

## **In Search of a Fair Bet in the Lottery**

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Note: This work should be seen as preliminary and incomplete.

## **In Search of a Fair Bet in the Lottery**

ABSTRACT: Although state-operated lotto games have the worst average expected payoffs among common games of chance, because the jackpot can accumulate, the maximum expected payoff is potentially unlimited. It is possible, therefore, that lotto can exhibit a positive expected return.

This paper examines 18,000 drawings in 34 American lotteries and finds approximately 1% of these drawings provided players with a fair bet. Furthermore, if it were possible for a bettor to purchase every possible combination, most lotteries commonly experience circumstances where such a purchase would provide a positive return with 11% of the drawings providing a fair bet to the player.

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## **Introduction to Lottery Games**

It is generally conceded that state lotteries have among the worst average expected payoffs among games of chance. While sports betting returns 91%, slot machines return 89%, bingo returns 74% and blackjack returns 97%, state lotteries return only 40% to 60% gross revenues to players in the form of prizes on average. Under specific theoretical conditions, however, it has been hypothesized that certain types of lottery games can have much higher payoffs even exceeding 100%. This paper examines a large number of state and multi-state lottery games to determine if lotteries ever provide a “fair bet” to the players, i.e. a bet with a positive net expected return.

State lottery associations generally offer three basic types of lottery games. First, they sell “instant win” lottery tickets where players scratch off tickets after purchase to immediately reveal whether a player has won a prize. Next, are the “on-line” games without a roll-over component. In these games, commonly called “Numbers” games, the player selects a set of numbers and watches a drawing later in the day or in the week during which the lottery association reveals the winning combinations. Any monies earned from ticket sales in a particular week or during a particular game go only to payoff winners in that particular game. No money is carried over to the next week so that the expected payoff from the game is the same regardless of past or current ticket sales. Similarly, “instant win” games also have the same expected payoff regardless of ticket sales. The industry average payout is approximately 55% on instant games and this first type of on-line game.

The final broad type of on-line lottery has a jackpot prize pool which accumulates money over time known as a “lotto” game. If no one hits the jackpot in any particular drawing, the

money in the pool rolls over into the jackpot prize drawing for the next period. If by chance no one wins the jackpot in a number of successive drawings, the jackpot prize can potentially become quite large. As the expected payout from this type of lottery varies with past and current ticket sales, this is the type of lottery that could have a positive net payoff, and therefore this paper will examine this type of lottery.

“Lotto” games, generally consist of an individual picking a set five or six numbers from a group of 35-55 choices. More recently lottery associations have created games where players choose numbers from multiple sets of choices in order to lengthen the odds of winning the jackpot. Lotto games generally have two payoff components. First, individuals who correctly pick some, but not all, of the winning numbers receive prizes that do not depend on the jackpot amount. These lower-tier winners either receive a fixed dollar payout or a parimutuel payout based on the current period ticket sales and the number of current period winners. This lower-tier prize component is usually set at roughly 10% to 30% of the ticket price and varies based on the specific rules of the game.

The second component is the jackpot prize. A portion of ticket sales, usually 20-40% of gross ticket sales, is diverted into the jackpot prize fund. A player who matches all numbers exactly wins the amount in the fund. If more than one ticket matches all the numbers, the money in the fund is divided among the number of winning tickets. If no ticket matches the winning numbers, the money in the fund is carried over until the next period and is added on to the ticket sales from that next period. Many lottos also guarantee a minimum payout so that players still receive large payouts even during the first few drawings of a new jackpot cycle.

Excluding a handful of small cash lottos, traditionally state lotteries require winners to

take jackpot winnings in payments over an extended period, usually between 20 and 30 years. The player's winnings from the jackpot prize fund are invested in interest bearing accounts from which the winner receives annuity payments over a number of years. Lottery associations announce the jackpot to be the nominal sum of these annuity payments. Due to changes in federal law, many lottery associations recently have begun to allow jackpot winners to take their prize winnings in a single lump-sum cash payment if desired. Since the annuity payments include interest earnings, this cash payment is always lower than the sum of the annuity payments, and therefore lottery associations continue to advertise the sum of the annuity payments as the size of the current jackpot in hopes of spurring higher ticket sales. As noted by Matheson and Grote [2003], lotto ticket purchases do not seem to be enhanced by artificially inflating the jackpot in this manner.

