Homework Problems for Calculus I (Professor Steven Miller, Steven.J.Miller@williams.edu)

Calculus: Early Transcendentals: Edwards and Penney

http://www.math.brown.edu/UTRA/
http://www.math.brown.edu/∼sjmiller/90/index.htm

1 Chapter 1

Section 1.1: Functions, Graphs and Models

- 1. Find $f(-a)$, $f(a^{-1})$, $f(\sqrt{a})$ and $f(a^2)$ when $f(x) = 1/x$.
- 14. Compute and simplify $f(a+h) - f(a)$ when $f(x) = x^2 + 2x$.
- 29. Find the largest domain of real numbers where $f(x) = \frac{2}{3-x}$ determines a real valued function.
- 34. Find the largest domain of real numbers where $f(x) = \sqrt{\frac{x+1}{x-1}}$ determines a real valued function.
- 42. An oil field containing 20 wells has been producing 4000 barrels of oil daily. For each new well that is drilled, the daily production of each well decreases by 5 barrels per day. Write the total daily production of the oil field as a function of the number x of new wells drilled.
- 47. An open-topped box is to be made from a square piece of cardboard of edge length 50in by cutting out four squares, one in each corner. If each square has side x, find a formula for the volume of the box obtained by folding the four flaps upward (as a function of x).
- Suggested Problem: 36. Express the area A of a square as a function of its perimeter P.

- Suggested Problem: 38. Express the volume V of a sphere as a function of its surface area S.

Section 1.2: Graphs of Equations and Functions

- 5. Write an equation of the line L described and sketch its graph: L passes through $(2, -3)$ and $(5, 3)$.

- 39. Sketch the graph of $f(x) = \frac{1}{(x-1)^2}$.

- 65. Use the method of completing the square to graph $y = 96t - 16t^2$ and find the maximum value.

- Suggested Problem: 16. Sketch the translated circle $9x^2 + 9y^2 - 6x - 12y = 11$ and find the center and radius.

- Suggested Problem: 44. Sketch $f(x) = 1/\sqrt{1-x}$.

- Suggested Problem: 79. For a FedEx letter weighing at most one pound the charge C is \$8 for the first 8 ounces and \$.8 for each additional ounce. Sketch the graph of the function C of the total number x of ounces, and describe it symbolically in terms of the greatest integer function.

Section 1.3: Polynomials and Algebraic Equations

- 14. Sketch the graph of $f(x) = \frac{x}{x^2-9}$.

- 22. Sketch the graph of $f(x) = x^3 - 3x + 2$. In particular, find (approximately) the zeros.

Section 1.4: Transcendental Functions

- 14. Find $f(g(x))$ and $g(f(x))$ if $f(x) = x^2 + 1$ and $g(x) = \frac{1}{x^2+1}$.

- 22. Find a function of the form $f(x) = x^k$ and a function g such that $f(g(x)) = h(x)$ where $h(x) = (4-x)^3$.

- 49. [Do not do.]

- Suggested Problem: 1, 7, 8. [Do not do.]

- Suggested Problem: 20. Find $f(g(x))$ and $g(f(x))$ where $f(x) = 1 - x^2$ and $g(x) = \sin x$.

2 Chapter 2: Prelude to Calculus

Section 2.1: Tangent Lines and Slope Predictors

- 5. Find the slope at a and the tangent line at $x = 2$ of $f(x) = 4x - 5$.
- 7. Find the slope at a and the tangent line at $x = 2$ of $f(x) = 2x^2 - 3x + 4$.
- 15. Find all points of the curve $y = 10 - x^2$ such that the tangent line is horizontal (i.e., has a slope of zero).
- Suggested Problem: 33. One of the two lines that pass through the point $(3, 0)$ of the parabola $y = x^2$ is the x-axis. Find an equation for the other line.

