

## MAT A31: REMARKS ON 2ND MIDTERM EXAM

In preparation for the midterm, you should be comfortable with the following as a minimum.

- (1) Your TA's name, and your tutorial number (the cover sheet of the exam is worth 5 points).
- (2) Proof that  $\sqrt{2}$  exists. **You will be asked to prove a substantially similar statement on the exam, so you need to understand the entire proof.** If you can't explain the proof – verbally – to someone else without looking in your notes, you don't understand it.
- (3) Proof of the density of  $\mathbb{Q}$  in  $\mathbb{R}$ . **You will be asked to prove a substantially similar statement on the exam, so you need to understand the entire proof.** Same comment as above.
- (4) Induction, Strong Induction, Well-Ordering Property: what they say, how to use them, how to prove that they're all equivalent.
- (5) Properties of  $\mathbb{R}$  (algebraic, order, and completeness). I will list these for you on the exam, so you don't need to memorize them. However, you should be very familiar with them – otherwise, using them will take too long. On the exam, I will explicitly specify when you need to use the properties to prove something, and when it's not necessary.
- (6) Supremum and Infimum – what they are, how to find them, how to prove that you're right.
- (7) The absolute value: definition and properties (e.g. triangle inequality).
- (8) Interval notation. Nested intervals theorem: the proof, and how to use the theorem.
- (9) The archimedean property: how to use it, how to prove it.
- (10) Countable vs. uncountable sets, and one-to-one correspondence. You do not need to understand this as rigorously as the other material, but you should be able to explain why two sets have the same size, or else why they do not. You should also understand the proofs that  $\mathbb{N}$ ,  $2\mathbb{N}$ ,  $\mathbb{Z}$ , and  $\mathbb{Q}$  are all countable, as well as the proof that  $[0, 1]$  is uncountable.