### **Payoffs from Lotteries**

The expected payoff from the purchase of a single lotto ticket in drawing  $t$ ,  $ER_t$ , is described by several studies including Krautmann and Ciecka [1993], Ciecka, *et al* [1996], Clotfelter and Cook [1993], Gulley and Scott [1993; 1995], and most recently and most completely by Matheson [2001]. The most common function used to describe  $ER_t$  derives directly from the definition of expected value which states that the expected return from a lottery ticket is simply the probability of winning a particular prize times the value of the prize won summed over all prize levels. Equation (1) describes the before tax expected value of a single lottery ticket after accounting for the possibility of multiple winners of the jackpot prize.

$$(1) \quad ER_t = \sum^i w_i DV_{it} + w_j DV_{jt} \sum_{m_t=0}^{B_t} \frac{P_{m_t}}{(m_t+1)} \quad \text{where}$$

$w_i$  is the probability of winning lower-tier prize  $i$ ,

$DV_{it}$  is the discounted present value of lower-tier prize  $i$  at time  $t$ ,

$w_j$  is the probability of winning the jackpot prize,

$DV_{jt}$  is the discounted present value of the jackpot prize at time  $t$ ,

$m_t$  is the number of tickets bought by competing players at time  $t$  matching the jackpot prize,

$p_{m_t}$  is the probability that exactly  $m_t$  other tickets match the jackpot prize,

$B_t$  is the number of other ticket buyers for the drawing in period  $t$ .

The  $w_i$ 's and  $w_j$  can be calculated in straight forward manner for any lotto based on the game matrix of the specific lotto [See Packer, 1981]. For lower-tier prizes with a fixed prize value,  $DV_{it}$  is a fixed dollar amount set by the lottery association and no further calculations or assumptions are necessary. Roughly one-third of lottery associations, including both of the large multi-state lotteries, Powerball and the Big Game Mega Millions, use fixed dollar amounts for all lower-tier prizes. For lower-tier prizes where the payout is parimutuel, it is convenient to assume that the expected payout from the prize will be the average expected payout which assumes that lottery players are equally likely to choose any combination of numbers.<sup>1</sup> Using

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<sup>1</sup>It is fact that certain combinations of numbers (birthdays, vertical or diagonal columns on the play slip, etc.) are more commonly played than other combinations and therefore by playing rarer combinations (such as numbers all above 28 or 31) a ticket buyer can earn an expected return on the lower tier prizes above the average expected payout. For example, an

this assumption,  $DV_{it} = (1-g)\alpha_i / w_i$  where  $g$  is the vigorish (the amount of total ticket sales kept by the lottery association as government revenue) and  $\alpha_i$  is the percentage of the prize pool allocated to lower-tier prize  $i$ . The majority of state lotto games use parimutuel payoffs for lower-tier prizes. For both types of prize structures, the  $DV_{it}$ 's generally do not require a conversion from advertised value to discounted present value since lower-tier prizes are nearly always paid in a single lump sum rather than in an annuity fashion.

As noted previously, it is much more common for lottery associations to report the sum of the jackpot annuity payments as the jackpot amount rather than the discounted sum of these payments. The value of the advertised jackpot prize,  $AV_{jt}$ , can be converted into discounted present dollars using Equation (2) where  $n$  is the number of annuity payments and  $r_k$  is the interest rate for riskless security with a maturity of  $k$  years.

$$(2) \quad DV_{jt} = \frac{AV_{jt}}{n} \sum_{k=0}^{n-1} (1+r_k)^{-k}$$

Some lotteries including the Colorado Lotto, the New York Lotto, and the California Super Lotto pay an annuity that increases in size over the time period rather than a fixed annual payment. The conversion from an advertised to a discounted jackpot is more complex in these cases but straightforward. The authors will provide further details upon request. For the annuity

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examination of 668 drawings in the Texas Lotto (a 6 of 50 Lotto) shows that the average payout for choosing 5 out of 6 numbers correctly was \$1,661 and \$105 for choosing 4 of 6 correctly. However, in the 44 drawings where the smallest number drawn was 29 or higher, the average payouts were \$2,122 and \$131 respectively while in the 106 drawings where the highest number drawn was 28 or lower, the average payouts were \$1,303 and \$86 on average. See Clotfelter and Cook [1989, p. 81], MacLean, et.al. [1992], or Thaler and Ziemba [1988] for further discussion.

lengths and the prevailing interest rates over the time periods in the data set, the advertised jackpot was between 1.5 and 2.5 times larger than the discounted present value of the jackpot.

Finally, the binomial function is used to calculate the probability that exactly  $m$  tickets purchased by other bettors match the winning jackpot numbers and is a direct function of  $B_t$ . Equation (3) describes this function.

$$(3) \quad p_{m_t} = \frac{B_t! w_j^{m_t} (1-w_j)^{(B_t-m_t)}}{m_t! (B_t-m_t)!}$$

Using the Poisson distribution as an approximation to the binomial distribution, Gulley and Scott [1993; 1995], Clotfelter and Cook [1993], and Matheson [2001] combine Equations (1) and (3) into Equation (4).

$$(4) \quad ER_t = \sum w_i DV_{it} + \frac{DV_{jt}}{B_t} (1 - e^{-B_t w_j})$$

The final consideration necessary for the proper calculation of the expected return of the purchase of a lottery ticket is the issue of taxation. Following Matheson [2001], one must subtract applicable taxes from the expected return of a lottery ticket since lottery winnings are fully taxable as income at least at the federal level. In addition, the purchase price of the lottery ticket is tax deductible but only to the extent of any lottery winnings. For the purchase of a single ticket, this essentially means that all winnings are taxable but that the price of the ticket is tax



deductible if you win a prize. Even the smallest lower-tier prizes are generally larger than the price of the ticket so any time a prize is won the price of the ticket can be fully deducted. The inclusion of taxes changes Equation (4) to

$$(5) \quad ER_t = \left[ \sum w_i DV_{it} + \frac{DV_{jt}}{B_t} (1 - e^{-B_t w_j}) (1 - \theta) \right] + \left[ \sum w_i + w_j \right] \theta \tau$$

where  $\theta$  is the tax rate and  $\tau$  is the price of a ticket.