Section 2.2: The Limit Concept

- 1. Find $\lim_{x \to 3}(3x^2 + 7x - 12)$; show all steps.
- 4. Find $\lim_{x \to -2}(x^3 - 3x + 3)(x^2 + 2x + 5)$; show all steps.
- 6. Find $\lim_{t \to -2} \frac{t+2}{t^2+4}$; show all steps.
- 21. Find $\lim_{z \to -2} \frac{(z+2)^2}{z^4-16}$; show all steps.
- 37. Find the slope-predictor function for $f(x) = x^3$ and then find the equation of the tangent line at $x = 2$.
- Suggested Problem: 27. Find $\lim_{x \to 1} \frac{x^3-1}{x^4-1}$.
- Suggested Problem: 38. Find the slope-predictor function for $f(x) = 1/x$ and then find the equation of the tangent line at $x = 2$.

Section 2.3: More About Limits

- 1. Find $\lim_{\theta \to 0} \frac{\theta^2}{\sin \theta}$
- 2. Find $\lim_{\theta \to 0} \frac{\sin^2 \theta}{\theta^2}$
- 7. Find $\lim_{x \to 0} \frac{\sin 5x}{x}$

- 25. Use the squeeze law and find $\lim_{x\to 0} x^2 \cos(10x)$
- 32. Use one-sided limit laws to find the limit or prove it does not exist: $\lim_{x\to 4^-} \sqrt{4-x}$.
- Suggested Problem: 70. The sign function $\operatorname{sgn}(x)$ is defined as follows: $\operatorname{sgn}(x) = x/|x|$ if $x \neq 0$ and 0 if $x = 0$. Use the sign function to define two funtions f and g whose limits as $x \to 0$ do not exist, but such that (a) $\lim_{x\to 0} [f(x) + g(x)]$ does not exist; (b) $\lim_{x\to 0} f(x) \cdot g(x)$ does exist.

Section 2.4: The Concept of Continuity

- 1. Apply the limit laws to show that $f(x) = 2x^5 - 7x^2 + 13$ is continuous for all x.
- 6. Apply the limit laws to show that $h(x) = \sqrt[3]{1-5x}$ is continuous for all x.
- 15. Determine if $f(x) = 2x + \sqrt[3]{x}$ is continuous for all x.
- 20. Determine if $g(z) = \frac{1}{z^2-1}$ is continuous for all z.
- 49. Find c so that $f(x) = x + c$ if $x < 0$ and $4 - x^2$ if $x \geq 0$ is continuous for all x.
- Suggested Problem: 47. Consider $f(x) = 1 + x^2$ if $x < 0$ and $\frac{\sin x}{x}$ if $x > 0$. Determine if there is a way to define this function at $x = 0$ to make it continuous.
- Suggested Problem: 52. Find c so that $f(x) = c^3 - x^3$ if $x \leq \pi$ and $c \sin x$ if $x > \pi$ is continuous for all x.
- Suggested Problem: 61. [Do not do.]
- Suggested Problem: 63. Suppose that f and g are two continuous functions on $[a, b]$ such that $f(a) = g(b) = p$ and $f(b) = g(a) = q$ where $p \neq q$. Sketch typical graphs of such functions. Apply the Intermediate Value Theorem to the function $h(x) = f(x) - g(x)$ to show that $f(c) = g(c)$ for some c in (a, b).
- Suggested Problem: 66. Apply the Intermediate Value Property to show that every real number has a cube root.

3 Chapter 3: The Derivative

3.1: The Derivative and Rates of Change

- Find the derivative of $f(x) = 3x^2 - 4x + 1$ by using the definition of the derivative (i.e., calculate the limit).

Section 3.2: Basic Differentiation Rules

- 3. Find the derivative of $f(x) = (2x + 3)(3x - 2)$.
- 8. Find the derivative of $f(x) = 4x^4 - \frac{1}{x^2}$.
- 16. Find the derivative of $f(x) = \frac{2x^3 - 3x^2 + 4x - 5}{x^2}$.
- 25. Find the derivative of

$$g(x) = \frac{\frac{1}{x} - \frac{2}{x^2}}{\frac{2}{x^3} - \frac{3}{x^4}} = \frac{x^{-1} - 2x^{-2}}{2x^{-3} - 3x^{-4}}.$$

