Hence, in equilibrium banks achieve a better land allocation, increase the land price, and provide liquidity. Since banks are unconstrained, markets are complete and this equilibrium is efficient.

II. Equilibrium with Equity Issuance Constraints

With limited ability to issue equity, banks' decisions depend on their level of equity, and so equilibrium depends on banks' aggregate level of equity. As banks build up equity, land allocations will generally move from $\underline{\lambda}$ to $\bar{\lambda}$ and the prices of the intermediate goods will converge. Banks will pay dividends when land is allocated as if $m = 0$, even though the economy is not stationary. In some ways, as the financial sector has more equity, the effective growth penalty in the economy decreases. This section also considers equilibrium welfare.

A. Solving for Dynamic Equilibrium

I use stochastic continuous-time methods to solve for global equilibrium dynamics. I will look for a recursive (or Markov), rational-expectations equilibrium with a finite-dimensional state variable that is a diffusion, and prices that are smooth functions of the state variable. I first solve the optimization problems for banks and households to derive the properties of equilibrium processes. I then define the state variable, which is the aggregate level of bank equity as a fraction of the economy, and derive its law of motion. Finally, I solve for the equations that define other equilibrium variables as a function of the state variable.

BANKS' PROBLEM. — Banks solve a dynamic optimization problem because dividends and equity are constrained to be positive. By homogeneity and price-taking,[11] the maximized value of a bank with equity n_t^b can be written as

$$\theta_t n_t^b \equiv \max_{\{\omega_t^{1b}\geq 0, \omega_t^{2b}\geq 0, d\zeta_t^b \geq 0\}} \mathbb{E}_t\left[\int_t^\infty e^{-r(s-t)} d\zeta_s^b\right], \tag{8}$$

where θ_t is the marginal value of equity, i.e., the proportionality coefficient that summarizes how market conditions affect the value of the bank's value function per dollar of equity. The marginal value of equity equals 1 plus the multiplier on the equity-issuance constraint and reflects the aggregate condition of the financial sector.[12] We can use θ_t to characterize a bank's problem (Bellman Equation).

LEMMA II.1 (Bank's Bellman Equation): *Let* $\{q_t, t \geq 0\}$ *be a price process for which the maximal payoff a bank can attain is finite.*[13] *Then the process* $\{\theta_t\}$

[11] If a bank's equity doubles it can double its value function by doubling asset holdings and doubling the dividend strategy.

[12] It makes sense to interpret $\theta_t n_t^b$ as the market value of equity, where n_t^b is the book value.

[13] This will be true in equilibrium, but it is not true for any given price process.

satisfies (8) under a strategy $\{\omega_t^{1b}, \omega_t^{2b}, d\zeta_t^b\}$ *if and only if*

$$r\theta_t n_t^b = d\zeta_t^b + \mathbb{E}[d(\theta_t n_t^b)], \tag{9}$$

when n_t *follows (6) and the transversality condition* $\mathbb{E}[e^{-rt}\theta_t n_t] \to 0$ *holds. This strategy is optimal if and only if*

$$r\theta_t n_t^b = \max_{\{\omega_t^{1b}\geq 0, \omega_t^{2b}\geq 0, d\zeta_t^b \geq 0\}} \left\{ d\zeta_t^b + \mathbb{E}[d(\theta_t n_t^b)] \right\}, \tag{10}$$

subject to the dynamic budget constraint (6).

We can further characterize the optimality conditions in the following way.

PROPOSITION II.1: *Consider a finite process*

$$\frac{d\theta_t}{\theta_t} = \mu_t^\theta dt + \sigma_t^\theta dW_t, \tag{11}$$

with $\sigma_t^\theta \leq 0$*. Then* $\theta_t n_t$ *represents the maximal future expected payoff that a bank with book value* n_t *can attain, and* $\{\omega_t^1, \omega_t^2, d\zeta_t\}$ *is optimal if and only if*

1) $\theta_t \geq 1 \ \forall t$*, and* $\zeta_t > 0$ *only when* $\theta_t = 1$*,*
2) $\mu_t^\theta = \phi_L$*,*
3) $\bar{r}_t^{jb} - r^L \leq -\sigma_t^\theta(\sigma + \sigma_t^q)$ *with strict equality when* $\omega_t^j > 0$*,*
4) *The transversality condition* $\mathbb{E}[e^{-rt}\theta_t n_t] \to 0$ *holds under* $\{\omega_t^1, \omega_t^2, d\zeta_t\}$*.*

I look for an equilibrium with $\sigma_t^\theta \leq 0$, so that banks' marginal value of equity increases after bad aggregate shocks, and in equilibrium it will be so. This restriction is intuitive; however, it is conceivable that equilibrium in which this restriction does not hold may exist. Hence, $-\sigma_t^\theta(\sigma + \sigma_t^q)$ represents the bank's required risk-premium (or level of risk aversion). Banks will not pay dividends when $\theta_t \geq 1$, and θ_t can never be less than one because banks can always pay out the full value of equity instantaneously, guaranteeing a value of at least n_t^b.

Households earn the same net return as banks when cultivating good 2, i.e., $\bar{r}_t^{2b} - r^L = \bar{r}_t^{2h} - r$. Since $\sigma^\theta \leq 0$, and since households do not require a risk-premium to cultivate good 2, banks will never cultivate good 2, $\omega_t^2 = 0$. But since $\bar{r}_t^{1b} - r^L > \bar{r}_t^{1h} - r$, banks will always buy land to produce good 1. In equilibrium households alone will cultivate good 2, and $\mathbb{E}\left[dr_t^{2h}\right] = rdt$ always. Thus we will have the following equations for returns in equilibrium:

$$\frac{p_{1t}}{q_t} + g + \mu^q + \sigma\sigma^q - r = -\sigma^\theta(\sigma + \sigma^q), \quad \frac{p_{2t}}{q_t} + g + \mu^q + \sigma\sigma^q = r. \tag{12}$$

As a result, there will be times when households cultivate good 1 (when $\sigma^\theta(\sigma+\sigma^q)$ is sufficiently large).

Let ψ_t be the fraction of the land owned by banks, i.e.,

$$\psi_t = \frac{1}{Y_t}\int y_{1t}^b db, \tag{13}$$

where $y_{1t}^b = \omega_t^{1b} n_t / q_t$. Note $\psi_t \leq \lambda_t$.

The State Variable and its Evolution. — With constrained equity issuance, the level of equity in the banking sector matters for equilibrium. Define $N_t = \int n_t^b db$ as aggregate bank equity. Because land productivity grows geometrically and the bank problem is homogenous, the equilibrium state-variable of interest is aggregate bank equity as a fraction of total value of land, or a variant of the "wealth distribution."[14] It is convenient to use the following state variable, which normalizes bank equity by the value of land:

$$\eta_t = \frac{N_t}{q_t Y_t}. \tag{14}$$

[14] The state variables in the economy are the stock of land Y_t and the aggregate level of bank equity N_t, which determine the households' and banks' problems and market clearing. In the Online Appendix I argue why the allocation of land is not a state-variable.

