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MATA31 – Calculus I for Mathematical Sciences

Problem Set 6 (due the week of Nomber 19th – 23rd)

At the top of your assignment, please write your full name and student number. Also, please copy (by hand) the following statement onto the top of your assignment, and sign it:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.
[signature]

A. Prove that every infinite set has a countably infinite subset. (Thus, $\mathbb{N}$ has the smallest possible size any infinite set can have.)

B. Suppose that $A \subseteq B$ and A is uncountable. Prove that B must be uncountable.

C. Consider the function $f : \mathbb{R} \to \mathbb{R}$ defined by

$$f : x \mapsto \frac{x}{\sqrt{1+x^2}}.$$

Prove that this gives a one-to-one correspondence between $\mathbb{R}$ and the interval $(-1, 1)$.

D. Given $a, b \in \mathbb{R}$ with $a < b$, find an explicit one-to-one correspondence between the open intervals (a, b) and $(0, 1)$.

E. i. Suppose $A \subseteq \mathbb{R}$ is an infinite set, and $x \notin A$. Prove that A and $A \cup \{x\}$ have the same size (i.e. are in one-to-one correspondence).

ii. Suppose $A \subseteq B \subseteq \mathbb{R}$ such that A is infinite and $B \backslash A$ is finite. Prove that A and B are in one-to-one correspondence. (Thus, for example, the open interval $(0, 1)$ and the closed interval $[0, 1]$ have the same size.)

F. Bartle & Sherbert:
1.1 # 8, 9, 18
1.3 # 13
2.4 # 6, 8, 9, 10