- 42. Write the equation of the tangent line to the curve $y = f(x)$ at P, expressing your answer in the form $ax + by = c$, when $y = 3x^2 - 4$ and $P = (1, -1)$.
- Suggested Problem: 55. Find an equation for the straight line that passes through the point $(1, 5)$ and is tangent to the curve $y = x^3$.
- Suggested Problem: 61. Prove the derivative of $f(x)^3$ is $3f(x)^2 f'(x)$ by using the product rule multiple times.
- Suggested Problem: 66. Find constants a, b, c, d such that the graph of $f(x) = ax^3 + bx^2 + cx + d$ has horizontal tangents at $(0, 1)$ and $(1, 0)$.
- Suggested Problem: 73. Sketch the graph of $f(x) = |x|^3$ and determine where it is differentiable.

Section 3.3: The Chain Rule

- 1. Find dy/dx when $y = (3x + 4)^5$.

- 7. Find dy/dx when $y = (2-x)^4(3+x)^7$.
- 39. Find dy/dx two different ways (using the chain rule and without using the chain rule) when $y = (x^2-1)^2 = x^4 - 2x^2 + 1$.
- 42. Find dy/dx two different ways (using the chain rule and without using the chain rule) when $y = (x+1)^{-2} = \frac{1}{x^2+2x+1}$.
- 54. Find $f'(-1)$ given $f(y) = h(g(y))$, $h(2) = 55$, $g(-1) = 2$, $h'(2) = -1$ and $g'(-1) = 7$.
- 33. Suggested Problem: Find the derivative of $h(v) = \left[v - \left(1 - \frac{1}{v}\right)\right]^{-2}$.
- 49. Suggested Problem: [Do not do.]
- 52. Suggested Problem: Each edge of an equilateral triangle is increases at 2 cm/s. At what rate is the area of the triangle increasing when each edge is 10 cm?
- 63. Suggested Problem: Suppose u is a function of v, v is a function of w and w is a function of x, and all these functions are differentiable. Explain why it follows from the chain rule that $\frac{du}{dx} = \frac{du}{dv} \cdot \frac{dv}{dw} \cdot \frac{dw}{dx}$.

Section 3.4: Derivatives of Algebraic Functions

- 1. Differentiate $f(x) = 4\sqrt{x^5} + \frac{2}{\sqrt{x}}$.
- 17. Differentiate $f(x) = (2x^2 - x + 7)^{3/2}$.
- 31. Differentiate $f(x) = x(3-4x)^{1/2}$.
- 47. Find all points where the tangent line to $y = x^{1/2} - x^{3/2}$ is either horizontal or vertical.
- 57, 59, 62. [Do not do.]
- 63. The period of oscillation P (in seconds) of a simple pendulum of length L (in feet) is $P = 2\pi\sqrt{L/g}$ where $g = 32$ ft/s^2. Find the rate of change of P with respect to L when $P = 2$.

- 71. Consider the cubic equation $x^3 = 3x + 8$. If we differentiate each side with respect to x, we find $3x^2 = 3$, which has the two solutions $x = 1$ and $x = -1$; however, neither of these is a solution to the original equation. What went wrong?

- Suggested Problem: 43. Differentiate $g(t) = \sqrt{t + \sqrt{t + \sqrt{t}}}$.

- Suggested Problem: 58, 60, 61. [Do not do.]

- Suggested Problem: 65. Find the two points on the circle $x^2 + y^2 = 1$ at which the slope of the tangent line is -2.

Section 3.7: Derivatives of Trigonometric Functions

- 1. Differentiate $f(x) = 3\sin^2 x$.

- 7. Differentiate $f(x) = \frac{\sin x}{x}$.

- 8. Differentiate $f(x) = \cos^3 x \sin^2 x$.

- 13. Differentiate $f(x) = 2x \sin x - 3x^2 \cos x$.

- 20. Differentiate $g(t) = 1/\sqrt{\sin^2 t + \sin^2(3t)}$.

- 41. Find dx/dt when $x = \tan(t^7)$.

- 45. Find dx/dt when $x = t^7 \tan(5t)$.