LEMMA II.2: *The equilibrium law of motion of η will be endogenously given as*

$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dW_t + d\Xi_t, \tag{15}$$

where $d\Xi_t$ is an impulse variable creating a regulated diffusion. Furthermore,

$$\begin{aligned}
\mu_t^\eta &= -\frac{(\psi_t - \eta_t)}{\eta_t}(\sigma + \sigma^q)(\sigma^\theta + \sigma + \sigma^q) + \left(\frac{p_{1t}}{q_t} + (\lambda_t - \psi_t)m - (1 - \psi_t)\phi_L\right), \\
\sigma_t^\eta &= \frac{(\psi_t - \eta_t)}{\eta_t}(\sigma + \sigma^q), \\
d\Xi_t &= \frac{d\zeta_t}{N_t},
\end{aligned}$$

where $d\zeta_t = \int d\zeta_t^b db$.

I derive the evolution of η_t using Ito's Lemma and the equations for returns and budget constraints. The details are in the Online Appendix.

Aggregate leverage is given by $L_t = \frac{\psi_t - \eta_t}{\eta_t}$. All else equal, higher leverage increases volatility and decreases the drift of the state variable. Additionally, the allocation of land affects the drift of η_t through the terms $\frac{p_{1t}}{q_t} + (\lambda_t - \psi_t)m$. When less land is used for good 1, p_{1t} is higher. Household production of good 1, though bad for economic growth, improves the relative capitalization of banks. These effects, which occur during "bad times," will tend to stabilize the economy.

EQUILIBRIUM AS A SYSTEM OF DIFFERENTIAL EQUATIONS. — Equilibrium consists of a law of motion for η_t and asset allocations and prices as functions of η. The asset prices are $q(\eta), \theta(\eta)$, and the flow allocations and goods prices are $\lambda(\eta), \psi(\eta), p_1(\eta), p_2(\eta)$. I solve for equilibrium by converting the equilibrium conditions into a system of differential equations ("ODE") in the asset prices q and θ. Given $q(\eta), q'(\eta)$ and $\theta(\eta), \theta'(\eta)$ I can get equilibrium returns and allocations to get $q''(\eta), \theta''(\eta)$. I solve the ODE using appropriate boundary conditions. The derivations are in the Online Appendix.

PROPOSITION II.2 (Equilibrium): *The equilibrium domain of the functions* $q(\eta), \theta(\eta)$*, and* $\lambda(\eta), \psi(\eta), p_1(\eta), p_2(\eta)$ *is an interval* $[0, \eta^*]$*. Function* $q(\eta)$ *is increasing,* $\theta(\eta)$ *is decreasing, and the following boundary conditions hold:*

(1) $\theta(\eta^*) = 1$*; (2)* $q'(\eta^*) = 0$*; (3)* $\theta'(\eta^*) = 0$*; (4)* $q(0) = \underline{q}$*; (5)* $\lim_{\eta \to 0^+} \theta(\eta) = \infty$.[15] *Over* $[0, \eta^*]$*,* $\theta_t \geq 1$ *and* $d\zeta_t^b = 0$*, and* $d\zeta_t^b > 0$ *at* η^* *creating a regulated barrier for the process* η_t*. I refer to* η^* *as the* stochastic steady state.

If the price function is twice-continuously differentiable, then equations (5) and (11) are functions of η:

$$\frac{dq_t}{q_t} = \mu^q(\eta_t)dt + \sigma^q(\eta_t)dW_t, \quad \frac{d\theta_t}{\theta_t} = \mu^\theta(\eta_t)dt + \sigma^\theta(\eta_t)dW_t, \tag{16}$$

where the drift and variance terms are determined by the derivatives of $q(\eta)$ and $\theta(\eta)$. (For the remainder of the paper, the dependence on the state-variable η_t is suppressed for notational ease.) The derivations are in the Online Appendix.[17]

B. Numerical Solution

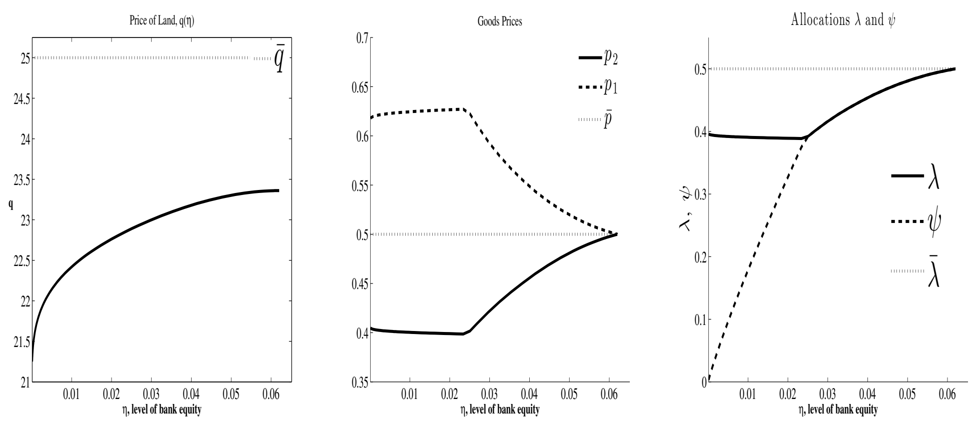
I solve the model numerically with the following parameters: $g = .02$, $r = .04$, $\sigma = .02$, $\alpha = .5$, $\phi_L = .02$, $m = .01$. While these choices are not the result of careful calibration, they produce reasonable results. These parameters imply that $\bar{q} = 25$, $\underline{q} = 20.346$, and $\bar{\lambda} = .5$, $\underline{\lambda} = .4$. The qualitative results in the paper are largely robust to parameter choices and there is discussion of the role of m in Section IV.[18]

[15] There is a level η^* where $\theta(\eta^*) = 1$. By smooth pasting, $q'(\eta^*) = 0$, and $\theta'(\eta^*) = 0$.[16] At $\eta = 0$ households are the sole agents in the economy, which yields the boundary condition $q(0) = \underline{q}$. Finally, $\lim_{\eta \to 0^+} \theta(\eta) = \infty$, because when $\eta = 0$, the economy is stationary forever and banks can earn a positive return, with leverage, from buying land to produce good 1.

[17] See also Brunnermeier and Sannikov (2014) for details on the computational solution method.

