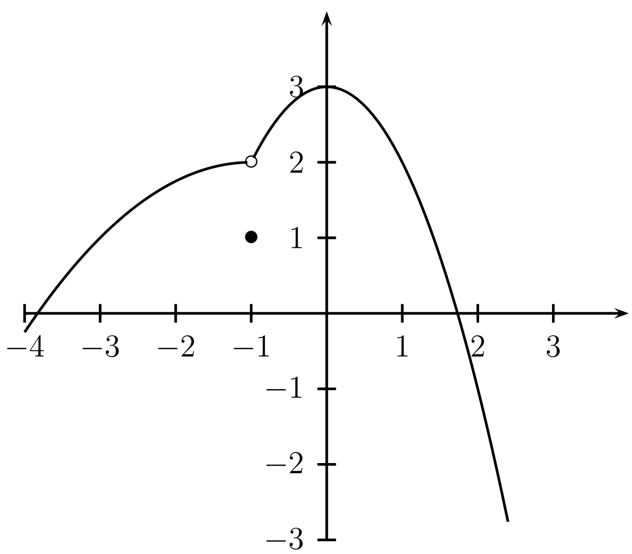
MATA32 – Winter 2010

Quiz 6: Solutions

Name: KEY

1. The picture below shows a graph of $y = f(x)$. Does $\lim_{x \to -1} f(x)$ exist? If so, evaluate it; if not, explain why not.



As indicated in the picture, $f(-1) = 1$. However, as x approaches -1 from the left, the value of $f(x)$ is approaching 2; in other words, $\lim_{x \to -1^-} f(x) = 2$. Similarly, $\lim_{x \to -1^+} f(x) = 2$. Since the left and right limits both exist and agree, we conclude that $\lim_{x \to -1} f(x)$ exists, and equals 2.

Continued on reverse...

2. (a) Consider $\lim_{x \to -2} \dfrac{x+2}{x^2-4}$. If the limit exists, evaluate it. If the limit does not exist, explain why not.

First, note that this is a $0/0$ case, which means the answer could be anything ... so we have to work out the answer carefully. Here we go:

$$\begin{aligned} \lim_{x \to -2} \frac{x+2}{x^2-4} &= \lim_{x \to -2} \frac{x+2}{(x+2)(x-2)} \\ &= \lim_{x \to -2} \frac{1}{x-2} \\ &= -\frac{1}{4} \end{aligned}$$

(b) Consider $\lim_{x \to 2} \dfrac{x+2}{x^2-4}$. If the limit exists, evaluate it. If the limit does not exist, explain why not.

In this case, blindly plugging in $x = 2$ would give $4/0$, which automatically tells us that the answer is either $+\infty$, $-\infty$, or DNE. To figure out which one, we plug in values slightly to the left and to the right of 2 and see what we get (they must be either $+\infty$ or $-\infty$). If we plug in $x = 1.9$, we would have a positive number in the numerator, and a negative in the denominator; this tells us that $\lim_{x \to 2^-} \dfrac{x+2}{x^2-4} = -\infty$. Plugging in $x = 2.1$ still yields a positive numerator, but this time, the denominator is also positive; in other words, $\lim_{x \to 2^+} \dfrac{x+2}{x^2-4} = +\infty$. Since the left and right limits do not agree, the two-sided limit as $x \to 2$ does not exist.