- Suggested Problem: 21. Find dy/dx when $y = \sin^2(\sqrt{x})$.

- Suggested Problem: 23. Find dy/dx when $y = x^2 \cos(3x^2 - 1)$.

- Suggested Problem: 38. Find dy/dx when $y = \sqrt{x} \sin\left(\sqrt{x + \sqrt{x}}\right)$.

- Suggested Problem: 50. Find dx/dt when $x = \sec(t)/(1 + \tan(t))$.

- Suggested Problem: 61. Write an equation of the line that is tangent to $y = x\cos x$ at $x = \pi$.

- Suggested Problem: 71. Find the derivatives of $\cot x$, $\sec x$ and $\csc x$.

- Suggested Problem: 77. An observer on the ground sights an approaching plane flying at constant speed and altitude 20000ft. From his point of view, the plane's angle of elevation is increasing at .5 degrees per second when the angle is 60 degrees. What is the speed of the plane?

Section 3.8: Exponential and Logarithmic Functions

- 1. Differentiate $f(x) = e^{2x} = \exp(2x)$.
- 7. Differentiate $g(t) = te^{\sqrt{t}} = t\exp(\sqrt{t})$.
- 18. Differentiate $f(x) = \sqrt{e^{2x} + e^{-2x}}$.
- 33. Differentiate $f(x) = \ln(\cos x)$.
- 39. Find the derivative of $f(x) = \ln\left[(2x+1)^3(x^2-4)^4\right]$ (hint: apply log laws before differentiating).
- 59. Find the tangent line to $y = xe^{2x} = x\exp(2x)$ when $x = 1$.
- Suggested Problem: 63: Take several derivatives of $f(x) = e^{2x}$ and guess what the n^{th} derivative will be.
- Suggested Problem: 64: Take several derivatives of $f(x) = xe^x$ and guess what the n^{th} derivative will be.
- Suggested Problem: 69, 71. [Do not do.]
- Suggested Problem: 72. Show that $\log_2 3$ is irrational.

Section 3.5: Maxima And Minima Of Functions On Closed Intervals

- 1. Determine whether or not $f(x) = 1-x$ has a maximum or minimum value (or both) on the interval $[-1, 1)$.
- 9. Determine whether or not $f(x) = \frac{1}{x(1-x)}$ has a maximum or minimum value (or both) on the interval $[2, 3]$.
- 18. Find the maximum and minimum values for $g(x) = 2x^3 - 9x^2 + 12x$ for $x \in [0, 4]$.

- 28. Find the maximum and minimum values for $f(x) = |2x - 3|$ for $x \in [1, 2]$.
- 47, 48, 52. [Do not do.]
- Suggested Problem: 41. Suppose $f(x) = Ax + B$ and $A \neq 0$. Explain why the maximum and minimum values of f on any closed interval must occur at the endpoints of that interval.
- Suggested Problem: 42. Suppose f is continuous on $[a, b]$ and differentiable on (a, b) with $f'(x)$ never zero for $x \in (a, b)$. Explain why the maximum and minimum values of f on any closed interval must occur at the endpoints of that interval.
- Suggested Problem: 49, 50, 51. [Do not do.]

Section 3.6: Applied Optimization Problems

- 1. Find two positive real numbers x and y such that their sum is 50 and their product is as large as possible.
- 7. The sum of two positive numbers is 48. What is the smallest possible value of the sum of their squares.
- 13. Find the maximum possible area of a rectangle with diagonals of length 16.
- 20. Suppose you are to make a rectangular box with a square base from two different materials. The material for the top and the four sides of the box costs \$1/ft^2; the material for the base costs \$2/ft^2. Find the dimensions of the box of greatest possible volume if you are allowed to spend \$144 for the material to make it.
- 47. A company has plants located at the points $A = (0, 1)$, $B = (0, -1)$ and $C = (3, 0)$. The company plans to construct a distribution center at the point $P = (x, 0)$. What value of x minimizes the sum of the distances from P to A, B and C?
- Suggested Problem: 15. The volume V in cubic centimeters of 1kg of water at temperature T between 0° and 30°C is very closely approximated by $V = 999.87 - .06426T + .0085043T^2 - .0000679T^3$. At what temperature does water have its maximum density.