[18] The growth rate of $g = 2\%$ is roughly the growth rate of GDP per capita, $r = 4\%$ is a common discount rate; results are not very sensitive to these parameters. The parameter ϕ_L determines the attractiveness of debt finance, and therefore plays an important role in determining the stochastic steady state η^*. While 2% is a large estimate for liquidity premium, it is reasonable to interpret this parameter as capturing the other attractive features of debt (see footnote 7 in Section 2). The parameters ϕ_L and α together determine how much leverage banks use. He and Krishnamurthy (2012) document average leverage for commercial banks and broker-dealers of 8.3 and 25 respectively, and Acharya et al. (2011)

Figure 1 plots prices $q(\eta)$ and $p_1(\eta), p_2(\eta)$ and allocations $\psi(\eta)$ and $\lambda(\eta)$. As bank equity η increases, their portfolios grow, and as a result the price of land increases, the allocation of land becomes more efficient, and the prices of goods converge. The land allocation at η^* is the same as when banks do not face equity constraints (i.e., $\lambda(\eta^*) = \bar{\lambda}$) and the risk-premium to producing good 1 is zero. There is systemic risk ($\sigma^\eta > 0$), but land is allocated as if the economy were stationary with banks able to issue equity costlessly. Households produce good 1 when bank equity is sufficiently low.[19]



Land price, q_t Goods prices p_{1t}, p_{2t} Land allocations, λ_t, ψ_t

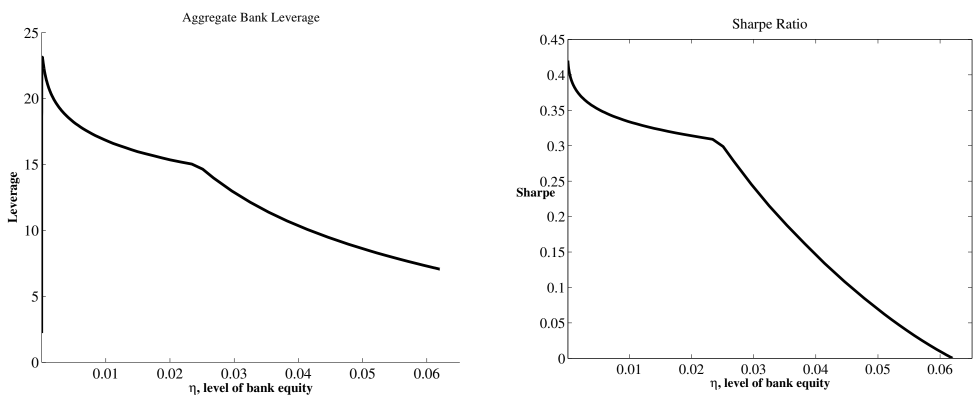
Figure 1. : Equilibrium Prices and Allocations

Figure 2 plots aggregate bank leverage and the Sharpe ratio of banks' assets. As equity decreases, banks hold smaller portfolios, though leverage increases. Importantly, leverage is bounded: local to $\eta = 0$, leverage decreases as banks become

show that when leverage is measured as total assets over common equity, leverage rose over 2000–2007 from 15 to 22 for commercial banks and from 17-35 for investment banks. In my model leverage ranges from 7 to 23. The most important parameter is volatility. A value $\sigma = 2\%$ is roughly the volatility of GDP/TFP. Hassan, Karels and Peterson (1994) find evidence that the volatility of banks' assets is between .9-2.3%, and He and Krishnamurthy (2014) employ a value of 3% in their model. Higher volatility causes states with low bank equity to occur with higher frequency, which increases the average level of leverage and the average Sharpe ratio and which decreases welfare.

[19]One interesting feature is that for low values of η, λ falls below $\underline{\lambda}$, the level without banks. This is because $q_t > \underline{q}$. There, flow output is actually worse than it would be without banks. Additionally, liquidity provision varies as ψ varies.

extremely risk averse (to avoid bankruptcy). The increase in the Sharpe ratio as equity decreases is driven by changes in p_{1t} and q_t. Remember that households continue to price land through good-2 production, which yields expected return r. In other words, the decrease in q_t does not alone produce excess returns; banks earn excess returns precisely because p_{1t} increases;[20] returns to intermediated assets exceed returns to direct investments. The Sharpe ratio equals $-\sigma_t^\theta$, banks' instantaneous risk-aversion.[21]



Aggregate Bank Leverage

Banks' Sharpe Ratio

Figure 2. : Leverage and Sharpe Ratio of Banks' Assets

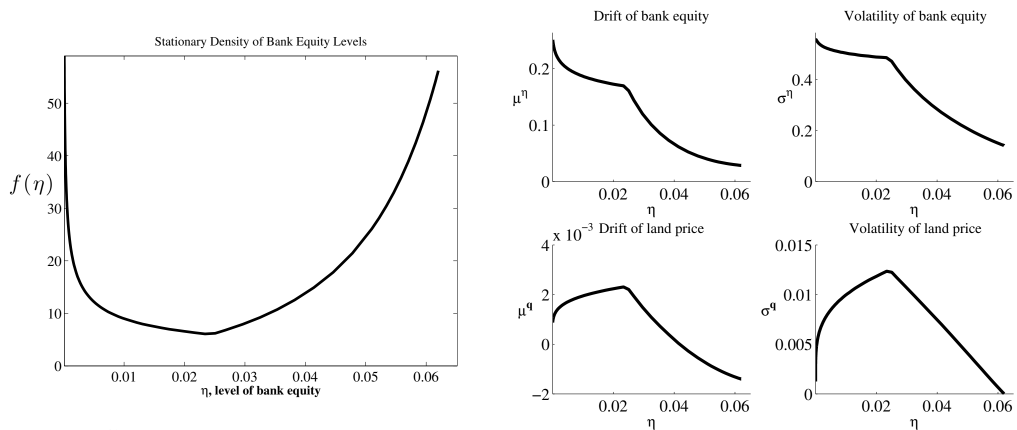
Equilibrium is typically stable but a sequence of large shocks leads to protracted periods of bad outcomes (low flow utility) and higher endogenous volatility. Figure 3 plots the stationary distribution, denoted by $f(\eta)$,[22] and equilibrium drifts and volatilities for η_t and q_t. The economy is typically near the stochastic steady

[20]Since $p_{1t} > p_{2t}$ when $\eta < \eta^*$, a decrease in q_t increases the dividend yield for banks more than for households.

[21]The average Sharpe ratio is 14.4% and the Sharpe at η^* is 0. Since agents are risk-neutral, it is no surprise that the model does not match empirical Sharpe ratios. However, the model does a fair job at matching the average Sharpe ratio of investments *relative* to household investments. In general the Sharpe ratio can be decomposed into a fundamental component that would arise in a representative agent economy and a component reflecting financial risk. In an economy with a representative agent with risk-aversion of $\gamma = 10$, the fundamental Sharpe ratio would be $\gamma\sigma = 20\%$. (Note that standard models have difficulty matching equity premia without using large degrees of risk-aversion. In He and Krishnamurthy (2014), the Sharpe ratio in the unconstrained region is 32%.) Adding $-\sigma_t^\theta$ on top of that would yield an average Sharpe ratio of 34.4%, which is not far from typical asset pricing calibrations.

