# ***** Sorry - Solutions will not be posted *****

**FINAL EXAMINATION**

**MATA32 - Calculus for Management I**

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Date: December 10, 2008
Duration: 3 hours

## Provide the following information:

**Surname (PRINT):** _____

**Given Name(s) (PRINT):** _____

**Student Number :** _____

**Signature:** _____

## Read these instructions:

1. This examination has 13 numbered pages. It is your responsibility to ensure that at the beginning of the exam, all of these pages are included.
2. If you need extra space for any question, use the back of a page or the blank page at the end of the exam. Clearly indicate the location of your continuing work. You may write in pencil, pen, or other ink.
3. You may use one standard hand-held calculator. All other electronic devices, extra paper, notes, and textbooks are forbidden at your workspace.

## Print letters for the Multiple Choice Questions in these boxes:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | | |

## Do not write anything in the boxes below.

| **A** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | **TOTAL** |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | | |
| 48 | 9 | 6 | 11 | 10 | 11 | 12 | 13 | 15 | 15 | 150 |

## The following may be helpful:

$$S = P(1+r)^n \qquad S = Pe^{rt} \qquad S = R\left[\frac{(1+r)^n - 1}{r}\right] \qquad A = R\left[\frac{1-(1+r)^{-n}}{r}\right]$$

**Part A: Multiple Choice Questions** For each of the following, **clearly print the letter of the answer you think is most correct in the boxes on the first page**. Each right answer earns 4 points and no answer/wrong answers earn 0 points. No justification is required.

1. If $w > 0$ is a constant and $f(x) = \ln(wx^2 + w^2x + w)$ then $f\,'(1)$ is equal to

   (a) $2 + w$ (b) 1 (c) $1 - w$ (d) none of (a) - (c)

2. If $y\,' = 3\sqrt{x} - 1$ and $y(1) = 2$ then $y(0)$ is equal to

   (a) 0 (b) 7/3 (c) 1 (d) $-3/2$ (e) none of (a) - (d)

3. The area of the region lying between $y = x^2$ and $y = -2$ where $0 \le x \le 3$ is

   (a) 21 (b) 33 (c) 18 (d) 24 (e) none of (a) - (d)

4. $\int \dfrac{2e^{3x}+1}{e^x}\,dx$ equals

(a) $2e^{2x}-e^x+C$ (b) $e^{2x}+e^{-x}+C$ (c) $e^{2x}+e^x+C$ (d) $e^{2x}-e^{-x}+C$

5. The fewest whole number of months it takes a principal to increase by exactly 79% at 5.3% APR interest compounding continuously is

(a) 133 (b) 132 (c) 131 (d) none of (a) - (c)

6. The exact value of $\int_0^1 \dfrac{3x}{3x^2+1}\,dx$ is

(a) $\ln(3)$ (b) $\ln(\sqrt{3})$ (c) $\ln(2)$ (d) $\dfrac{ln(27)}{2}$ (d) none of (a) - (d)

7. If $g(x) = 2x^3 + 6x^2$ then

(a) $g$ has an inflection point at $x = -1$
(b) $g$ has a relative minimum at $x = 2$
(c) $g$ has a relative maximum at $x = 2$
(d) both (a) and (b) are true
(e) both (a) and (c) are true

8. If $y = (3x + e)^{2x}$ then $y\,'(0)$ equals

(a) 0 (b) $2\ln(3 + e)$ (c) 2 (d) none of (a) - (c)

9. The premium on an insurance policy is \$500 per quarter payable at the start of each quarter. If the policy holder wishes to pay two year's premiums in advance, how much (rounded up to the nearest dollar) should be paid provided that interest is 6% compounding quarterly ?

(a) $\$3,743$ (b) $\$3,800$ (c) $\$3,300$ (d) $\$3,662$

10. If $f(x) = 3e^{-3x}$ then for all $n \geq 1$, $f^{(n)}(0)$ equals

(a) $(-1)^n 3^{n+1}$ (b) $(-1)^{n+1} 3^{n+1}$ (c) $-(3^{n+1})$ (d) none of (a) - (c)

11. If $\int h(x)\, dx = \dfrac{1}{2-x} + 3$ then $h'(x)$ equals

(a) $\dfrac{1}{2-x}$ (b) $-\dfrac{2}{(2-x)^3}$ (c) $\dfrac{2}{(2-x)^3}$ (d) $-\dfrac{1}{(2-x)^2}$ (e) none of (a) - (d)

12. Exactly how many of the following four mathematical statements are always true:

(i) The definite integral represents area under a curve or beneath a curve.
(ii) The definite integral is a function obtained by taking the limit of a special sum.
(iii) The definite integral of a function $g(x)$ is a function $G(x)$ such that $\int G(x)\, dx = g(x)$
(iv) $\int_a^b f(x)\, dx$ is defined to be $F(b) - F(a)$ where $F$ is any antiderivative of $f$

(a) 4 (b) 3 (c) 2 (d) 1 (e) 0

**(Be sure you have printed the letters for your answers in the boxes on the first page)**

**Part B: Full-Solution Questions** Write clear, full solutions in the spaces provided. Full points will be awarded only if your solutions are correct, complete, and sufficiently display appropriate concepts from MATA32.