- Suggested Problem: 27. A printing company has eight presses, each of which can print 3600 copies per hour. It costs \$5 to set up each press for a run and $10 + 6n$ dollars to run n presses for 1 hour. How many presses should be used to print 50,000 copies of a poster most profitably?

- Suggested Problem: 42. Given: there is exactly one point on the graph of $y = \sqrt[3]{3x-4}$ that is closest to the origin. Find it.

- Suggested Problems: 33, 48, 49, 62. [Do not do.]

Section 3.9: Implicit Differentiation and Related Rates

- 5. Find dy/dx by implicit differentiation when $\sqrt{x} + \sqrt{y} = 1$.

- 13. Find dy/dx by implicit differentiation when $2x + 3e^y = e^{x+y}$.

- 15. Use implicit differentiation to find the equation of the tangent line to $x^2 + y^2 = 25$ at the point $(3, -4)$.

- 37. Sand is being emptied from a hopper at a rate of $10\text{ft}^3/\text{sec}$. The sand forms a conical pile whose height is always twice its radius. At what rate is the radius of the pile increasing when the height is 5ft?

- 44. The volume V (in cubic inches) and pressure p (in pounds per square inch) of a certain gas satisfies the equation $pV = 1000$. At what rate is the volume of the sample changing if the pressure is $100\text{lb}/\text{in}^2$ and is increasing at the rate of $2\text{lb}/\text{in}^2$ per second?

- Suggested Problems: 23, 38, 50.

Section 3.10: Newton's Method

- 1. Use Newton's Method to find the solution of the equation $f(x) = x^2 - 5 = 0$ on the interval $[2, 3]$ to four decimal places.

- Suggested Problem: 21. (a) Show that Newton's method applied to the equation $x^3 - a = 0$ yields the formula $x_{n+1} = \frac{1}{3}\left(2x_n + \frac{a}{x_n^2}\right)$ for approximating the cube root of a. (b) Use this formula to find $\sqrt[3]{2}$ to five decimal places.

4 Chapter 4: Additional Applications of the Derivative

Section 4.2: Increments, Differentials and Linear Approximation

- 2. Write dy in terms of x and dx when $y = 3x^2 - \frac{4}{x^2}$.
- 11. Write dy in terms of x and dx when $y = \sin(2x)\cos(2x)$.
- 28. Use a linear approximation $L(x)$ to an appropriate function $f(x)$ with an appropriate value of a to estimate $\sqrt{80}$.
- 42. Use linear approximation to estimate the change in the surface area of a sphere if the radius is increased from 5 inches to 5.2 inches.

Section 4.3: Increasing and Decreasing Functions and the MVT

- 15. Determine the open intervals on the x-axis where $f(x) = 6x - 2x^2$ is increasing and where i is decreasing.
- 17. Determine the open intervals on the x-axis where $f(x) = x^4 - 2x^2 + 1$ is increasing and where i is decreasing.
- 25. Show $f(x) = x^2 - 2x$ on $[0, 2]$ satisfies the conditions of Rolle's Theorem and find all x that satisfy the conclusions of that theorem.
- 26. Show $f(x) = 9x^2 - x^4$ on $[-3, 3]$ satisfies the conditions of Rolle's Theorem and find all x that satisfy the conclusions of that theorem.
- 33. Show that $f(x) = 3x^2 + 6x - 5$ on $[-2, 1]$ satisfies the hypotheses of the mean value theorem and find all c in $[-2, 1]$ that satisfy the conclusions of that theorem.
- 45. A car is driving along a rural road where the speed limit is 70mph. At 3pm its odometer (measuring distance traveled) reads 8075 miles. At 3:18pm it reads 8100 miles. Prove that the driver violated the speed limit at some instant between 3pm and 3:18pm.
- Suggested Problems: 1, 4, 5, 16, 18, 27, 48, 61. [Do not do.]