[22]The function $f(\eta)$ is defined by equation (8) in the Online Appendix.

state; however, if aggregate bank equity falls sufficiently, banks rebuild equity slowly and so the economy may spend a lot of time with a weakly capitalized financial sector.[23] The time it takes to return to the stochastic steady state, i.e., to move from bad times to good times, increases quickly far away from η^*, and so the economy can spend a significant amount of time in bad states.[24]



Stationary Distribution

Drifts and Volatilities of q_t, η_t

Figure 3. : Instability: Stationary Distribution and Drifts and Volatilities

As Figure 3 shows, asset price volatility rises as η moves away from η^*, but it drops off as households produce good 1 directly. Near η^*, volatility is low and the state variable's drift is high; thus, the economy is constantly "drawn to" η^* even as it is reflected. At η^* there is systemic risk because of bank leverage, but there is no endogenous asset price risk. As η decreases, price volatility rises as banks rebalance their portfolios with greater frequency arising from greater leverage. When p_1 is high enough, households invest directly in good 1 (even though they face a growth discount) which prevents further increases in p_1. The drift and

[23]How much the stationary distribution "tilts" to the right or left depends on the parameters chosen; the distribution need not be U-shaped. For example, as σ increases, the density increases near 0 and decreases near η^*. Decreasing ϕ_L decreases the financing benefit of debt—steady state leverage decreases—and so η^* increases. Lower ϕ_L increases the density near η^*, and a higher ϕ_L increases the density near 0.

[24]I derive this formally, and calculate and plot results in the Online Appendix.

volatility of η drop sharply near 0 because bank equity becomes a small portion of the economy and thus absolute fluctuations are small.

C. Welfare

Welfare corresponds to the equilibrium expected present discounted value of flow utility over consumption and liquidity given the current effective land and bank capitalization:

$$V(\eta_\tau, Y_\tau) = \mathbb{E}_\tau \left[\int_\tau^\infty e^{-r(t-\tau)} \left(C_t + \phi_L D_t \right) dt \right]$$
$$\text{with } C_t + \phi_L D_t = z(\eta_t) Y_t$$
$$\frac{d\eta_t}{\eta_t} = \mu_t^\eta dt + \sigma_t^\eta dW_t + d\Xi_t$$
$$\frac{dY_t}{Y_t} = g_Y(\eta_t) dt + \sigma dW_t,$$

where $z(\eta_t) = \lambda_t^\alpha (1-\lambda_t)^{1-\alpha} + \phi_L(\psi_t - \eta_t) q_t$ is output plus liquidity and $g_Y(\eta_t) = g(1-\psi_t) + \psi_t g_B - (\lambda_t - \psi_t)\, m$ is aggregate productivity growth.[25]

Because households are risk neutral and their investments earn expected return r and $r - \phi_L$ for deposits, expected discounted utility is equal to wealth. Household wealth includes the land they own and the debt and equity invested in banks. The total wealth is $q_t Y_t + (\theta_t - 1) N_t = (1 + (\theta_t - 1)\, \eta_t)\, q_t Y_t$. [26] Thus, $J(\eta_t) = (1 + (\theta_t - 1)\, \eta_t)\, q_t$ is household welfare per unit of land and $\theta(\eta_t) q(\eta_t) \eta_t$ defines the value of bank equity per unit of land. Expected discounted payoffs depend on the current state because agents discount the future and η_t can be quite persistent. Additionally, $J(\eta)$ solves

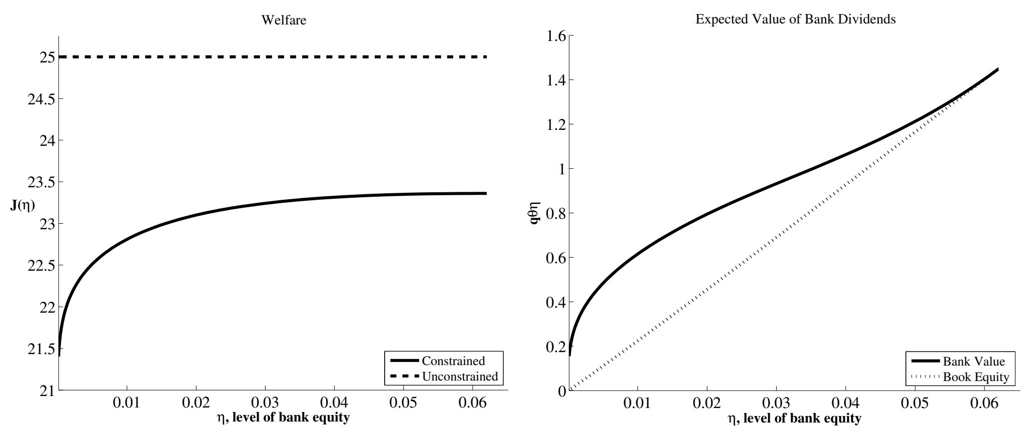
$$rJ(\eta) = z(\eta) + J'(\eta)\eta\mu^\eta + J(\eta) g_Y(\eta) + \frac{1}{2} J''(\eta)(\eta\sigma^\eta)^2 + J'(\eta)\eta\sigma^\eta\sigma. \tag{17}$$

[25] Remember that banks are owned by households; they are not competing agents. As a result, banks' payoffs are not included in welfare calculations. When households receive dividends from banks, households reinvest in assets or in deposits to earn the liquidity premium.

[26] Their land is worth $(1-\psi_t) q_t Y_t$; their debt is worth $\psi_t q_t Y_t - N_t$; bank shares are worth $\theta_t N_t$, since the expected value of dividends from banks is $\theta(\eta_t) N_t = \theta(\eta_t) q(\eta_t) \eta_t Y_t$.

The derivation is in the Online Appendix.

Figure 4 plots welfare and bank value per stock of land. If banks could freely issue equity, welfare would equal $\bar{J}(\eta) = \bar{q}$, which is the maximum welfare attainable. The present value of dividends differs from bank's equity, which is the dotted line, because $\theta_t \geq 1$. Banks would issue new shares of equity if they could, increasing—not diluting—the value of existing shares. As well, the bank value is convex near η^*.[27]



Welfare Bank Value

Figure 4. : Equilibrium Welfare and Banks Value

Efficiency. — Equilibrium is not efficient, and it is not constrained efficient. Our first result is that, from a social perspective, banks tend to use too much leverage near the stochastic steady state because agents do not take into account how their portfolio choices affect the equilibrium evolution of aggregate bank equity and the equilibrium evolution of asset prices. In other words, banks do not internalize their effect on systemic risk.

[27]Banks care about how frequently η hits η^* and how long it takes the economy to recover; the flow activity in the economy for low η affects shareholders only indirectly. On the other hand, because households receive flow utility throughout, they are directly affected by flow economic activity for all values of η.

