1

Can you read this?

what about this?

Can you read this?

How's this?

Leo Goldmakher

2

Measure theory is about... measuring sizes of sets.

e.g. length:

- of $[0,1]$? 1
- of $(0,1)$? 1
- of $(0,\frac{1}{4}) \cup (\frac{1}{4},1)$? 1
- of $\mathbb{Q} \cap [0,1]$?

3

Maybe 0?

Because ~~#~~ in all previous examples can have a 1-1 correspondence w/ $[0,1]$, but this doesn't.

<u>BUT</u>: $[0,2]$ has 1-1 corresp w/ $[0,1]$.

4

Maybe $\mathbb{Q} \cap [0,1]$ has length 1, since it's dense?

Certainly, length ≤ 1. b/c subset of $[0,1]$.

Each pt. in rationals has length 0; so add up to get 0.

But irrationals!?!

5

Maybe length isn't additive?

1+1=1.

Ben K

Countable # of additions of length 0 sets should be length 0.

Maybe?

6

Length might be subadditive, but <u>not</u> superadditive.

Maybe think probabilistically instead!

We don't know how precisely to find length of $\mathbb{Q} \cap [0,1]$.

1

BUT we do know how to find lengths of intervals! So:

Cover all pts of $\mathbb{Q}\cap[0,1]$ w/ intervals

– e.g. $[0,1]\supseteq\mathbb{Q}\cap[0,1]$

$\Rightarrow \ell(\mathbb{Q}\cap[0,1])\le 1$

2

$\frac{1}{2}\in(-\sqrt{2},\sqrt{2})$

$\frac{1}{2}\in(0,1)$

$\mathbb{Q}\cap[0,1]$ is ctble, so can write

$\mathbb{Q}\cap[0,1]=\{q_1,q_2,q_3,\dots\}$

Pick interval $I_1\ni q_1$ of length $\frac{1}{4}$

— " — $I_2\ni q_2$ of length $\frac{1}{8}$

— " — $I_3\ni q_3$ of length $\frac{1}{16}$

⋮

$\Rightarrow \mathbb{Q}\cap[0,1]\subseteq\bigcup_{k=1} I_k$

3

$\Rightarrow \ell(\mathbb{Q}\cap[0,1])\le \sum_{k=1}^{\infty}\ell(I_k)=\frac{1}{2}$

Claim: $\ell(\mathbb{Q}\cap[0,1])=0$

Proof: Given $\varepsilon>0$.

Enumerate $\mathbb{Q}\cap[0,1]=\{q_1,q_2,q_3,\dots\}$

4

Pick interval $I_1\ni q_1$ w/ $\ell(I_1)=\frac{\varepsilon}{4}$.

Pick interval $I_2\ni q_2$ w/ $\ell(I_2)=\frac{\varepsilon}{8}$

⋮

Pick interval $I_k\ni q_k$ w/ $\ell(I_k)=\frac{\varepsilon}{2^{k+1}}$

$\Rightarrow \mathbb{Q}\cap[0,1]\subseteq\bigcup_{k=1}^{\infty} I_k$

5

$\Rightarrow \ell(\mathbb{Q}\cap[0,1])\le \ell\left(\bigcup_{k=1}^{\infty} I_k\right)$

$\le \sum_{k=1}^{\infty}\ell(I_k)=\frac{\varepsilon}{2}$

Thus $\ell(\mathbb{Q}\cap[0,1])<\varepsilon$ $\forall\varepsilon>0$.

$\Rightarrow \ell(\mathbb{Q}\cap[0,1])=0$. ∎

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Ben M's conjecture:

$\ell(A):=\inf$ over all possible coverings of A by intervals of sum of all the intervals in the cover.

~~non-zero length~~? or maybe open?

1

What properties should length enjoy? Length should be a function

$$\ell : \mathcal{P}(\mathbb{R}) \longrightarrow [0,\infty]$$

power set of $\mathbb{R}$

2

s.t.

(1) $\ell(A \cup B) = \ell(A) + \ell(B)$.

when $A \cap B = \emptyset$

(2) $\forall A, B$,

$$\ell(A \cup B) \leq \ell(A) + \ell(B)$$

~~(3) Even~~

3

4

5

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