Section 4.4: The First Derivative Test and Applications

- 1. Apply the first derivative test to classify the critical points of $f(x) = x^2 - 4x + 5$.
- 11. Apply the first derivative test to classify the critical points of $f(x) = x + \frac{9}{x}$.
- 27. Determine two real numbers with difference 20 and minimum product possible.
- 29. Find the point (x, y) on the line $2x + y = 3$ that is closest to the point $(3, 2)$.
- Suggested Problems: 30, 31, 34, 42. [Do not do.]

Section 4.6: Higher Derivatives and Conductivity

- Sketch $f(x) = -\frac{1}{3}x^3 + 5x^2 - 16x + 2004$.
- 4. Compute the first three derivatives of $g(t) = t^2 + \sqrt{t+1}$.
- 13. Compute the first three derivatives of $f(x) = \sin x \cos x$.
- 33. Apply the second derivative test to find the local maxima and minima of $f(x) = x^3 - 3x + 1$, and apply the inflection point test to find all inflection points.
- 35. Apply the second derivative test to find the local maxima and minima of $f(x) = xe^{-x}$, and apply the inflection point test to find all inflection points.
- Suggested Problems: 77, 80, 82, 83, 89. [Do not do.]

Section 4.8: Indeterminate Forms and L'Hôpital's Rule

- 1. Find $\lim_{x\to 1} \frac{x-1}{x^2-1}$.
- 2. Find $\lim_{x\to\infty} \frac{3x-4}{2x-5}$.
- 9. Find $\lim_{x\to 0} \frac{e^x - x - 1}{x^2}$.
- 42. Find $\lim_{x\to 0} \frac{\sqrt{1+3x}-1}{x}$.
- Suggested Problems: 62, 68, 70, 73. [Do not do.]

5 Chapter 5: The Integral

Section 5.2: Antiderivatives and Initial Value Problems

- 1. Evaluate $\int(3x^2 + 2x + 1)dx$.
- 3. Evaluate $\int(1 - 2x^2 + 3x^3)dx$.
- 16. Evaluate $\int(t + 1)^{10}dt$.
- 37. Solve the initial value problem $dy/dx = \sqrt{x}$ when $y(4) = 0$.
- Suggested Problems: 58, 68.

Section 5.3: Elementary Area Computations

- 3. Write $\sum_{j=1}^{5} \frac{1}{j+1}$ in expanded notation.
- 9. Write $1 + 4 + 9 + 16 + 25$ in summation notation.

  For problems 19 and 22, use the following identities: $\sum_{i=1}^{n} 1 = n$, $\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$, $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$.
- 19. Find $\sum_{i=1}^{10}(4i - 3)$.
- 22. Find $\sum_{k=1}^{6}(2k - k^2)$.
- 39. Let R denote the region that lies below the graph $f(x) = 9 - x^2$ on $[0, 3]$ over the interval $[0, 3]$. Calculate both the upper and lower estimates for the area when $n = 5$ (i.e., when there are five equal partitions).
- Suggested Problems: 52, 53. [Do not do.]

Section 5.6: The Fundamental Theorem of Calculus

- 1. Find the average value of $f(x) = x^4$ on $[0, 2]$.
- 13. Evaluate $\int_{-1}^{3} dx$.
- 27. Evaluate $\int_{4}^{8} \frac{1}{x}dx$.

- 33. Rosanne drops a ball from a height of 400ft. Find the ball's average height and its average velocity between the time it is dropped and the time it strikes the ground.
- Suggested Problems: 41, 43, 66. [Do not do.]

Section 5.5: Evaluation of Integrals

- 1. Evaluate $\int_0^1 (3x^2 + 2\sqrt{x} + 3\sqrt[3]{x})dx$.
- 13. Evaluate $\int_{-1}^1 x^{99} dx$.
- 33. Evaluate $\int_0^{\pi/2} \cos(3x)dx$.
- 54. Without evaluating the integral, prove $\frac{\pi}{8} \leq \int_0^{\pi/4} \frac{1}{1+\cos^2 x} dx \leq \frac{\pi}{6}$.
- Suggested Problems: 59, 60. [Do not do.]