To see this, we take the derivative of the right-hand side of equation (17) with respect to ψ_t, yielding

$$(18)\quad \mathcal{L}(\psi_t) \equiv \frac{\partial z(\psi,\lambda)}{\partial \psi} + J'(\eta)\eta\frac{\partial \mu^\eta}{\partial \psi} + J\frac{\partial g_Y}{\partial \psi} + \left(J''(\eta)\eta\sigma^\eta(\psi,\lambda,\eta) + J'\eta\sigma\right)\frac{\partial \sigma^\eta}{\partial \psi}.$$

The expression $\mathcal{L}(\psi_t)$ measures the impact of a marginal change in lending on social welfare.[28] The first term is the marginal change in flow utility (output and liquidity); the remaining terms capture how increasing banks' portfolios affects the evolution of the economy. Notice that $\mathcal{L}(\psi_t)$ explicitly captures the effect of changing ψ_t on the evolution of bank equity, something agents in the economy do not do.

The level of leverage is socially optimal when $\mathcal{L}(\psi_t) = 0$, i.e., when the marginal social costs (which may include higher volatility and decreased drift) exactly equal the marginal social benefits (which may include higher flow utility from better land allocation and increased liquidity). We can prove that the marginal social value of leverage is negative near η^*. Thus, a regulator looking to make a marginal change in leverage would want to decrease leverage relative to the competitive equilibrium.

PROPOSITION II.3: *In a competitive equilibrium, the marginal social value of aggregate bank leverage at the stochastic steady state* η^* *is negative. Welfare would improve if bank leverage near* η^* *were marginally lower than the competitive-equilibrium level.*

PROOF:

[28]See Klimenko, Pfeil and Rochet (2015) for a similar exercise. Their paper derives a similar result in a model in which banks have costly equity issuance and determine the supply of credit. Their model does not have a durable factor of production, which allows them to solve the Social Planner's problem, but as a result there are no dynamic amplification mechanisms through asset prices.

We need to show that $\mathcal{L}(\psi_t) < 0$ near η^*. Near η^*, $\psi(\eta) = \lambda(\eta)$, so that

$$\begin{aligned}\frac{\partial z(\psi,\lambda)}{\partial \psi} &= \alpha\left(\frac{1-\lambda_t}{\lambda_t}\right)^{1-\alpha} - (1-\alpha)\left(\frac{\lambda_t}{1-\lambda_t}\right)^{\alpha} + \phi_L q_t \\ &= p_1 - p_2 + \phi_L q_t,\end{aligned}$$

and $\frac{\partial g_Y}{\partial \psi} = -\phi_L$. By smooth-pasting, $J'(\eta^*) = 0$; plugging in terms, we have

$$\mathcal{L}(\psi_t) = p_1 - p_2 + J''(\eta^*)\eta\sigma^{\eta}(\psi,\lambda,\eta)\frac{\partial \sigma^{\eta}}{\partial \psi} - \phi_L(J(\eta^*) - q(\eta^*)).$$

Rearranging equation (17) yields $J(\eta^*) = \frac{z(\eta^*)}{r-g_Y} + \frac{1}{2(r-g_Y)}J''(\eta^*)(\sigma^{\eta}(\psi,\eta))^2$. The first term is the present discounted value if the system did not move from η^*. Since welfare is strictly less than $\frac{z(\eta^*)}{r-g_Y}$, $J''(\eta^*) < 0$. $J(\eta) \geq q(\eta)$ because $\theta_t \geq 1$. When $\lambda_t = \psi_t$, $p_1 - p_2 = -\sigma_t^{\theta}(\sigma + \sigma^q)q_t$. By Lemma II.2, $\sigma^q = \frac{q'(\eta)}{q(\eta)}\frac{(\psi-\eta)\sigma}{1-\frac{q'(\eta)}{q(\eta)}(\psi-\eta)}$,[29] and therefore $\frac{\partial \sigma^{\eta}}{\partial \psi} > 0$. In competitive equilibrium $\theta'(\eta^*) = 0$, so that $\sigma_t^{\theta} = 0$, which implies $p_1 = p_2$, and hence $\mathcal{L}(\psi_t) < 0$. Since the marginal social value of leverage is negative, welfare in competitive equilibrium would improve if leverage near η^* decreased on the margin.

This local result can be extended to cases when households are risk-averse and banks maximize more general preferences. (The statement of the proposition and its proof are in the Online Appendix.) The intuition is that banks choose portfolios without internalizing systemic risk. Banks are willing to pay dividends at η^* and banks are instantaneously risk neutral, which might be unobjectionable since households are risk-neutral. But the economy has endogenous risk in addition to the fundamental risk; a regulator would like banks and households to internalize their impact on that risk.

It is important to stress two things. First, the value function $J(\eta)$ depends on the evolution of η_t, which depends on q_t and θ_t over the entire state-space. Thus,

[29]see Online Appendix for derivation.

regulating leverage away from η^* would change $J(\eta)$ near η^* (and also $J'(\eta)$ and $J''(\eta)$ as well as $J''(\eta^*)$). The proposition would still hold relative to the new equilibrium so long as leverage were not regulated near η^*. Second, this result is very much a marginal, local result: the policy exercise is to regulate leverage only local to the stochastic steady state, and welfare improves only for small decreases in leverage very local to η^*. With this in mind, we now turn to global leverage regulation.

III. Limiting Leverage

In this section I analyze the effect of limiting leverage to investigate two things: first, to positively investigate the effect of leverage on stability; second, to normatively investigate how a regulator could improve equilibrium. I consider when leverage cannot exceed an exogenously fixed level, and also when leverage is endogenously determined by value-at-risk constraints.

In the absence of equity issuance constraints, leverage limits are unequivocally bad for welfare.[30] With limited equity issuance, leverage constraints will have nontrivial effects on welfare because leverage affects stability. Limited equity issuance creates a distortion between the marginal value of equity and the marginal utility of consumption ($\theta_t > 1$). Leverage determines the size of the distortion in each state, the probabilities of states, and how the economy transitions between states. Households would be made better off by giving up one unit of consumption to invest an extra unit in bank equity—which they can't do—and so welfare depends critically on the evolution of bank equity.

