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MATA31 – Calculus I for Mathematical Sciences

Problem Set 1 (due the week of September 24th – 28th)

At the top of your assignment, please write your full name and student number. Also, please copy (by hand) the following statement onto the top of your assignment, and sign it:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

[signature]

1. In this problem, you will explore how rational and irrational numbers interact.
 - (a) Suppose $a, b \in \mathbb{Q}$. Prove that $a + b$ and ab are both rational. Must $a/b \in \mathbb{Q}$?
 - (b) Suppose $a \in \mathbb{Q}$ and $\beta \notin \mathbb{Q}$. What can you say about the rationality of $a + \beta$? Of $a\beta$?
 - (c) Suppose $\alpha, \beta \notin \mathbb{Q}$. Is it possible for $\alpha + \beta$ to be rational? What about $\alpha\beta$? What about α/β ?
 - (d) Prove that there exist $\alpha, \beta \notin \mathbb{Q}$ such that $\alpha^\beta \in \mathbb{Q}$.
2. For each of the following sets, give the simplest description of the set that you can. Explanations required!
 - (a) $\{\alpha \in \mathbb{R} : n\alpha \in \mathbb{Z} \text{ for some } n \in \mathbb{N}\}$
 - (b) $\left\{ \sum_{n=0}^N a_n 2^n : N \in \mathbb{N}, a_n \in \{0, 1\} \text{ for every } n \leq N \right\}$
 - (c) $\left\{ \sum_{n=0}^N a_n 3^n : N \in \mathbb{N}, a_n \in \{0, 1\} \text{ for every } n \leq N \right\}$
3.
 - (a) Prove that $\sqrt{3} \notin \mathbb{Q}$.
 - (b) Prove that $\sqrt{6} \notin \mathbb{Q}$.
4.
 - (a) Suppose $a, b \in \mathbb{Z}$. Prove that any solution to the equation $x^3 + ax + b = 0$ must either be an integer, or else be irrational.
 - (b) Prove that $\{\log_2 n : n \in \mathbb{N}\}$ consists entirely of integers and irrational numbers.