Instructor: Leo Goldmakher

NAME: ____________________

SECTION: ____________________

Williams College
Department of Mathematics and Statistics

MATH 250 : LINEAR ALGEBRA

Problem Set 9 – due Thursday, May 12th

INSTRUCTIONS:
This assignment must be turned in *as a hard copy* to the mailbox of your TA (on your left as soon as you enter Bronfman from Science Quad, labelled by last name), by **9pm** sharp. Assignments turned in later than this, but before 5pm on Friday, will also be graded, but the grade will be reduced by one mark. *Assignments submitted later than Friday at 5pm will be returned without being marked.*

Please print and attach this page as the first page of your submitted problem set.

PROBLEM	GRADE
9.1	
9.2	
9.3	
9.4	
Total	

Please read the following statement and sign below:

I understand that I am not allowed to use the internet to search for problems or solutions. I also understand that I must write down the final version of my assignment without reference to notes copied from anyone else's speech or written text. I pledge to abide by the Williams honor code.

SIGNATURE:____________________

Problem Set 9

9.1 Suppose V and W are vector spaces. A linear map $T : V \to W$ is called *left-invertible* iff there exists a linear map $L : W \to V$ such that $L \circ T = I_V$, and is called *right-invertible* if and only if there exists a linear map $R : W \to V$ such that $T \circ R = I_W$. (Here I_V denotes the identity map on V, i.e. $I_V(\vec{v}) = \vec{v}$ for all $\vec{v} \in V$.) Prove that a linear map $T : V \to W$ is invertible (according to our definition from class) if and only if T is both left- and right-invertible.

9.2 If V is isomorphic to W, we write $V \simeq W$. Prove that $\simeq$ is an equivalence relation.

9.3 Find an example of a finite-dimensional vector space V and a subset $W \subseteq V$ such that W is a vector space, but is *not* a subspace of V.

9.4 Given V is a finite-dimensional vector space.

(a) Suppose W is a subspace of V. Prove that $W = V$ iff $\dim W = \dim V$. [*Hint: Use problem **8.1***]

(b) Suppose $T : V \to V$ is a linear map. Prove that T is an isomorphism if and only if $\ker T = \{\vec{0}\}$.