### **Empirical Estimates of Expected Payoffs from Lotteries**

The question of interest in this paper is whether the expected return from the purchase of a lottery ticket ever exceeds its price. Since price of a lotto ticket and the odds of winning remain fixed regardless of the size of the jackpot, and since the jackpot fund,  $DV_{jt}$ , for any lotto game can increase in an unconstrained fashion if there are many ticket buyers but no jackpot winners over a successive number of drawings, the expected return  $ER_t$  is potentially limitless. The complicating factor, however, is the problem that as the advertised jackpot grows, the number of ticket buyers increases as well. The increased number of ticket buyers increases the probability that the winning number will be shared by two or more tickets. Thus, the increase in expected return due to the increase in the size of the jackpot is tempered by the prospect of potentially having to share this larger jackpot among several winners.

The past literature has been mixed in its views on whether or not state lotteries ever provide fair bets to their participants. Clotfelter and Cook [1989, 1990] stress that lotteries have an average return of approximately 50%, emphasizing the average loss while down-playing the

potential for positive expected returns. While Clotfelter and Cook do acknowledge the possibility of lotteries that provide a fair bet, they pronounce that “such occasions are rare indeed” [Clotfelter and Cook, 1990, p. 109]. Scott and Gulley [1995] consider three different state lotteries, finding little evidence of positive expected returns, justifying their argument that if such returns did continually exist, the lottery market would not be efficient. Basing their results on the possibility of playing rare numbers rather than opportunities presented by large jackpots, Thaler and Ziemba [1988] recognize that expected returns can be positive for state lotteries and examine the conditions by which this will occur. They are quick to caution that an individual attempting to capitalize on these positive expected returns will likely go bankrupt before his or her winning combination of numbers are actually drawn, a viewpoint shared by MacLean, Ziemba and Blazenko [1992] among others. Finally, Krautmann and Ciecka [1993] and Ciecka, Epstein and Krautmann [1996] also examine the phenomenon of fair bet lottos but find only one such opportunity in the lotteries which they considered after accounting for taxation.

In order to estimate the *ex ante* expected return from the purchase of a lotto ticket, one must not only estimate the odds of winning but also the odds of sharing the grand prize with one or more other tickets. Hence, an estimate of number of tickets purchased by other bettors in the lotto drawing,  $B$ , must be obtained. Numerous papers have attempted to generate estimates of ticket sales including Krautmann and Ciecka [1993], Ciecka, *et al* [1996], Gulley and Scott [1993; 1995], and Forrest, *et al* [2002]. Most studies find that ticket sales can be quite accurately forecasted based on factors such as the rollover amount, the rollover jackpot squared, the day of week of the next drawing, and time trends.

In order to facilitate the examination of a large number of lotto games, this paper will instead examine the *ex post* expected return from the purchase of a lotto ticket based on actual ticket sales rather than forecasted ticket sales. Here, the *ex post* expected return only refers to knowledge of the number of other tickets sold, not, of course, knowledge of the winning numbers or the numbers selected by other players. While it is certainly true that the *ex post* and *ex ante* ticket sales (and hence *ex post* and *ex ante* returns) may differ from one another if players inaccurately estimate ticket sales, previous research has found that players can quite closely estimate ticket sales and do not generally make systematic forecasting errors. Given these results, it can be said that the *ex ante* and *ex post* estimates approximately match one another on any individual drawing and that on average over many drawings will exactly match. For simply ascertaining the relative frequency of fair bets in the lottery, the *ex post* method gives a good approximation with a significant reduction in computational difficulty.

To test for fair bets in the lottery, we have collected data on jackpot size, ticket sales, and game rules (including payoff percentages, game matrix, number of weekly drawings, or annuity length) from 34 state and multi-state lotto games in order to calculate the *ex post* expected payoff from these lotteries for each available drawing.

For each drawing, the after-tax return from the purchase of a single lotto ticket with randomly selected numbers is calculated using Equation (5). For each lottery the expected return from the lower tier prizes was determined using information provided by the individual lottery associations. As mentioned previously, for lower-tier prizes with a parimutuel payout, we assumed that all number combinations were equally likely to be chosen, and therefore the expected prize is the same regardless of the numbers chosen. The expected payoff for prize  $i$  was

set equal to the percentage of total ticket sales allocated to the specific prize pool divided by the probability of winning prize  $i$ . As some lottos award free lotto tickets as a lower-tier prize, we have assumed that the value of a free \$1.00 lotto ticket was \$0.50.

