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Problem Set 2 (due October 18, 2010 at the start of lecture)

Please review the policies listed in the syllabus.

2.1 An artist's rendition of a certain Babylonian tablet is given in Figure 2.1 (see attached picture).

(a) Interpret the tablet; what is the significance of the numbers?

(b) What does your answer to (a) indicate about the mathematical knowledge of the Babylonians? Justify your answer.

2.2 In lecture we proved that $\sqrt{2}$ is irrational (the argument we gave, as well as a geometric one, appears in Eves). In this exercise, you will explore some ramifications of this.

(a) $\sqrt{2} \approx 1.414213\ldots$ Is this decimal expansion finite? What else can you say about the decimal expansion?

(b) Prove that $\sqrt{3}$ is irrational.

(c) Give an example of an integer k such that k^2 is a multiple of 12, but k is not.

(d) Prove that $\sqrt{12}$ is irrational. (Take part (c) into account!)

(e) Prove that $\sqrt{2} + \sqrt{6}$ is irrational.

2.3 In this exercise, you will explore the Babylonian number system a bit more.

(a) Recall from lecture that around 300 BC, a symbol for zero was introduced... but only used in the middle of a number, never at the end. Write 1512025 in cuneiform, both how it would have appeared in pre- and post-300 BC. In each case, give three other interpretations of which number the cuneiform might represent.

(b) As seen above (and discussed in lecture), the Babylonian system can easily lead to ambiguity. Discuss why the Babylonians might have been content with their restricted use of zero. In other words, why does using zero in the middle of a number potentially have a greater impact on the interpretation of that number than using zero at the end? Use examples to justify your response.

(c) Write 1/2, 1/3, 1/4, 1/5, 1/6, 1/8 in (post-300 BC) cuneiform; these fractions are said to be *regular* base 60, because they have finite sexagesimal expressions. Which of these fractions are regular base 10 (i.e. have finite decimal expansions)? Do you notice a pattern? Which of these fractions would you expect to be regular base 14? (You do not have to actually write down the fractions base 14.)

(d) Write 1/11 in cuneiform. Indicate a repeating block by putting a bar over it (this notation is anachronistic, but convenient nonetheless).

(e) Write 29/15 and 29/20 in (post-300 BC) cuneiform. Now write 29×3 (i.e. 87) and 29×4 (i.e. 116) in (post-300 BC) cuneiform. What do you observe? Explain why this happens.

(f) Use your observations from part (e) to write down 37×15 in cuneiform, *without* first multiplying the two numbers!

2.4 Compare and contrast Egyptian, Babylonian, and modern numerals. What are some advantages / disadvantages of each system?