The SMALL Program at Williams College

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1. Introduction

The SMALL program seeks to introduce undergraduates to the excitement and challenge of doing original research in mathematics. Too often, we have seen talented students lose interest in mathematics because they perceive it as a lifeless subject within which one simply studies the work of long-dead mathematicians. In SMALL, they get to work on current projects under the direction of faculty who are themselves actively engaged in cutting edge research. They see the vitality of mathematics and their own potential for making a contribution. They understand the ultimate goal of learning, namely to utilize that learning in order to make their own advances and further the pursuit of knowledge.

Although SMALL began as a program for Williams students, outside students have come to play an ever larger and more important role. Since its founding in 1988, the program has guided over 300 students in mathematics research. SMALL alumni have gone on to earn Ph.D.s in mathematics and related fields, and to other activities such as teaching mathematics in high school. (SMALL is an acronym from the names of the founding faculty, Silva, Morgan, Adams, Lenhart, Levine.) Over 60 research articles coming out of SMALL have been published or been accepted for publication in mathematics journals. More than 25 SMALL alumni have earned Ph.D.s in mathematics or mathematics related fields, and many are on the mathematics regular faculty at colleges and universities, while many others are in postdoctoral or visiting positions. Still other alumni are on university faculties other than mathematics. Of the 80 SMALL participants in the five years 1999 - 2003 more than 50 are enrolled or have completed Ph.D. programs in mathematics or related fields. SMALL has also contributed to the increasing interest in mathematics among our students at Williams, so that currently we have 50 senior mathematics majors, about 10% of our senior class.

The faculty who have participated in the program include: Colin Adams, Duane Bailey, Deborah Bergstrand, Gerald Bope, Carsten Botts, Edward Burger, Elizabeth Camp, Charles Chace, Satyan Devadoss, Richard De Veaux, Thomas Garrity, Gary Lawlor, David Levine, William Lenhart, Susan Loepp, Robert Mizner, Frank Morgan, Allison Pacelli, Cesar Silva, Kris Tapp, Alice Underwood, David Witte, and Janine Wittwer. The first director of the program was Frank Morgan; the

other directors have been Colin Adams, Deborah Bergstrand, Thomas Garrity and Cesar Silva.

The research topics have included knot theory, hyperbolic manifolds, minimal surfaces, symmetry groups, combinatorics, graph theory, computational geometry, algebraic geometry, dynamics and ergodic theory, parallel processing, topology of robotics, CR structures, Riemannian geometry, neural networks, bayesian statistics, and commutative algebra.

The funding for the program has come mainly from the National Science Foundation and Williams College. This article is based in part on previous NSF grant applications.

2. Structure of the Program

The SMALL project lasts for nine weeks in June through August. The students are assigned into groups of about four, each group working with an individual faculty member. The students learn to work as a group, assigning sub-projects to individuals, and helping each other.

Each group, along with the faculty advisor, decides on the daily routine. Some meet once or even twice a day at a scheduled time with the faculty member, while others prefer a more open door policy where the students come to see the faculty member whenever they have questions. When not talking to the faculty advisor, the groups are working together or individually in the Mathematics and Statistics Library (which is surrounded by the faculty offices), down in the Math Computer lab or in one of several classrooms reserved for their use. See the pictures at http://www.williams.edu/Mathematics/small_CES.html for a sense of the facilities.

A variety of weekly activities are held for all the students. Every Tuesday at 10:00 am convocation is held. Announcements are made, sometimes followed by short progress talks by the student groups. Tuesday at noon the students, together with at least another 150 students working on research in the sciences over the summer, attend research talks by faculty from the sciences. Lunch is provided by the College. On Wednesdays at 1:00, there is a mathematics colloquium talk by a faculty member from inside or outside Williams. These talks are specifically directed at the students, although the topic is usually current research. At 4:00 on Fridays, there is a tea, giving students a chance to talk to each other and to faculty about their progress over the week. In addition, there are a variety of social events organized by students and faculty that increase the opportunities for interaction. In past summers, the enthusiasm has been so great that students have often worked more than the standard forty-hour work week, including working late into the night and over the weekends.

Over the summer, students present their work to the rest of SMALL. Our students have participated in a poster presentation by all summer science students. In addition, we have had students present their work in numerous other venues, including MAA and AMS national and regional math conferences, as well as the Hudson River Undergraduate Math Conference. Near the end of the summer we have often travelled to MathFest, where the students report on their results in the MAA and PME student talk sessions. Over the years many students have won awards, including the top CUR MAA and PME awards.

