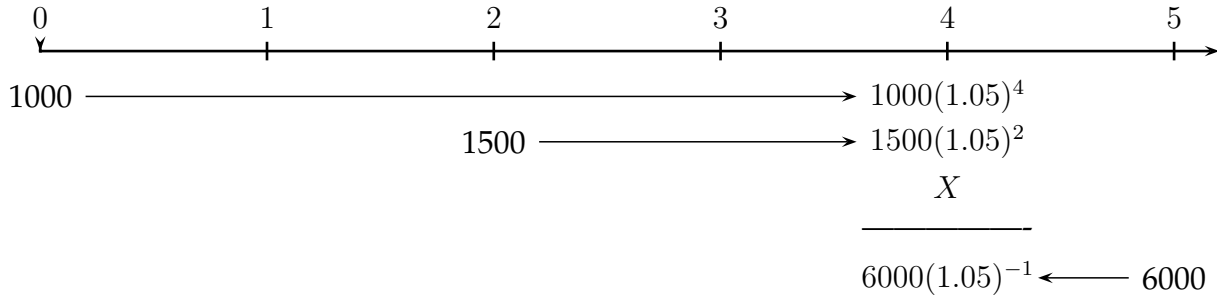


MATA32 – Winter 2010

Quiz 4: Solutions

Name: _____ KEY

1. A debt of \$6,000 due five years from now is instead to be paid off by three payments: \$1000 now, \$1500 in two years, and a final payment at the end of four years. What would this final payment be if an interest rate of 5% compounded annually is assumed?



Let X be the amount of the third installment. As indicated on the timeline above, the future value of the first installment four years from now is $1000(1.05)^4$. Similarly, the future value of the second installment is $1500(1.05)^2$. On the other hand, the total debt of \$6000 has a value of $6000(1.05)^{-1}$ in the fourth year. Therefore, to balance the debt with the payments, we must have

$$1000(1.05)^4 + 1500(1.05)^2 + X = 6000(1.05)^{-1}.$$

Solving for X gives $X \approx 2845.029$. Therefore the amount of the third installment should be \$2845.03.

2. Find the sum of the geometric series

$$\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + \cdots + \frac{1}{2^8}$$

The first term is $a = \frac{1}{4}$, the constant ratio is $r = \frac{1}{2}$, and the number of terms is $N = 8 - 2 + 1 = 7$. Therefore, the sum of the series is

$$a \times \frac{1 - r^N}{1 - r} = \frac{1}{4} \times \frac{1 - \left(\frac{1}{2}\right)^7}{1 - \frac{1}{2}} = 1 - \frac{1}{2^7} = \frac{127}{128}$$