The expected return from the jackpot prize was calculated using individual lottery rules, prevailing interest rates, and the *ex post* revelation of actual ticket sales. The expected value of a winning lottery ticket can be calculated by calculating the probability of sharing the jackpot with one or more other winners. Of course, the sum of the annuity payments from the jackpot must be converted to present dollars using interest rates. Interest rates from government constant maturity series bonds with varying maturity lengths were used in order to closely match the actual interest rates lottery officials would receive when purchasing bonds that would pay for the lottery annuity payments. For all lotteries a tax rate of  $\theta = 0.3$  was used.

Again, the expected values are for a ticket with randomly selected numbers. Since the actual distribution of numbers selected by other buyers is not entirely uniform, it is possible for an informed buyer to improve upon their return by selecting infrequently chosen numbers [Thaler and Ziemba, 1988]. This ability to earn above normal returns is limited by the amount to which the distribution of numbers played deviates from a uniform distribution. Since roughly 70% of all lotto tickets sold use computer generated numbers which can be reasonably assumed to follow a uniform distribution, any supernormal expected returns are limited to the deviation from uniformity by the 30% of tickets that are sold to players who select their own numbers. Furthermore, as lotto jackpots grow, the percentage of players selecting their own numbers falls, further reducing any ability of players to select advantageous numbers during periods of high expected returns. Still, any expected returns should be seen as lower bounds for the game. Table

1 shows the lotteries examined, the range of dates for drawings, the total number of drawings, the total number of drawings presenting a fair bet, the percentage of such fair bets, and the maximum expected return per dollar played.

The results both confirm and counter the prevailing literature. Overall, it is shown that fair bets are indeed rare occurrences with roughly 1% of drawings providing a player with a fair bet. On the other hand, the instances of fair bets may be significantly more common than previously believed. Half of the games studied showed at least one instance of a fair bet, and numerous games provided players with even odds on a relatively frequent basis. Several of the states exhibited even odds in 4% or more of the drawings.

It is also worthwhile to note that among the lotteries providing fair bets, several have maximum net expected payoffs well in excess of the price of the ticket with Indiana, Kansas, Kentucky and Missouri having a maximum expected gain of 40% or more and Oregon having a maximum expected return of over \$2.20 on the purchase of a single one dollar ticket. Another fact that can be observed in Table 1 is the lotteries with positive maximum expected payoffs tend to be in smaller states. The eye-popping jackpots advertised in the Powerball and Big Game Lotteries as well as those in the bigger states such as New York, California, Texas, and Florida, attract large numbers of buyers diminishing the expected value of the ticket. As hypothesized by Forrest, *et al*, [2002], players seem to react to big jackpots rather than big expected returns.

### **Expected Payoffs from the “Trump Ticket”**

It has been suggested that there may be conditions during which it may be profitable to corner a lottery game by purchasing every possible combination of numbers for a given drawing.

Krautman and Ciecka [1993] and Matheson [2001] dub this strategy the “Trump Ticket.” Calculating the expected payoffs requires some additional calculations. Assuming that other lottery players’ decisions on whether to buy tickets remain constant regardless of whether another player buys the Trump Ticket, the purchase of a Trump Ticket does not affect the probability of any single ticket winning the jackpot nor does it change the expected number of winning tickets among the other buyers in the particular drawing. The purchase does, however, increase the size of the jackpot that the jackpot winner(s) receives. Since the purchase of the Trump Ticket necessitates a large purchase of tickets, if a specific portion of ticket sales is allocated to the jackpot prize pool, as in most games, the purchase of the Trump Ticket will cause a significant increase in the size of the jackpot. Mathematically,  $DV_{jt}^{TT} = DV_{jt} + \tau \alpha_j / w_j$  where  $DV_{jt}^{TT}$  is the discounted present value of the jackpot after the purchase of the Trump Ticket,  $DV_{jt}$  is the discounted present value of the jackpot before the purchase of the Trump Ticket,  $\tau$  is the price of a single lottery ticket,  $\alpha_j$  is the percentage of gross sales allocated to the jackpot pool, and  $w_j$  is the probability of winning the jackpot prize. Since all number combinations are chosen under a Trump Ticket strategy, it is not necessary to assume that other players’ number selections are uniformly distributed.

The issue of taxation again must be considered. As with the purchase of a single ticket, any winnings are fully taxable at the rate  $\theta$ , but the Trump Ticket purchaser may deduct the cost of the tickets purchased to the extent of any winnings. If the purchaser’s winnings exceed the cost of the Trump Ticket then the winnings less the cost of the Trump Ticket are taxable. If the purchaser’s winnings are less than the cost of the Trump Ticket, then the full cost of the Trump Ticket is not deductible, but the purchaser will not have to pay taxes on any of the winnings,

either.

