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Problem Set 1 (due September 27, 2010 at the start of lecture)

Please review the policies listed in the syllabus.

1.1 In lecture, we decided (after some experimentation) that

every positive integer can be written as the sum of distinct powers of 2. (*)

(a) Write 133 as the sum of distinct powers of 2.

(b) Describe the "greedy algorithm" for finding the decomposition of a general positive integer into distinct powers of 2. (You may find it helpful to look over our algorithm for decomposition into distinct Egyptian fractions.)

(c) Carefully explain why the algorithm must terminate after a finite number of steps?

(d) Carefully explain why the algorithm gives a decomposition into *distinct* powers of 2.

(e) Produce an example of a number which *cannot* be decomposed as a sum of distinct powers of 3. How can you modify the statement (*) above so that one can express any positive integer in terms of powers of 3?

1.2 In this exercise, you will explore Egyptian fractions a little more. Recall that an Egyptian fraction decomposition of a fraction a/b is a representation of a/b as a sum of distinct Egyptian fractions. The *length* of the decomposition is the number of terms (i.e. $8/15 = 1/3 + 1/5$ is a decomposition of length 2).

(a) Find two different Egyptian fraction decompositions of 19/90, each of length two.

(b) What is the shortest Egyptian fraction decomposition you can find for 7/17?

(c) Show that any Egyptian fraction has an Egyptian fraction decomposition of length 2.

(d) Recall that the Erdős-Straus conjecture states that $4/n$ admits an Egyptian fraction decomposition of length 3, for every odd $n \geq 5$. Check the conjecture for $n = 5$, 7, 9, 11, and 13.

1.3 (a) Write 4078 in Egyptian hieroglyphs.

(b) Write 7/9 in Egyptian hieroglyphs.

1.4 (a) Use the multiplication algorithm described in the Rhind papyrus to compute 34×95. Then do the same multiplication using the usual (modern) technique.

(b) Redo both parts of (a) for the product 30×79.

(c) Compare and contrast the modern method with the Egyptian. What are some advantages and disadvantages of each? Under which circumstances is one faster or easier than the other?

1.5 (a) Using what we discussed in lecture, deduce the Egyptian approximation of π (write the approximation to 4 decimals).

(b) Find your own approximation to π by drawing an $n \times n$ grid of squares over a circle. What value of n must you choose to obtain a better approximation to π than the Egyptians had?