Sometimes some or all of the groups participate in other schools or conferences. For example, in 2002 the students spoke at the beginning of the summer about

their research project at an MAA meeting at Williams. Many of the students have presented posters at the AMS annual meetings. Some groups have travelled to other meetings; for example, the Geometry Group participated in the Clay Institution summer graduate school at MSRI in 2001, and in a 10-day graduate school in Paris in summer 2004.

All the students are housed at a the college. Starting with summer 1991, Williams has provided us with a college house for the exclusive use of our program. Having all of the students living together in a single building allows for the mathematical interaction to expand beyond the boundaries of the workday. The college has also provided us with computing facilities, classroom space, and also often additional funds for supporting students, including supplies, tea, and additional student stipends. See "Is an REU for You?,"by D. Haunsperger and S. Kennedy, *Math Horizons*, February 1998, for a description of the SMALL program from the point of view of one of the participants. A more recent account may be found in "REU Spotlight: Williams College–The SMALL Program," by Jennifer Novak and Eric Schoenfeld, *Math Horizons*, November 2005.

3. Student Selection and Project Evaluation

We aim to have a national representation of students. We mail about 200 posters to various colleges and universities in the region and the nation. We maintain a current web page with information on the program and all the information that is need to apply. We will continue to make strong efforts to recruit minorities and women. About 33% of the 80 students in SMALL during the years 1999 - 2003 were women. We hope to continue to attract talented women and to attract more underrepresented minorities than we have been able to so far.

Every summer, we assign two students the duty of formulating an evaluation of SMALL. They create and disseminate a survey filled out by all participants and then write up an evaluation. We have found that information very useful in fine-tuning the program, and we plan to continue to use this system of evaluation. We are tentatively planning to invite an outside faculty member to visit SMALL and write an evaluation from the faculty perspective.

4. Plans of SMALL alumni

SMALL students come from a diverse group of colleges and universities including Carleton, Emory, Stanford, Pomona, Cornell, MIT, Berkeley, Texas A&M, Harvard, Colby, U. Virginia, Missouri Baptist, New College, Wesleyan, U. Chicago, U. Texas, U. Notre Dame, Brandeis, Augustana, College of New Jersey, Swarthmore, U. Richmond, Johns Hopkins, Bucknell, Rose-Hulman, Princeton, and Williams. Out of the 80 students who participated in SMALL during 1999 - 2003, about 50 of them are pursuing or have completed Ph.D.s in mathematics or related fields (such as applied mathematics, physics, computer science, engineering, economics). The graduate schools where SMALL students are currently enrolled or have graduated from include Berkeley, Brown, Caltech, Chicago, Cornell, Columbia, Duke, Georgia Tech, Harvard, Maryland, Michigan, MIT, Northwestern, Penn, Princeton, Rutgers, Stanford, Texas, UCLA, UCSD, Washington and Wisconsin. At least 12 SMALL students have earned NSF Fellowships in mathematics, and another two students have earned NSF Fellowships in related fields. Four have earned honorable

mention in the Alice T. Schafer Prize for Excellence in Mathematics by an Undergraduate Woman, one was a runner-up in 2001 and one won the Schafer Prize in 2003. Of the 12 NSF Mathematics Fellowship winners, four are women. We expect these numbers to increase as more students graduate.

5. Research Publications

Every student research group has produced an internal paper, and most of the groups have submitted a paper externally. Many of the papers have already appeared in refereed mathematics journals and many others have been accepted for publication, while others still are in the submittal or preparation stage. For example, of the 80 students who participated in SMALL in the years 1999 - 2003, 64 students are co-authors of one or more published research papers. The result of research that was either generated by the SMALL Project or followed up on work begun in SMALL has appeared or is to appear in journals such as Acta Arithmetica, Acta Crystallographica, American Mathematical Monthly, Annals of Mathematics, Canadian Mathematical Bulletin, Colloquium Mathematicum, Communications in Algebra, Computational Geometry, Ergodic Theory and Dynamical Systems, The Fibonacci Quaterly, Houston Journal of Mathematics, Illinois Journal of Mathematics, Integers: Electronic Journal of Combinatorial Number Theory, Journal of the Australian Mathematical Society, Journal of Combinatorial Theory, Journal of Geometric Analysis, Journal of Knot Theory and its Ramifications, Journal of Discrete and Computational Geometry, Journal of Number Theory, Michigan Journal of Mathematics, New York Journal of Mathematics, Networks, Pacific Journal of Mathematics, Proceedings of the American Mathematical Society, Real Analysis Exchange, Technometrics, Topology and its Applications, Rocky Mountain Journal of Mathematics, Transactions of the American Mathematical Society. All of the papers were either authored or co-authored by students. For a full list of publications the reader may refer to http://www.williams.edu/Mathematics/small_CES.html.

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