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MATH 374 : TOPOLOGY

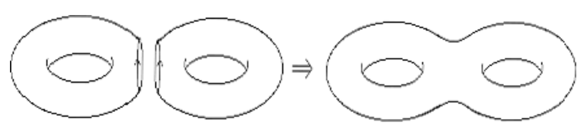
Problem Set 11 – due Friday, December 6th

INSTRUCTIONS:

This assignment is due at 4pm on Friday, to be submitted to the mailbox outside my office. (This week there will be no late penalty for submitting Friday.) However, assignments will not be accepted after 4pm on Friday.

11.1 In class I asserted that $\mathbb{RP}^2$ can be visualized as S^2 from which you remove a disk and glue (along the boundary of the hole) a Mobius strip. Give a gluing diagram argument to justify this.

We've seen many examples of *surfaces* (2-dimensional objects—we'll discuss a formal definition of this on Monday). For example, the sphere S^2, the torus T^2, the real projective plane $\mathbb{RP}^2$, the Klein bottle K, and the Möbius strip M are all surfaces. Given two surfaces X and Y, recall that there's a natural way to combine them: their *connected sum* (denoted $X\#Y$) is the surface you get by deleting the interior of a disk from each surface, and then gluing the boundaries of these two holes together. Here's an illustration of $T^2\#T^2$:



$T^2\#T^2$ is a two-holed torus

11.2 Above we saw an illustration that $T^2\#T^2$ is a two-holed torus. Use gluing diagrams to show this.
[*Hint: You might wish to make use of Problem* **10.7** *from the last problem set.*]

11.3 We saw in class that gluing two Möbius strips along their boundary produces the Klein bottle K. It's tempting to deduce that $M\#M \approx K$, but this is not the case. Explain why not.

11.4 Show that $\mathbb{RP}^2\#T^2 \approx \mathbb{RP}^2\#K$.

11.5 Show that $\mathbb{RP}^2\#\mathbb{RP}^2 \approx K$. (It follows that $\mathbb{RP}^2\#T^2 \approx \mathbb{RP}^2\#\mathbb{RP}^2\#\mathbb{RP}^2$.)

11.6 Compute $\chi(T^2\#T^2)$ and $\chi(T^2\#T^2\#T^2)$. Make a conjecture about $\chi(\underbrace{T^2\#T^2\#\cdots\#T^2}_{n \text{ copies}})$.

11.7 In class I asserted that $\mathbb{Z}_{\text{furstenberg}} \approx \mathbb{Q}$, since $\mathbb{Z}_{\text{furstenberg}}$ satisfies the hypotheses of Sierpinski's theorem: it's clearly countable, and we saw in problem **10.5** that it's metrizable. Prove that $\mathbb{Z}_{\text{furstenberg}}$ has no isolated points.