

Instructor: Leo Goldmakher

NAME: _____

SECTION: _____

Williams College
Department of Mathematics and Statistics

MATH 250 : LINEAR ALGEBRA

Problem Set 3 – due Thursday, March 3rd

INSTRUCTIONS:

This assignment must be turned in *as a hard copy* to the mailbox of your TA (on your left as soon as you enter Bronfman from Science Quad, labelled by last name), by **9pm** sharp. Assignments turned in later than this, but before 5pm on Friday, will also be graded, but the grade will be reduced by one mark. *Assignments submitted later than Friday at 5pm will be returned without being marked.*

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	GRADE
3.1	
3.2	
3.3	
3.4	
3.5	
3.6	
Total	

Please read the following statement and sign below:

I understand that I am not allowed to use the internet to search for problems or solutions. I also understand that I must write down the final version of my assignment without reference to notes copied from anyone else's speech or written text. I pledge to abide by the Williams honor code.

SIGNATURE: _____

Problem Set 3

3.1 Compute each of the following.

$$(a) \begin{pmatrix} 2 & 3 \\ -1 & 6 \end{pmatrix} \begin{pmatrix} 5 \\ 4 \end{pmatrix} \qquad (b) \begin{pmatrix} -1 & 3 \\ 4 & 2 \end{pmatrix} \begin{pmatrix} 1 \\ 3 \end{pmatrix} \qquad (c) R_{3\pi/4}(2, 1)$$

$$(d) \rho(R_{\pi/3}(3, 4)), \text{ where } \rho : \mathbb{R}^2 \rightarrow \mathbb{R}^2 \text{ is the reflection across the horizontal axis.}$$

3.2 Below are matrices corresponding to functions mapping $\mathbb{R}^2$ to $\mathbb{R}^2$. Describe each function geometrically. (For example, a geometric description of R_θ might be: it rotates the plane counterclockwise around the origin by angle θ .)

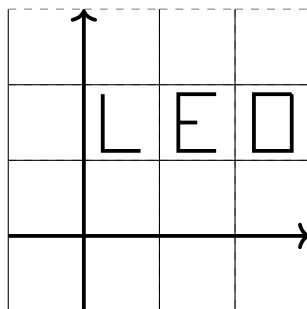
$$(a) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \qquad (b) \begin{pmatrix} 2 & 0 \\ 0 & 5 \end{pmatrix} \qquad (c) \begin{pmatrix} -2 & 0 \\ 0 & 5 \end{pmatrix} \qquad (d) \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}$$

3.3 Determine the matrix of $F : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by $F(x, y) := (2x - 3y, x + y)$.

3.4 Suppose $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear function with matrix $\begin{pmatrix} 3 & -5 \\ 2 & 4 \end{pmatrix}$, and $g : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is a linear function with matrix $\begin{pmatrix} 6 & -1 \\ -8 & 7 \end{pmatrix}$. Consider the functions $h : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ and $k : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ defined by

$$h(x, y) := f(x, y) + g(x, y) \qquad \text{and} \qquad k(x, y) := f(g(x, y)).$$

- (a) Determine $h(1, 0)$, $h(0, 1)$, $k(1, 0)$, and $k(0, 1)$.
 - (b) Prove that h is linear, and determine the matrix of h .
 - (c) Prove that k is linear, and determine the matrix of k .
- 3.5** Suppose $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear such that $T(0, 1) = (0, 1)$ and $T(1, 0) = (\frac{1}{4}, \frac{\sqrt{3}}{4})$.
- (a) Determine the matrix of T .
 - (b) Let $S_1 := \{(x, y) : 0 \leq x \leq 1, 1 \leq y \leq 2\}$, $S_2 := \{(x, y) : 1 \leq x \leq 2, 1 \leq y \leq 2\}$, and $S_3 := \{(x, y) : 2 \leq x \leq 3, 1 \leq y \leq 2\}$. Carefully write the first letter of your first name in S_1 , the second letter of your first name in S_2 , and the third letter of your first name in S_3 . (See below for illustration.) What would these three letters look like after applying T to them? Draw a clear picture.



Leo's name before applying T

3.6 Suppose $f : \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is linear with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$.

- (a) Show that if $ad - bc = 0$, then there exists a line $\mathcal{L}$ passing through the origin such that all outputs of f lie on $\mathcal{L}$.
- (b) Conversely, show that if there exists a line $\mathcal{L}$ such that $f(x, y)$ is on $\mathcal{L}$ for every (x, y) , then $ad - bc = 0$.