MATA32 – Winter 2010
Practice Midterm Problems
(from previous midterms)

The following formulas will be given on the exam:

$$a + ar + ar^2 + \cdots + ar^{N-1} = a \times \frac{1 - r^N}{1 - r}$$

$$P(x) = P_0 \left(1 + \frac{r}{N}\right)^{Nx}$$

$$P_{\text{cont}}(x) = P_0 e^{rx}$$

1. The future value of \$132.73 earning annual interest of 2% compounding continuously at the end of 246 months (to the nearest dollar, rounded up) is:

2. If $f(x) = \dfrac{x^2 - 3x - 18}{x^2 + 3x} + e^4 e^x$, determine the exact value of $\lim\limits_{x \to -3} f(x)$.

3. If an investment increases by exactly 255% over 16 years under a constant periodic interest rate of r% compounding quarterly, what is the nominal annual interest rate (to two decimal places)?

4. The premiums on an insurance policy are \$40 per month, payable at the end of each month. If the policy holder wishes to pay for three year's premiums in advance, how much (rounded up to the nearest dollar) should be paid, provided that the interest rate is 6% APR compounding monthly?

5. A total debt of \$1000 due now, \$4000 due 2 years from now, and \$6000 due 5 years from now is to be repaid in three payments:

   (1) the first payment is made now;
   (2) the second payment (which is 80% of the first) is made at the end of 30 months from now;
   (3) the third payment (which is 60% of the second) is made at the end of 4 years from now.

   Interest is 4% APR compounding semiannually. Calculate the amount of each of the three payments. Round your final answers up to the nearest dollar. A time diagram and an equation of value is required to receive full marks.

6. Suppose R dollars is deposited into an annuity at the end of each quarter year. Interest is 5.2% APR compounding quarterly.

   (a) Determine the effective rate of interest, expressed as a percentage (rounded up to three decimal places).

(b) If it is assumed that the annuity is empty to begin with, find the least number of quarters it will take for the annuity to have a (future) value of $500R$.

7. Find (to the nearest cent) the present value of $60 due in three years at 3.6% APR compounded quarterly.

8. To three decimals, what interest rate compounded continuously is equivalent to 5.1% APR compounded every three months?

9. If $h(x) = \dfrac{x^2 - 9x + 20}{x^2 - 3x - 4}$, determine the value of $\lim\limits_{x \to 4} h(x)$.

10. Determine the least number of months required for a principal of P dollars to increase by 40% at a 4.2% APR compounded semiannually.

11. A bank account gives 5.2% APR compounding weekly. Assuming the account starts empty, what is the least full dollar amount your parents must deposit in the account now so that you can make a withdrawal of $50 at the end of each week for the next 52 weeks?

12. Determine $\lim\limits_{x \to \infty} \left(\dfrac{x^3}{x^2 - 5x} - x \right)$.

13. What nominal rate compounded every other month gives an effective rate of r_e?

14. A debt of $8000 due 4 years from now and $4000 due 8 years from now is to be repaid by three payments:

(1) the first payment is at the end of 2 years from now;
(2) the second payment, which is 3/4 of the first, is made at the end of 38 months from now;
(3) the third payment, which is 2/3 of the second, is made at the end of 75 months from now.

If interest is 4.8% APR compounding monthly, calculate the value of each payment. (Round the final answer up to the nearest dollar. A time-diagram and equation of value is required for full marks.)

15. Imagine winning a large lottery. There are two banks in which to consider investing your winnings: Bank A pays 5.54% APR compounding monthly and Bank B pays 5.52% APR compounding daily (365 days per years).

(a) Which bank is the better choice to invest your lottery winnings? Justify your answer.

(b) Assume Bank C offers r% compounded quarterly. What value of r makes Bank A and Bank C equally attractive for the investment of your winnings? Round your final answer to three decimals.

16. Find the annual continuously compounding interest rate that would cause a principal to increase by exactly 132% at the end of 4234 days (365 days = 1 year). Express your answer as a percentage rounded to two decimals.

17. Find the value of the constant c so that $\lim_{x \to 2} f(x)$ exists, where

$$f(x) = \begin{cases} \frac{x^3 - 8}{x - 2} & \text{if } x < 2 \\ c^2 x^2 & \text{if } x > 2 \end{cases}$$

Justify your answer.

18. Evaluate each of the following limits:

   (a) $\lim_{x \to 3} \dfrac{1 - \sqrt{x - 2}}{x - 3}$

   (b) $\lim_{x \to 0^-} \left(\dfrac{1}{|x|} + \dfrac{1}{x} \right)$

19. A bank account pays interest at 6% APR compounding quarterly. On your 17th birthday, you deposit $1000 into the empty account. Beginning with the first quarter after your 20th birthday, you make a $750 deposit into the account at the end of each quarter up to and including your 40th birthday. Then, starting with the first quarter after your 40th birthday, you deposit at the end of each quarter $1500 up to and including your 62nd birthday. There are no further deposits after your 62nd. Calculate how much you'll have in the account on your 67th birthday. Round your final answer up to the nearest dollar.

ANSWERS:

1. $200

2. $3 + e$

3. 8.00%

4. $1315

5. 1st payment: $4507
   2nd payment: $3605
   3rd payment: $2163

6. (a) 5.303%
   (b) 156 quarters

7. $53.88

8. 5.068%

9. $-1/5$

10. 102

11. $2533

12. 5

13. $6\left(\sqrt[6]{1 + r_e} - 1\right)$

14. 1st payment: $4851;
    2nd payment: $3639;
    3rd payment: $2426.

15. (a) Bank A
    (b) 5.557%

16. 7.25%

17. $\pm\sqrt{3}$

18. (a) $-1/2$
    (b) 0

19. $956,050