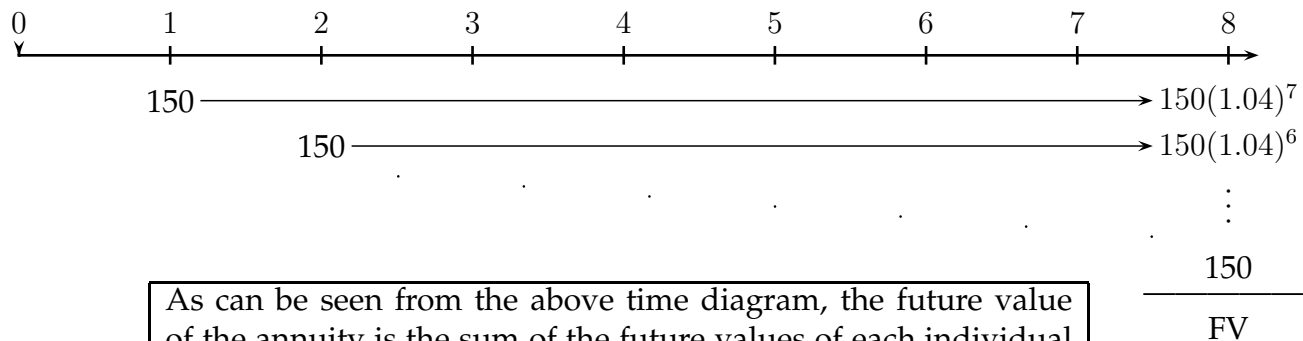


MATA32 – Winter 2010

Quiz 5: Solutions

Name: _____ KEY

1. Find the amount (future value) of an annuity consisting of payments of \$150 at the end of every six months for 4 years, at the rate of 8% compounded semiannually.



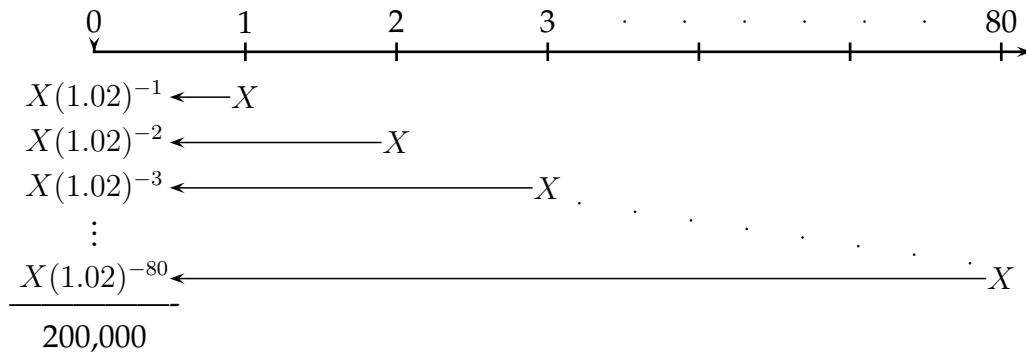
As can be seen from the above time diagram, the future value of the annuity is the sum of the future values of each individual contribution of \$150, i.e.

$$\begin{aligned} \text{FV} &= 150 + 150(1.04) + 150(1.04)^2 + \cdots + 150(1.04)^7 \\ &= 150 \times \frac{(1.04)^8 - 1}{0.04} \\ &\approx 1382.134 \end{aligned}$$

(the book would call this $150 s_{\overline{8}|0.04}$). We conclude that the amount of the annuity is \$1382.13.

Continued on reverse...

2. A person amortizes a loan of \$200,000 for a new home by obtaining a 20-year mortgage at the rate of 8% compounded quarterly.



(a) Determine the quarterly payment.

As indicated in the picture above, since we are dividing 20 years into quarterly pay periods, we will pay 80 installments all together. Let's say each installment is X dollars. From the above picture, we see that for the total payments to balance the loan plus interest, we must have that the sum of the present values of each installment equals the present value of the loan, i.e.

$$\begin{aligned} 200,000 &= X(1.02)^{-1} + X(1.02)^{-2} + X(1.02)^{-3} + \dots + X(1.02)^{-80} \\ &= X a_{\overline{80}|0.02} \end{aligned}$$

where as usual,

$$\begin{aligned} a_{\overline{80}|0.02} &= (1.02)^{-1} + (1.02)^{-2} + (1.02)^{-3} + \dots + (1.02)^{-80} \\ &= (1.02)^{-1} \times \frac{1 - (1.02)^{-80}}{1 - (1.02)^{-1}} \\ &\approx 39.745 \end{aligned}$$

It follows that each quarterly payment is $X \approx \frac{200,000}{39.745} \approx 5032.14$ dollars.

(b) Determine the total interest charges.

The person pays 80 installments of \$5032.14 each, i.e. a total of $80 \times 5032.14 = 402,571.20$ dollars. Since he originally owed \$200,000, the total interest charge is $402,571.20 - 200,000 = 202,571.20$ dollars.