COURSE INFORMATION

MATH 402 – Measure Theory

Course homepage: `https://web.williams.edu/Mathematics/lg5/402/`

Instructor: Leo Goldmakher (either 'Leo' or 'Professor Goldmakher' is fine)
email: `Leo.Goldmakher@williams.edu`
Office hours: Mondays 9am–10am, Wednesdays 3pm–5pm, and by appointment. (You do not need to make an appointment for the Monday / Wednesday times listed above.) Access link available on Glow.

Lectures: Tuesdays and Fridays, 3:15–4:30, on Zoom (access link available on Glow).

Syllabus: How large is the unit square? One might measure the number of individual points in the square (uncountably infinite), the area of the square (1), or the dimension of the square (2). But what about for more complicated sets, e.g., the set of all rational points in the unit square? What's the area of this set? What's the dimension? In this course we'll come up with precise ways to measure size—length, area, volume, dimension—that apply to a broad array of sets. Along the way we'll encounter topics such as Lebesgue measure and Lebesgue integration, Hausdorff measure and fractals, space-filling curves and the Banach-Tarski paradox. We will also investigate Hilbert spaces, mathematical objects that combine the tidiness of linear algebra with the power of analysis and are fundamental to the study of differential equations, functional analysis, harmonic analysis, and ergodic theory, and also apply to fields like quantum mechanics and machine learning.

Textbook: *Real Analysis* by Stein and Shakarchi (Princeton Lectures in Analysis III). See course website for other resources.

Discussion sessions: Like any other worthwhile activity, measure theory is *hard.* Some of the concepts and problems are going to be manifestly difficult, and you will need to persist in the face of feeling totally lost, in particular re-reading the text and your notes and reaching out to me, the TAs, and your peers for inspiration.

But the far greater challenge in this course is dealing with those difficulties and subtleties you **aren't** aware of. How can you fix a problem you don't know is a problem? The single most effective method is:

Try to explain the material to someone else.

For this reason I'm dividing the class into many groups of three students. Each triplet will meet (at least) once per week outside of class. These discussion sessions are explicitly **not** for discussing problems, but rather, for talking about the material from class.

Problem sessions: I highly recommend getting together with your peers to collaborate on the problems. That said, to maximize your understanding of the material, I urge you to work on the problems on your own first, and only then brainstorm with others. Note that there will not be any dedicated TA-led problem sessions; I encourage students to take initiative and organize student-run sessions.

Precepts: Each triplet of students will have a 50-minute meeting ('precept') once per week with either the TA or the instructor. At these meetings, the TA or instructor will give you individualized feedback on the previous week's problem set, and go over any material that you would like to discuss.

Teaching Assistant: Alex Trevithick (amt6)

Assessment: Your grade will be calculated based on several components:

1. Problem sets – 15% total

   This course will have weekly problem sets. Some of the problems will be straightforward once you master the concepts from class / reading, while others will require additional creativity. **The problem sets are intended to be challenging**. The goal is to struggle with every question; it's OK not to solve every problem on the assignment, so long as you make a serious attempt at all of them. However, I have one hard rule: **please do not search for problems, solutions, or examples online.**

   Problem sets are due on Mondays by 6pm (submission via Glow). Late assignments may be submitted by 3pm on Tuesday; however, 5% will be deducted for submission past Monday 6pm. *Assignments will not be accepted after Tuesday at 3pm under any circumstances.*

   Problem sets will be graded based purely on effort. However, in you weekly precept you will receive detailed feedback on your work.

2. Lecture summary – 15%

   Each student (some in pairs) will be responsible for writing up a detailed lecture summary in LaTeX at some point during the semester. (You are not expected to come in with a knowledge of LaTeX; guidance will be provided.) Tuesday's lecture summary will be due by Wednesday at 10pm; Friday's lecture summary will be due by Monday at 10am.

3. Oral midterm – 35%

   There will be an oral midterm (probably during the week October 5th–9th). Format to be discussed in class.

4. Final exam – 35%

   There will be a final exam (format to be determined) during the final exam period.

5. Precept attendance – 1 percentage point added per two sessions attended.

   Once your overall course grade has been calculated, you will receive one additional percentage point per two precepts attended. Partial points will not be awarded. *Attendance is counted at the preceptor's discretion*; if the preceptor feels you did not actively participate in the session, you will not receive credit for that session.

Team work and plagiarism: I strongly encourage you to brainstorm with other students as you work on your problem sets. However, **you must write up the solutions on your own without copying from any text (written or spoken)**. For example, if you take notes during a meeting based on a solution explained to you by another student, *do not copy from these notes* when writing up your assignment! When you read this sentence, please send an email to our TA Alex with subject line consisting of the single word ephemeral. To avoid a slippery slope, I encourage you to write up your problems sets in physical isolation from any other student and from any notes you've taken while with other students.

Internet usage: The internet is an amazing resource, but I urge you to use it wisely. In particular, I request that you do not search for problems or examples. **Looking up definitions is OK, looking up (or asking about) problems online is not**. It is better to struggle on your own and *not* solve the problem than to simply copy a solution. When it comes to exams, please don't use the internet for any class-related reason apart from email or accessing the official course website.

Peer Tutoring (free!): The Peer Academic Support network provides free peer tutoring. Step-by-step instructions for finding and scheduling tutoring sessions are on the Peer Tutoring Program website:

academic-resources.williams.edu

If you have any questions about tutoring at Williams, please email msrc@williams.edu

Attendance expectations: The goal of this class, apart from learning about the subject of real analysis, is to learn how to *think* like a practicing research mathematician (specifically, like an analyst). During our class meetings we will endeavor to collaboratively discover the material as a class. In particular, efficiency of information transfer will not be our paramount goal; instead, free discussion will drive the class. Right ideas, wrong ideas, pre-ideas, and all questions are encouraged, as all these are vital to mathematical discovery.

In keeping with these goals, I expect that you will try to participate synchronously in the class on Zoom, to the utmost that your situation allows. (Of course allowances can be made in special circumstances; please email me if you see an issue with your attending the class on Zoom.) Videos of the class will be recorded and made available for review purposes, but this does not replace the experience of participating in the collaborative discovery process. While attending the class, I expect your video to be turned on; if this is an issue for bandwidth or other reasons, please let me know.