Table 2 shows the maximum expected return per dollar played for both a single ticket and a Trump Ticket purchase for every lotto game as well as the number of Trump Ticket drawings providing a fair bet. In comparing Tables 1 and 2, the first obvious conclusion is that Trump Ticket purchases are more often associated with positive expected returns than are single ticket purchases. As noted by Matheson [2001], the purchase of a Trump Ticket always has a higher expected return per dollar played than the purchase of a single ticket for two reasons. First, the purchase of the Trump Ticket increases the size of the jackpot without changing the expected number of other players matching the jackpot ticket leading to a higher expected payout from the grand prize. Second, because the purchase of the Trump ticket guarantees at least a share in the winning jackpot (as well as lower tier prizes), the purchaser of the Trump Ticket has a much higher chance of being able to deduct the price of the tickets from applicable taxes than the purchaser of a single ticket. Therefore, a significantly greater number of the lotteries studied provide opportunities for positive expected returns for the Trump Ticket purchaser than for the single ticket purchaser. With only one exception, each lottery examined shows at least one instance of the Trump Ticket providing greater than even odds.

The other startling aspect of Table 2 is simply the extraordinarily high number of times that the Trump Ticket presents a fair bet. Overall, 11% of the drawings examined provided an even odds bet for the purchase of the Trump Ticket with one-third of the games presenting an fair bet during at least 20% of draws. The size of the potential winnings is also surprising with many games offering an after-tax expected rate of return of over 50% at their highest point.

## **Explaining the Results**

In the past, many have addressed the issue of why people purchase lottery tickets in the first place. Considering the results in the tables above, a better question appears to be why more people don't purchase lottery tickets. To consider this, however, individual ticket purchasers must be considered separately from the Trump Ticket purchasers because it will be argued that the two groups may have different incentives and constraints.

For an individual, there are two approaches. The first, and most plausible explanation, is that the above analysis avoids the issue of risk aversion. Even if the expected return is positive on average for an individual ticket purchaser for a given lottery draw, the actual return for most ticket holders will still be zero once the lottery drawing is completed. Balancing this risk of money won versus money lost is only a straightforward comparison of expected gains versus costs if an individual is risk neutral. Since the marginal utility of the single dollar lost to gambling is likely to be higher than the marginal utility of the one-millionth (or one-hundred millionth) dollar won, utility maximizing players may choose not to play despite a drawing with a positive net expected value. While clearly a lesser problem, the risk aversion issue also applies to the purchase of the Trump Ticket. Although the purchaser of the Trump Ticket is guaranteed a portion of the winning prize, the number of other tickets with which the Trump Ticket will have to share the jackpot is still subject to random chance. Therefore, even in the presence of a very high expected return, the purchaser of the Trump Ticket may lose money if the number of other winners is unexpectedly high.

One way to measure the level of risk aversion is to consider the risk premium that an individual would be willing to pay to avoid the risk, comparable to an insurance premium paid to



avoid the risk of loss on personal property. In the case of purchasing a risky asset or lottery ticket, the analysis is reversed: the premium is the amount of extra return that must be paid on a risky asset in order to induce an individual to purchase the risky asset rather than a less-risky alternative. Considering Table 1, this argument may indeed hold for the expected returns close to one dollar, Georgia or Washington for example. What about the expected returns of \$1.30 or better in several states or an expected return in Oregon of \$2.20? Must the expected return be in excess of 30% or as much as 120% in order to induce a risk-averse individual to purchase a lottery ticket? In the case of Oregon, \$741,729 in sales were recorded for its record \$18 million jackpot, compared to \$207,690 several weeks earlier before the jackpot began to rollover for the first time starting at the \$1 million level. That is, just over 3.5 times the number of tickets were sold for a jackpot that was 18 times higher. Although sales did grow somewhat, the increase seems extremely low given the 120% expected return on the purchase at an \$18 million dollar jackpot level relative to a negative expected return at the \$1 million level.

Even though it is difficult to say precisely what size premium must be available to provide a risk-averse individual with the incentive to purchase a lottery ticket, the lack of buyers in the presence of returns in excess of 30% for a three day investment is difficult to explain by risk aversion alone. An easy explanation would be that individuals are unaware of how to perform an expected return calculation and do not know that there are positive expected returns in some lottery drawings. Although perhaps true, placing the majority of the explanation on this alone runs contrary to the belief in rational individuals engaging in economic decision-making. Even though individuals may not be able to or are unwilling to do complicated mathematical calculations in their head, they will consider such things as risk and return when making

decisions and changes in these arguments will affect marginal decision-making. Indeed, Scott and Gulley [1995] contend that any instance of positive net expected returns from a lottery game violate efficiency in gambling markets. It is also possible that the state lotteries where the purchase of a single ticket becomes a fair bet simply do a poor job of advertising the size of the jackpot so that many potential players are not aware of the size of the current jackpot.