Because banks do not internalize their impact on price volatility, banks are likely to collectively choose aggregate leverage that creates higher volatility and in-

[30]Consider the effect of a leverage constraint when banks can freely issue equity. Suppose bank leverage is constrained so that $\delta_t^b \leq Ln_t^b$, where L is the constraint. This constraint binds since, in the absence of equity constraints, banks choose infinite leverage without it. As a result, banks' cost of finance is $\frac{Lr^L+r}{L+1}$, which is less than r (the cost of equity) but greater than r^L (the cost of debt). As a result, land used for good 1 earns a higher expected return, implying a lower land price and a higher p_1. One can easily check that this implies that banks will hold smaller portfolios, and so land will be misallocated and liquidity provision decreases.

creases the frequency of bad states. (This is the balance sheet externality present in Brunnermeier and Sannikov (2014).) When banks are more often in bad shape, the economy more often has less intermediation. Regulating leverage changes the distribution and law of motion of the state of the economy, and leverage can be regulated in a way that improves welfare. While banks cannot increase equity instantly by issuing equity, leverage regulation can increase bank equity *in expectation.*

A. *The Model with Borrowing Constraints*

Suppose bank leverage, defined as debt divided by equity, cannot exceed L. Thus, for any bank

$$\delta_t^b \leq L n_t^b. \tag{19}$$

Banks maximize (10) subject to to the dynamic budget constraint (6) and the leverage constraint (19). Since $\omega_t^{2b} = 0$ optimally, the leverage constraint requires that $\omega_t^{1b} \leq L + 1$.

When borrowing constraints bind, the return to land used for good 1 will exceed banks' risk tolerance. Define $\mu_t^A = \frac{p_{1t}}{q_t} + g + \mu_t^q + \sigma\sigma_t^q$ as the expected return banks get from buying land to produce good 1. Then we can characterize the banks' problem as follows.

PROPOSITION III.1: *Let θ_t be given by equation (11), with $\sigma_t^\theta \leq 0$. Then $\theta_t n_t$ represents the maximal future expected payoff that a bank with book value n_t can attain, and $\{\omega_t^1, d\zeta_t\}$ is optimal if and only if*

1) *$\theta_t \geq 1$ $\forall t$, and $\zeta_t > 0$ only when $\theta_t = 1$,*
2) *$\mu_t^\theta = \phi_L - \omega_t^1 \left[\mu_t^A - r + \sigma_t^\theta(\sigma + \sigma_t^q)\right]$,*
3) *If $\omega_t^1 < L+1$, then $\bar{r}_t^{1b} - r^L \leq -\sigma_t^\theta(\sigma + \sigma_t^q)$ with strict equality when $0 < \omega_t^1$, and $\bar{r}_t^{1b} - r^L > -\sigma_t^\theta(\sigma + \sigma_t^q)$ only if $\omega_t^1 = L + 1$,*
4) *The transversality condition $\mathbb{E}[e^{-rt}\theta_t n_t] \to 0$ holds under $\{\omega_t^1, d\zeta_t\}$.*

Notice that when borrowing constraints do not bind, $\mu_t^A - r + \sigma^\theta(\sigma + \sigma_t^q) = 0$. The law of motion of η_t has the following modification.

LEMMA III.1: *With borrowing constraints, the law of motion of η_t is defined by*

$$\mu_t^\eta = \frac{(\psi_t - \eta_t)}{\eta_t}\left(\mu_t^A - r - (\sigma + \sigma^q)^2\right) + \left(\frac{p_{1t}}{q_t} + m(\lambda_t - \psi_t) - (1-\psi_t)\phi_L\right),$$

and σ_t^η and $d\Xi_t$ are as in Lemma II.2.

Numerical Solution. — We solve for equilibrium as before.[31] I limit leverage to $L = 12$ and $L = 8.4$, yielding the leverage outcomes shown in Figure 5. These policies can lead to higher welfare, as seen in Figure 6.

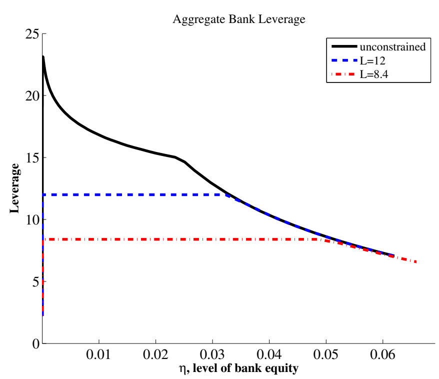


Figure 5. : Equilibrium Leverage

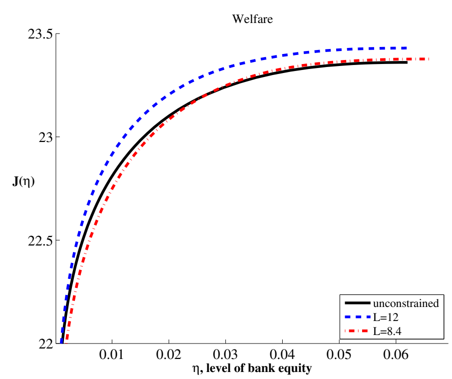


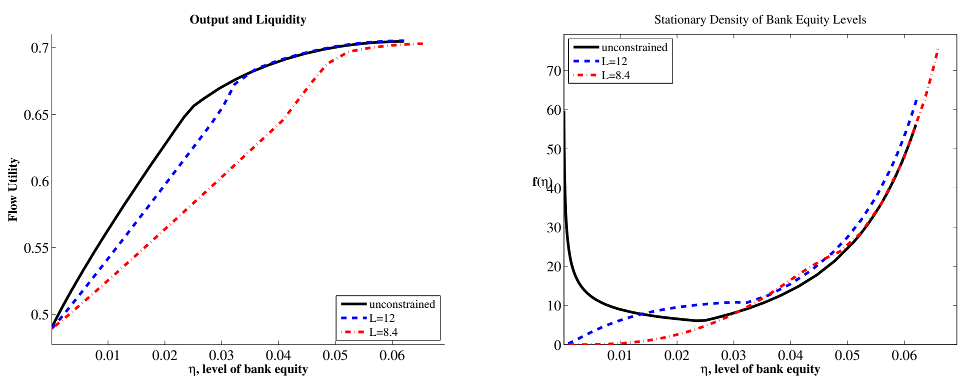
Figure 6. : Welfare with Leverage Limits

The effect of limiting leverage is not monotonic and it is not uniform across the state-space. One policy may be better when bank equity is initially low but not when bank equity is high; welfare depends on how quickly the economy recovers. Limiting leverage to $L = 12$ improves on the equilibrium allocation everywhere, but limiting leverage to $L = 8.4$ is better than the unconstrained case only when

[31]Except for local to $\eta = 0$, the dynamics over the full state-space are not sensitive to the value of $\theta(0)$ so long as it is large. For low $\theta(0)$, local to 0 banks take more risk, leverage skyrockets, and bankruptcy is possible. So long as $\theta(0)$ is large enough, banks decrease leverage near 0.

bank equity is very high (high η). Lowering leverage by a larger amount decreases welfare everywhere.[32]

Tighter leverage constraints monotonically improve stability, but at the expense of flow utility. Figure 7 plots flow utility and the stationary distributions given these leverage constraints. Flow output and liquidity suffer when banks are constrained, but stability is improved (the stationary distribution shifts toward η^*).[33] Leverage creates this fundamental tradeoff between activity and stability: limiting leverage worsens outcomes when constraints bind, but increased stability makes those states less likely to occur. For low η welfare is lower for $L = 8.4$ because when bank equity is low, misallocation is the most severe and stability is less important than moving to good states.[34]



Flow Utility　　Stationary Distribution

Figure 7. : Flow Utility and the Stationary Distribution with Leverage Limits

[32]The result that leverage regulation can improve welfare but that overly tight regulation can decrease welfare is robust to the parameter values in the paper. To illustrate, I calculate the welfare at the stochastic steady state for several different parameter values and calculate the percent welfare gains given leverage caps. The results are in the appendix in Table A1.

