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Williams College
Department of Mathematics and Statistics

MATH 140 : CALCULUS II

Problem Set 12 – due Wednesday, March 11th

INSTRUCTIONS:
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Please submit this at the *start* of Wednesday's class. Don't worry if you don't manage the get an answer for any particular question, but please give each problem an honest try (and record what you were able to accomplish, even if you didn't solve it). Eventually you should make sure to understand the problems, as some of them may appear on next week's in-class quiz. You are encouraged to collaborate with other students on these problems. However, please write up your solutions in isolation from one another.

12.1 In this problem we integrate the exponential function.

(a) Explain why $x + x^2 + x^3 + \cdots + x^N = \dfrac{x}{x-1} \cdot (x^N - 1)$.

(b) Use the limit definition of the integral to show that

$$\int_0^3 e^t \, dt = \lim_{\Delta \to 0} \frac{\Delta e^\Delta}{e^\Delta - 1}(e^3 - 1).$$

[*Hint: use part (a).*]

(c) Explain why $\lim\limits_{\Delta \to 0} \dfrac{\Delta}{e^\Delta - 1} = 1$. [*Hint: Consider the derivative of e^x.*]

(d) Evaluate $\displaystyle\int_0^3 e^t \, dt$.

12.2 Use the limit definition of the integral to show that $\displaystyle\int_0^{17} t \, dt = \dfrac{17^2}{2}$.

12.3 Use the limit definition of the integral to show that for any positive number x we have $\displaystyle\int_0^x t \, dt = \dfrac{x^2}{2}$.

12.4 Use the limit definition of the integral to show that for any positive number x we have $\displaystyle\int_0^x t^2 \, dt = \dfrac{x^3}{3}$.

12.5 Explain why $\displaystyle\int_8^{13} t^2 \, dt = \dfrac{13^3}{3} - \dfrac{8^3}{3}$.

[*Hint: Think about the geometric definition of this integral. Then use the previous problem.*]