There is also an alternative explanation. This explanation lies in the consideration of the purchase of a lottery ticket not as a form of investment, in which case only expected return matters, but rather as a form of recreation. When examined from this perspective, the correct approach for analysis is not how much an individual can expect to earn from a dollar spent, but rather a comparison of the marginal utility gained per dollar spent relative to alternative forms of recreational spending. Higher net expected returns may result in only slightly higher ticket sales if players derive most of their utility from the joy of gambling rather than the winnings themselves. Combined with the risk aversion factor, expected winnings would be appropriately discounted to allow for dollars to be spent on other forms of recreation. Therefore, while high jackpots may attract more dollars in sales from those who are buying tickets and may attract new buyers to the market, bettors will choose to spend only to the extent of their recreational spending budget and then only if it provides a greater expected utility per dollar than another activity. There is also a third group of consumers for whom lottery spending is not a factor in their recreational spending decision, who will choose not to purchase tickets regardless of the size of expected returns. Such individuals are those who are morally opposed to gambling per se or those who are opposed to the government's involvement in what they perceive as gambling activities.

While the previous arguments may explain why individual purchases of lottery tickets do not rise to eliminate the fair bet, it is not possible to use the “lottery ticket as a form of recreation” argument to explain the results of Table 2. Presumably, any individuals involved in a Trump Ticket purchase of all lottery tickets are looking to maximize their expected return and not looking for alternate forms of spending. A better explanation lies in the transaction and organization costs that are encountered in the act of purchasing all of the possible combinations prior to a given drawing. Such costs include the costs of financing the multi-million dollar purchase and the act of purchasing the tickets.

The cost of the financing the purchase is similar to the costs of making any large financial investment; although in this case there is the additional issue of the defending the legitimacy of using the acquired funds to purchase lottery tickets. The formation of a private consortium of investors avoids the principal-agent problem to some degree (by avoiding a financial intermediary like a bank) but still involves the problem of attracting private investors to the consortium. Even if this latter problem can be overcome and the finances can be obtained, however, the tickets still have to be physically purchased in order to ensure that the jackpot is won. Particularly for lotteries that have either bi-weekly or tri-weekly drawings, this presents a problem of actually purchasing the required number of tickets within the two or three days prior to the next drawing. In a 1992 drawing of the Virginia Lotto, an Australian consortium attempted to purchase the Trump Ticket, yet despite the aid of an agreement formed in advance with a large grocery chain, the consortium was only able to purchase 2.4 million of the possible 7.1 million number combinations by the time of the drawing [Krautmann and Ciecka 1993, p. 161].

These costs to organizing both investors and sellers of tickets in an unconventional

market necessarily will increase the costs to a Trump Ticket purchaser above the \$1.00 per ticket that it essentially costs the single-ticket purchaser (ignoring minor transportation and other transaction costs). The question remains, however, as to whether these costs are substantial enough to preclude other consortia from forming and succeeding in purchasing the Trump Ticket. As can be seen in Table 2, the expected returns in Kentucky, Kansas, and Oregon are over 100% for the largest jackpots, returns that are substantially higher than those provided in private markets (particularly considering this is not an annual rate of return). Given the problem with physically purchasing the tickets, however, this is not a guaranteed rate of return. The lower the percentage of combinations that can actually be purchased in time for the lottery drawing, the greater the problem of risk aversion on the part of the investors will become.

### **Conclusions and Recommendations**

The results presented in this paper suggest that it is not only theoretically possible for lotteries to exhibit periods where the purchase of a single lottery ticket has a positive net present value, it is in fact a regular, though uncommon, occurrence for lotteries, especially smaller lotteries, to exhibit this trait. Since the presence of a fair bet in the purchase of a single lottery ticket represents a violation of efficient markets, lottery associations where fair bets routinely occur should be able to increase ticket sales in the presence of these higher expected returns through public education and better advertising of high jackpots.

In addition, it is extremely common that the purchase of the Trump Ticket, i.e. the purchase of all available combinations, would provide a fair bet to the buyer. The existence of fair bets in the purchase of the Trump Ticket is likely due to the transaction costs associated with

the purchase of such a ticket. One recommendation would be that lottery associations consider allowing for the direct purchase of a Trump Ticket by investment consortiums. For example, the jackpot for the Mass Millions game reached levels that would have allowed for the profitable purchase of the Trump Ticket during two separate runs in 1998. The purchase of the Trump Ticket in both of these cases would have led to two one-time ticket sales of roughly \$14 million. Total ticket sales for the Mass Millions game were \$64 million in 1998. Of course, the purchase of the Trump Ticket would eliminate the potential for high ticket sales in subsequent drawings since the jackpot would revert to its beginning value following the consortium winning the prize. As a rough estimate, this factor would have reduced ticket sales by approximately \$22 million. The result is a net increase in ticket sales by the Mass Millions game of roughly \$6 million or 10% in 1998. Similarly, Oregon experienced four runs in 1998 where the jackpot exceeded the threshold for a profitable purchase of the Trump Ticket. The purchase of the Trump Ticket in each case would result in an additional \$14 million in ticket sales countered by a corresponding drop of \$6 million due to lower average jackpots. The \$8 million in estimated gains would have represented a 25% increase in the actual ticket sales for Oregon.