[33]One important feature not in this model is that in reality deleveraging often entails dumping assets into “illiquid” markets with slippage and large transaction costs. In such a world, deleveraging might increase volatility as sellers are forced to receive below-market prices, which would rapidly drive down prices, amplifying the fire-sale externality. In either case, the results of this model show that deleveraging also has a stabilizing force that decreases endogenous volatility, because after a deleveraging banks are in a better position to respond to the next shock.

[34]Forcing banks to recapitalize by issuing new equity is one way for bank equity to recover. Of course, there may be equilibrium consequences of this policy, but the results of the model suggest, in my opinion, that this would be wise.

There are two reasons the stationary distribution improves: (i) endogenous volatility is lower; (ii) bank equity recovers more quickly. Tighter leverage constraints lead to lower asset price volatility and lower systemic volatility (σ^η). Banks respond to good and bad shocks differently because they are limited in their ability to issue new equity but not to pay dividends. Bigger shocks are on average worse for banks and increase the likelihood that they have low equity.[35] A key reason that the system is more stable is that the price of good 1 is higher when leverage is constrained. In other words, intermediated investments yield higher returns when banks use less leverage.[36]

In fact, these forces are so strong near zero that leverage limits can completely kill the mode at zero. When we limit leverage: (i) there is lower volatility, which reduces the probability of getting near zero, and (ii) there are higher returns, which ensure that the economy moves away from zero quickly if it ever gets there.[37]

B. Equilibrium with Endogenous Borrowing Constraints

The results of the previous section suggest that leverage is too high both near and away from the stochastic steady state—but that is only true in an economy

[35] Leverage endogenously increases the volatility in an economy because leveraged balance sheets amplify the shocks hitting the leveraged agents and thus amplify how agents react after shocks; this is the insight of the Financial Accelerator literature. The effect of volatility on the distribution of capitalization is perhaps the least intuitive mechanism—but it is also the most significant as it is precisely what macroeconomic instability is about. With financial frictions, increasing equity is a slow process, and in the strictest case the only way to increase equity is to retain earnings. This leads to a key asymmetry in the evolution of capital ratios: they can fall quickly, but they typically rise slowly. Thus, changing the size of shocks can change the distribution of capital ratios. Large bad shocks are fundamentally different from large good shocks: after a large good shock banks can recapitalize by paying a dividend, but after a large bad shock banks can only patiently wait to rebuild equity using retained earnings. Section V in the Online Appendix provides a simple example to demonstrate this mechanism.

[36] Underlying the changes in flow utility and stability are changes in allocations, laws of motions, Sharpe ratios, and the amount of time it takes for the economy to recover. In these examples, tighter leverage restrictions actually lead to faster recovery rates. Plots of these variables, as well as a simulation illustrating the effects of leverage constraints on stability, are in the Online Appendix.

[37] Even without leverage constraints the stationary distribution need not be bimodal for certain model parameters. The mode at zero is larger for: larger exogenous volatility σ (higher likelihood of bad shocks bringing the system of zero); larger liquidity premium ϕ_L (intermediaries use more leverage); lower intermediation penalty m (banks earn lower excess returns). See Section IV for the effect of m and see Section V in the Online Appendix for the effect of σ on the stationary distribution. In contrast, in He and Krishnamurthy (2013) the distribution is never bimodal because banks hold risky assets by assumption and near zero earn large risk premium, which leads to fast recovery.

without endogenous borrowing constraints. Endogenous borrowing constraints may already limit leverage, and maybe too much. Empirical evidence suggests that bank leverage away from the steady state does not behave as in the model thus far.[38] To make a more careful analysis of leverage policies away from steady state requires considering the equilibrium consequences of endogenous borrowing constraints.

In light of empirical evidence, I consider an economy in which banks' assets are endogenously limited by a constant value-at-risk ("VaR") constraint, limiting the value of assets to a multiple of the inverse of the volatility of investments $(\sigma + \sigma_t^q)$. Since volatility is hump-shaped, VaR constraints cause leverage to be U-shaped *when constraints bind*. When the VaR is sufficiently tight (the value at risk is low), leverage is procyclical, and this is empirically realistic for investment banks; looser constraints produce acyclical leverage, which matches the behavior of commercial banks more closely. The positive effects of endogenous borrowing constraints are nearly identical to the results for exogenous and fixed leverage limits: tighter constraints improve stability but increase misallocation and decrease flow utility.

Incorporating endogenous borrowing constraints allow us to consider the effect of a policy of increasing leverage when leverage is endogenously very low (countercyclical regulation). When deleveraging is substantial (endogenous borrowing constraints are very tight), increasing leverage for very low η can improve welfare. Such a policy improves allocations when misallocation is most severe, without increasing instability by too much. This is especially true when banks are able to issue equity at a cost. The full details and results are in the Online Appendix.

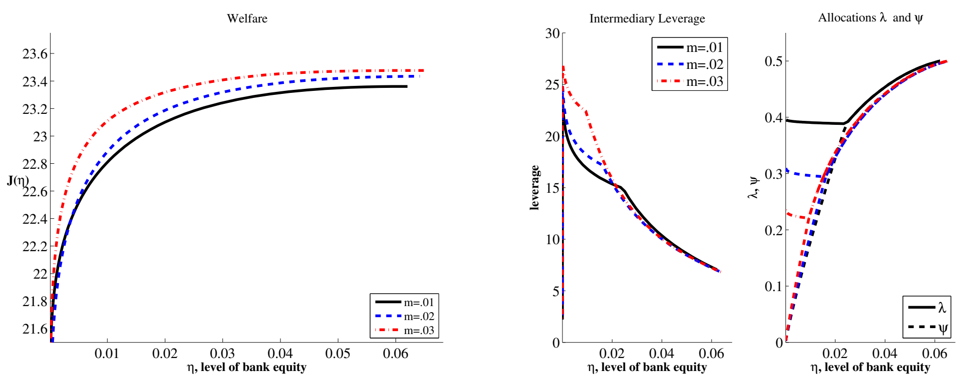
IV. An Intermediation Paradox

The instability caused by banks has first-order consequences for welfare. When the economic environment changes so that banks take on more risk, or so that they

[38]Adrian and Shin (2010, 2011) show that leverage for commercial and investment banks is typically acyclical and pro-cylical respectively; when volatility spikes, their leverage falls. They present very strong evidence that these institutions maintain a constant value-at-risk ("VaR").

are not compensated as much for their risk, instability increases. Paradoxically, when households are *worse* at producing good 1, household welfare can *improve*. If banks are sufficiently important, households are better off when banks are more important. This is because when banks have a larger advantage, they earn higher excess returns, and as a result the economy is more stable.[39]

I illustrate this result by varying m, which determines the growth advantage to intermediated investment. Figure 8 plots welfare, leverage, and land allocations for $m = .01, .02, .03$.[40] Welfare is higher for the larger m—which is when the growth effects of direct investment are the worst of all. Not shown, but unsurprisingly, bank value increases as m increases, and this is the main reason that welfare increases.