1. Find the point(s) on the curve $x^2 + xy + y^2 = 9$ for which the slope of the tangent is equal to 1. [9 points]

2. Assume $a > 0$ is a constant, $r$ is a function of the variable $t$, and $h = (1 + a)r - at$

   (a) Show that $\dfrac{dh}{dt} = (1 + a)\left[\dfrac{dr}{dt} - \dfrac{a}{1 + a}\right]$ [3 points]

   (b) Verify that if $\dfrac{dr}{dt} < \dfrac{a}{1 + a}$, then $h$ is a decreasing function of $t$ [3 points]

3. Find the area of the region bounded by the curve $y^2 = -x - 2$ and the lines $x - y = 5$, $y = -1$ and $y = 2$ [11 points]

(An accurate, labeled diagram is required for full points)

4. A marginal revenue function is given by $r' = 6q^2 + 4e^{-q} + 1$ where $q > 0$. Assuming that $r = p\,q$ for $q \geq 0$ and $r(0) = 0$ find the demand function $p = f(q)$ where $q > 0$

[10 points]

5. (a) Carefully define what it means for a function $f$ to be differentiable at a number $a$

[3 points]

(b) Use your answer in part (a) to show that $\lim_{x \to 0} \dfrac{(x+1)^{3/2} + 2x - 1}{x} = \dfrac{7}{2}$

(A solution using l'Hopital's rule (if you know that) will earn 0 points) [4 points]

(Question 5 continued)

(c) Carefully state the Fundamental Theorem of Calculus. [4 points]

6. On the axis below, sketch the graph of a continuous function $y = f(x)$ having **all** of the following properties. Points are awarded for accuracy and neatness. [12 points]

(i) $f(0) = 3$ and $f(3) = 0$

(ii) $f\,'(-2) = f\,'(1) = 0$

(iii) $f$ is not differentiable at $x = -1$

(iv) $f(x) > 0$ for all $x \in (-\infty, 3)$

(v) $\lim\limits_{x \to -\infty} f(x) = 0$ and $\lim_{x \to \infty} f(x) = -1$

(vi) $f\,''(x) > 0$ for all $x \in (-\infty, -3)$ and $f\,''(x) < 0$ for all $x \in (-1, 2)$

(vii) $(-3, 2)$ is a point of inflection of $f$

7. (a) Assume $R$ dollars are deposited into an ordinary annuity at the end of each month where interest is 9% APR compounded monthly. Calculate the least whole number of years that it will take the future value of the annuity to grow to $1,000R$ dollars. [5 points]

(b) Suppose now that $R$ dollars are deposited into another ordinary annuity at the end of each month for 15 years at an interest rate of $x$% APR compounded monthly. At the end of the 15 years it is known that the value of the annuity is $500R$. State the function $f(x)$ that you would apply Newton's method to in order to approximate the value of $x$. [4 points]

(c) What is the maximum amount of compound interest (expressed as a percentage, rounded to one decimal) that can be earned on a principal of $P$ dollars for exactly $2,000$ days at 7.4% APR? (use 1 year = 365 days) [4 points]

8. Evaluate and simplify.

(a) $\displaystyle\int \frac{x^3}{\sqrt{x^2+1}}\,dx$ [7 points]

(b) $\displaystyle\int_0^1 \frac{xe^x}{(x+1)^2}\,dx$ [8 points]

9. A marketing researcher has determined that when a product sells at $x$ dollars per unit, the number of units sold per week is given by the function $q = \dfrac{75}{x-m} + 80(5m - x)$ where $m > 0$ is the total manufacturing cost in dollars per unit and $m < x < 5m$.

   (a) Find the profit function $P(x)$ (in dollars per week) and simplify it. [5 points]

   (b) **Use calculus** to find the selling price $x$ that maximizes weekly profit. Sufficiently justify your solution. [6 points]

   (c) Use your answer to part (b) to determine the manufacturing cost $m$ that yields a maximum weekly profit of $3,455$ dollars. [4 points]

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