Of course, the ticket sale gains must be balanced against the possible loss of trust in the lottery by public who may feel that such direct purchase is akin to “fixing” the lottery. In addition, if ticket sales are fueled by stories of the “regular guy” hitting it big, it is likely that stories of rich investment consortiums getting even richer through taking advantage of such a direct purchase may depress sales further. In addition, the purchase of a Trump Ticket reduces the expected return of all other purchased tickets as other buyers are now guaranteed to have to share a potential jackpot with at least one other winner [Krautmann and Ciecka, 1993]. Should

the sale of such a ticket become known during the course of a drawing, purchases by other ticket buyers would likely decrease. Nonetheless, such a strategy may be worthwhile for an industry that is experiencing a decrease in sales following years of expansion.

## REFERENCES

Ciecka, James, Epstein, Seth and Krautmann, Anthony C. State Lotteries and Externalities to their Participants. *Atlantic Economic Journal*, December 1996, 349-60.

Clotfelter, Charles T. and Cook, Philip J. *Selling Hope*. Cambridge, MA: Harvard University Press, 1989.

Clotfelter, Charles T. and Cook, Philip J. On the Economies of State Lotteries. *Journal of Economic Perspectives*, Fall 1990, 105-19.

Cook, Philip J. and Clotfelter, Charles T. The Peculiar Scale Economies of Lotto. *American Economic Review*, June 1993, 634-43.

DeBoer, Larry. Lotto Sales Stagnation: Product Maturity or Small Jackpots? *Growth and Change*, Winter 1990, 73-7.

Forrest, D., Simmons, R. and Chesters, N. "Buying a Dream: Alternative Models of Demand for Lotto," *Economic Inquiry*, 40:3, July 2002, 485-96.

Krautman, Anthony C. and Ciecka, James E. When are State Lotteries a Good Bet? *Eastern Economic Journal*, Spring 1993, 157-64.

MacLean, L. C., Ziemba, W. T. and Blazenko, G. Growth Versus Security in Dynamic Investment Analysis. *Management Science*, November 1992, 1562-85.

Matheson, Victor A. When are State Lotteries a Good Bet? (Revisited) *Eastern Economic Journal*, Winter 2001, 55-70.

Matheson, Victor A. and Grote, Kent. Jacking up the Jackpot: Are Lotto Consumers Fooled by Annuity Payments? *Public Finance Review*, forthcoming.

Packel, Edward. *The Mathematics of Games and Gambling*. Washington, D.C.: The Mathematical Association of America, 1981.

Scott, Frank A. and Gully, O. David. The Demand for Wagering on State-Operated Lotto Games. *National Tax Journal*. March 1993, 13-22.

Scott, Frank A. and Gulley, O. David. Testing for Efficiency in Lotto Markets. *Economic Inquiry*, April 1995, 175-88.

Shapira, Zur and Venezia, Itzhak. Size and Frequency of Prizes as Determinants of the Demand for Lotteries. *Organizational Behavior and Human Decision Processes*, July 1992, 307-18.

Thaler, Richard H. and Ziemba, William T. Anomalies: Paramutuel Betting Markets: Racetracks and Lotteries. *Journal of Economic Perspectives*, Spring 1988, 161-74.

Thiel, Stuart E. Policy, Participation and Revenue in Washington State Lotto. *National Tax Journal*, June 1991, 225-35.



