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MATC01: GROUPS AND SYMMETRY

Problem Set 9 – due Monday, November 18th

INSTRUCTIONS:
To receive credit, you must turn this in during the first 5 minutes of lecture on the due date. Please print and attach this page as the first page of your submitted problem set.

PROBLEM	**MARK**
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Please read the following statement and sign below:

I understand that I am not allowed to use the internet to assist (in any way) with this assignment. I also understand that I must write down the final version of my assignment in isolation from any other person.

SIGNATURE:______________________________

Problem Set 9

9.1 Suppose $H \leq \Gamma$ (a group), and let a be any element of Γ.

(a) Show by example that aH is not necessarily a subgroup of Γ.

(b) Prove that if H is finite, then $|aH| = |H|$.

9.2 Suppose $H \leq \Gamma$ (a group), and let a be any element of Γ. Above, we saw that sets of the form aH are not necessarily subgroups. It turns out that a slight variant, $aHa^{-1} = \{aha^{-1} : h \in H\}$, is better behaved.

(a) Prove that $aHa^{-1} \leq \Gamma$.

(b) Prove that if H is finite, then $|aHa^{-1}| = |H|$.

(c) Prove that $H \subseteq gHg^{-1}$ for all $g \in \Gamma$ if and only if $H = gHg^{-1}$ for all $g \in \Gamma$.

9.3 Given $H \leq \Gamma$ (a group) and $a, b \in \Gamma$, prove that either $aH = bH$ or $aH \cap bH = \emptyset$. [*Hint: Start by assuming that* aH *and* bH *aren't disjoint. Why must* $aH = bH$*?*]

9.4 Given $H \leq \Gamma$ (a group), recall that we defined

$$[a] := \{g \in \Gamma : aH = gH\}.$$

(a) Prove that $xH = yH$ iff $[x] = [y]$. (Only $\Rightarrow$ was proved in lecture.)

(b) Prove that $xH = yH$ iff $x^{-1}y \in H$. (Only $\Leftarrow$ was proved in lecture.)

9.5 Recall from lecture that given $H \leq \Gamma$ (a group), we define

$$\Gamma/H := \{[a] : a \in \Gamma\}.$$

(a) Determine $\mathcal{G}_{\{\pm 1 \pm i\}}/K$, where $\mathcal{G}_{\{\pm 1 \pm i\}}$ is the group of symmetries of the square and $K = \{\mathbb{1}, \rho\}$. Your answer must be explicit and simplified (i.e. it should be a set, in which you explicitly list all the elements without duplicates). Don't worry if your answer is different from what we found in lecture.

(b) Determine $\mathcal{G}/A$, where $\mathcal{G}$ denotes the group of plane isometries and $A = \{T_h R_\theta : h \in \mathbb{C}, \theta \in [0, 2\pi)\}$. Again, your answer must be explicit and simplified.

9.6 Recall that given $H \leq \Gamma$ (a group), we proved the existence of a set $\overline{\Gamma} \subseteq \Gamma$ such that

$$\Gamma = \bigsqcup_{g \in \overline{\Gamma}} gH.$$

(a) Give two different examples of $\overline{\Gamma}$ for $\Gamma = \mathcal{G}_{\{\pm 1 \pm i\}}$ and $H = \{\mathbb{1}, \rho\}$.

(b) Give two different examples of $\overline{\Gamma}$ for $\Gamma = \mathcal{G}$ and $H = A$ as in 9.5(b).