Name:

Closed Book

Section:

1. (20 points) Let $\overrightarrow{P} = (1,0,-1)$, $\overrightarrow{Q} = (1,1,1)$ and $\overrightarrow{R} = (1,-2,1)$. Let $f(x,y,z) = xy^4z^9$, $g(x,y,z) = x - y + z$ and $h(u,v) = u^2 - v^2$.

   1. Find the cosine of the angle between $\overrightarrow{P}$ and $\overrightarrow{P}$, and find the equation of the plane containing $\overrightarrow{P}$, $\overrightarrow{Q}$ and $\overrightarrow{R}$.

   2. Compute the following quantities if possible; if not possible, state why not:
      - ⋄ (i) $(\overrightarrow{P} \times \overrightarrow{Q}) \times (\overrightarrow{P} \cdot \overrightarrow{R})$;
      - ⋄ (ii) $(\nabla f)(\overrightarrow{P}) \cdot \overrightarrow{Q}$.
      - ⋄ (iii) $f\left(g(1,2,3), g(2,3,1), g(3,1,2)\right)$.
      - ⋄ (iv) $f\left(\overrightarrow{P}, \overrightarrow{Q}, \overrightarrow{R}\right)$.

   3. You are at the point $\overrightarrow{P}$. In what direction should you move to see $g(x,y,z)$ increase the fastest?

   4. Let $A(u,v) = f(h(u,u), h(u,v), h(v,v))$. Using the Chain Rule, compute $\frac{\partial A}{\partial u}$ at the point $(1,1,1)$. You may check your answer by computing $\frac{\partial A}{\partial u}$ directly, but if you do not do the chain rule you will receive 0 points.

   5. Let $f(x,y) = 3x + y$ and $g(x,y) = x^2 + y^2$. Find the maximum and minimum values of $f(x,y)$ given that $g(x,y) = 1$.