Welfare Leverage and Land Allocations

Figure 8. : Household Welfare and Land Allocations for for $m = .01, .02, .03$

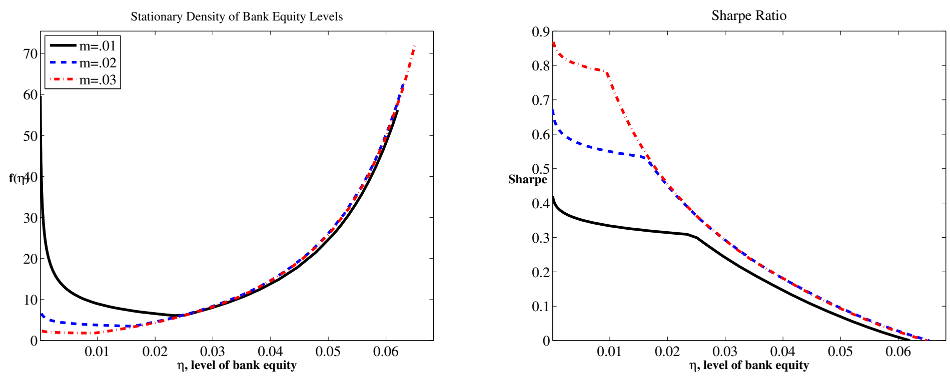
Stability is the crucial reason that welfare increases. Notice that when m is highest, the land allocations are the worst, but welfare is better. This is because banks earn higher excess returns when m is high.[41] Figure 9 plots the stationary

[39]Brunnermeier and Sannikov (2014) analyze how dynamics change as households' productivity differences improve, but they do not analyze the effects on welfare.

[40]Results are the similar with and without borrowing constraints.

[41]See Lemma II.2 for how m affects the law of motion for η, and remember that households will not invest in good 1 until banks' excess returns equal m, implying higher m gives higher excess returns, in

distribution and Sharpe ratios. When m is higher, the Sharpe ratio for a given η is higher; however, because the stationary distribution is shifted toward η^*, the *average* sharpe ratio actually decreases as m increases because the economy spends less time in states with low η and therefore high sharpe ratios.[42]



Stationary Distributions Sharpe Ratios

Figure 9. : Stationary Distributions and Sharpe Ratios for $m = .01, .02, .03$

This "paradox" is clearly not a monotonic relationship; there is not a "discontinuity at zero." If m is zero, then $J(0) = 25$, and so for m sufficiently small households would get welfare close to 25. (For example, welfare is strictly better if $m = .005$.) But when m is large enough, higher m improves welfare by improving stability, and so there is rarely inefficient direct investment by households.

V. Conclusion

I consider a general equilibrium model with a banking sector in order to analyze the tradeoffs between leverage and macroeconomic stability and the consequences for welfare. Leverage increases intermediation and provides liquid deposits, but it

general, to banks. Notice also that banks use more leverage the higher is m. This has the effect of increasing volatility, which all else equal would be worse for stability. See for example the "Volatility Paradox" in Brunnermeier and Sannikov (2014).

[42] The average Sharpe ratios are 14.4%, 12.9%, and 11.2% respectively for $m = .01, .02, .03$.

also increases asset-price volatility and destabilizes the financial sector. Financial sector volatility decreases the mean level of aggregate outcomes because equity levels fall on average faster than they rise. Equilibrium is inefficient because households and banks do not internalize their effects on how frequently banks are in trouble and on how quickly banks rebuild equity after bad shocks.

Regulating leverage alters the severity of aggregate outcomes, but it also affects the frequency and duration of aggregate outcomes by modifying the evolution of the financial sector. My results suggest that countercyclical macroprudential leverage regulation may be wise. With endogenous borrowing constraints, leverage may already be constrained below the optimal level; relaxing these constraints during crises can be beneficial when doing so improves allocations without excessively harming stability and recovery rates.

The results of this paper suggest that a policy of recapitalizing banks, which would mechanically decrease leverage, is a good one. This paper asks what is the best policy response when equity issuance is not an option. Forced equity issuance improves resource allocation, and recovery rates, though there could be general equilibrium effects on volatility and stability.

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Appendices

Parameter Robustness

Because equilibrium leverage can change significantly for different parameter values, it does not make sense to use the same leverage caps in different specifications (sometimes maximum leverage is below 12, for example). To compare across specifications, I normalize by the maximum leverage and the value of leverage at the stochastic steady state. For the baseline leverage regulations are $L_1 = 12$ and $L_2 = 8.4$, I define $\alpha_i = \frac{L_i - L(\eta^*)}{L_{max} - L(\eta^*)}$ to be the location of L_i relative to the maximum leverage and leverage at the steady state. Then the regulations used for a specification are $\hat{L}_i = \hat{L}(\eta^*) + \alpha_i(\hat{L}_{max} - \hat{L}(\eta^*)$, where for the parameter specification $\hat{L}(\eta^*)$ is the leverage at the stochastic steady state and $\hat{L}_{max}$ is the maximum leverage in equilibrium without leverage constraints.

Table A1 presents the results. When not specified, all other parameters are at baseline values.

Table A1—: Welfare Gains and Parameter values

Value	$J(\eta^*)$	$\hat{L}_{max}$	$\hat{L}(\eta^*)$	%Gain, $\hat{L}_1$	%Gain, $\hat{L}_2$
Baseline	23.36	23.13	7.06	0.30	0.07
$\sigma = .01$	24.03	44.64	12.56	0.26	0.01
$\sigma = .03$	22.87	15.00	5.13	0.26	0.07
$\sigma = .04$	22.50	10.95	4.13	0.25	0.07
$\alpha = .4$	24.25	23.56	7.33	0.12	-0.12
$\alpha = .3$	26.19	24.36	7.63	0.04	-0.17
$\phi_L = .015$	23.66	22.25	6.17	0.11	-0.10
$\phi_L = .025$	23.11	24.01	7.87	0.38	0.19
$m = .005$	23.53	21.95	8.30	0.19	0.02
$m = .015$	23.36	24.06	6.75	0.28	-0.01