Instructor: Leo Goldmakher

NAME: _____

**University of Toronto at Scarborough**
**Department of Computer and Mathematical Sciences**

**Problem Set 5 (due December 6th, 2010 at the start of lecture)**

**INSTRUCTIONS:** Please attach this page as the first page of your submitted problem set.

| PROBLEM | MARK |
|---|---|
| | |
| 5.1 | |
| 5.2 | |
| 5.3 | |
| 5.4 | |
| 5.5 | |
| 5.6 | |
| **Total** | |

## Problem Set 5

**5.1** Use Archimedes' method to approximate $\pi$, by inscribing a regular 12-sided polygon in a circle and taking the ratio of the perimeter to diameter. You must give your answer in exact form to receive full credit. [*Hint: If $ABC$ is a right triangle with $\measuredangle A = 15^\circ$ and $\measuredangle B = 75^\circ$, then*

$$\frac{BC}{AB} = \frac{\sqrt{6}-\sqrt{2}}{4}.$$

*(In the language of trigonometry, this is the same as saying* $\sin 15^\circ = \frac{\sqrt{6}-\sqrt{2}}{4}$*.) Feel free to use this fact without proving it on your assignment.* ]

**5.2** Compare and contrast the Chinese, Indian, and Greek concepts of number. How does this fit with their respective approaches to mathematics?

**5.3** In class, we discussed several examples of geometric series. We continue the topic in this problem.

(a) Prove the formula

$$a + ar + ar^2 + ar^3 + \cdots + ar^n = a \times \frac{1 - r^{n+1}}{1 - r}$$

where $a$ and $r$ are any two numbers. You may wish to look at the examples we covered in lecture.

(b) Use part (a) to show that $0.\overline{9} = 1$. (Recall that $0.\overline{9} = 0.9999\ldots$.)

**5.4** In class we discussed Archimedes' law of buoyancy, and mentioned the following story about King Hiero's crown. The king gave his goldsmith a quantity of gold from which to fashion a crown; however, when the crown was ready, he grew suspicious that the goldsmith was trying to cheat him by replacing some of the gold with silver. The story goes that Archimedes proposed a method by which one could determine whether the crown was made of pure gold, without altering the crown in any way. Devise a potential method Archimedes might have used to accomplish this. (Remember that Archimedes did not have access to modern technology!)

**5.5** Show that $24 \mid (p^2 - 1)$ for all primes $p \geq 5$.

**5.6** I have a lot of pennies. When I arrange them in groups of three, there is 1 penny left over; when I arrange them in groups of seven, there are 2 pennies left over; and when I arrange them in groups of eleven, there are 8 pennies left over. How many pennies do I have? Give two possible answers. [*Hint: 154 has a remainder of 1 when divided by 3, 99 has a remainder of 1 when divided by 7, and 210 has a remainder of 1 when divided by 11.*]