Table 1: Returns for Single Ticket purchase

Lottery	Dates of Data	Highest Observed Jackpot	Max. Expected Return (per \$1.00 played)	# of Draws	# of Positive Draws	% Positive Draws
Multi-state "Powerball"	4/22/92 - 1/15/03	\$315 million	\$0.727	1,121	0	0%
Multi-state "Big Game"	9/06/96 - 5/04/99	\$190 million	\$0.776	215	0	0%
Tri-State "Megabucks"	3/12/97 - 5/29/99	\$8.2 million	\$0.719	214	0	0%
Tri-State "Win Cash"	9/12/97 - 5/28/99	\$2.33 million	\$0.973	179	0	0%
Tri-West "Lotto"	2/04/95 - 1/31/98	\$1.63 million	\$1.067	307	2	0.7%
Multi-state "Wild Card"	2/04/98 - 7/28/01	\$2.06 million	\$0.681	364	0	0%
Arizona "Lotto"	11/28/98 - 5/22/99	\$10.1 million	\$0.921	51	0	0%
California "Super Lotto"	10/18/86 - 1/19/02	\$141 million	\$0.753	1,544	0	0%
Colorado "Lotto"	9/14/90 - 7/28/01	\$27 million	\$0.977	1,150	0	0%
Connecticut "Lotto"	9/20/94 - 8/07/01	\$26 million	\$1.251	719	10	1.4%
Delaware "All Cash"	10/27/98 - 5/18/99	\$1.13 million	\$0.888	88	0	0%
Florida "Lotto"	5/07/88 - 7/28/01	\$106.5 million	\$0.945	783	0	0%
Georgia "Lotto"	8/31/96 - 8/04/01	\$30.4 million	\$1.027	258	1	0.4%
Illinois	4/14/99 - 8/01/01	\$33 million	\$1.253	241	6	2.5%
Indiana	9/03/94 - 8/01/01	\$42 million	\$1.292	542	9	1.7%
Kansas "Cash"	8/18/96 - 5/12/99	\$2.00 million	\$1.565	428	21	4.9%
Kentucky "Lotto"	3/01/95 - 7/28/01	\$20 million	\$1.444	670	29	4.3%
Louisiana	4/19/98 - 5/22/99	\$2.05 million	\$0.660	114	0	0%
Maryland	1/03/98 - 7/14/99	\$18.5 million	\$1.144	160	5	3.1%
Mass. "Megabucks"	11/05/97 - 8/11/01	\$14.3 million	\$1.340	394	21	5.3%
Mass. "Millions"	11/06/97 - 8/13/01	\$30.6 million	\$1.145	394	6	1.5%
Michigan "Lotto"	9/04/96 - 7/28/01	\$40 million	\$1.159	497	10	2.0%
Minnesota "Gopher 5"	5/24/91 - 7/24/01	\$1.40 million	\$0.918	1,062	0	0%
Missouri "Lotto"	1/03/96 - 6/30/01	\$11.6 million	\$1.546	459	22	4.8%
New Jersey	7/03/95 - 4/05/99	\$35 million	\$1.086	393	1	0.3%
New York	4/14/99 - 8/01/01	\$45 million	\$0.691	375	0	0%
Ohio "Super Lotto"	1/12/91 - 7/28/01	\$54 million	\$1.004	1,099	1	0.1%
Oregon "Lotto"	4/19/95 - 5/19/01	\$18 million	\$2.204	636	32	5.0%
Pennsylvania "Pick 6"	9/12/98 - 8/04/01	\$73 million	\$0.843	303	0	0%
South Dakota "Cash"	7/03/96 - 8/11/01	\$0.34 million	\$0.884	530	0	0%
Texas "Lotto"	11/14/92 - 1/15/03	\$85 million	\$0.969	1,061	0	0%
Virginia "Lotto"	1/27/90 - 5/05/99	\$28 million	\$1.168	929	6	0.7%
Washington	1/01/97 - 5/26/99	\$24 million	\$1.042	251	2	0.8%
Wisconsin	6/20/92 - 5/15/99	\$16.5 million	\$0.812	<u>721</u>	<u>0</u>	<u>0%</u>
Total				18,252	184	1.0%

Table 2: Returns for the Trump Ticket purchase

<b>Lottery</b>	<b>Max. Expected Return: Single Ticket</b>	<b>Max. Expected Return: Trump Ticket</b>	<b># of Draws</b>	<b># of Positive Draws</b>	<b>% Positive Draws</b>
Powerball	\$0.727	\$1.036	1,121	10	0.9%
Big Game	\$0.776	\$1.120	215	2	0.9%
Tri- Megabucks	\$0.719	\$1.114	214	3	1.4%
Tri- Win Cash	\$0.973	\$1.443	179	33	18.4%
Tri-West	\$1.067	\$1.590	307	67	21.8%
Wild Card	\$0.681	\$1.234	364	21	5.8%
Arizona	\$0.921	\$1.434	51	14	27.5%
California	\$0.753	\$1.109	1,544	7	0.5%
Colorado	\$0.977	\$1.397	1,150	91	7.9%
Connecticut	\$1.251	\$1.767	719	202	28.1%
Delaware	\$0.888	\$1.438	88	30	34.1%
Florida	\$0.945	\$1.321	783	23	2.9%
Georgia	\$1.027	\$1.368	258	28	10.9%
Illinois	\$1.257	\$1.745	241	43	17.8%
Indiana	\$1.292	\$1.812	542	86	15.9%
Kansas	\$1.565	\$2.055	428	93	21.7%
Kentucky	\$1.444	\$2.014	670	227	33.9%
Louisiana	\$0.660	\$0.982	114	0	0%
Maryland	\$1.144	\$1.545	160	45	28.1%
Mass Mega	\$1.340	\$1.764	394	87	22.1%
Mass Millions	\$1.145	\$1.630	394	145	36.8%
Michigan	\$1.159	\$1.488	497	60	12.1%
Minnesota	\$0.918	\$1.338	1,062	76	7.2%
Missouri	\$1.546	\$1.911	459	102	22.2%
New Jersey	\$1.086	\$1.531	393	27	6.9%
New York	\$0.691	\$1.043	375	3	0.8%
Ohio	\$1.004	\$1.281	1,099	48	4.4%
Oregon	\$2.204	\$2.498	636	96	15.1%
Pennsylvania	\$0.853	\$1.173	303	27	8.9%
South Dakota	\$0.884	\$1.330	530	34	6.4%
Texas	\$0.969	\$1.189	1,061	52	4.9%
Virginia	\$1.168	\$1.670	929	127	13.7%
Washington	\$1.042	\$1.305	251	19	7.6%
Wisconsin	\$0.812	\$1.360	<u>721</u>	<u>84</u>	<u>11.7%</u>
Total			18,252	2,012	11.0%