HW Due Fri February 26

Section 2.3

2. Evaluate $\partial z/\partial x$, $\partial z/\partial y$ for given function

b)

$$z = \log\sqrt{1+xy} \ ; \quad (1,2)\ (0,0)$$
$$= \log(1+xy)^{1/2}$$
$$= \frac{1}{2}\log(1+xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{2}\cdot\frac{1}{(1+xy)}\cdot(y) = \frac{y}{2(1+xy)}$$

at $(1,2)$, $\partial z/\partial x = \dfrac{2}{2(1+2)} = \dfrac{2}{6} = \dfrac{1}{3}$

at $(0,0)$, $\partial z/\partial x = 0$

Similarly,

$$\frac{\partial z}{\partial y} = \frac{x}{2(1+xy)}$$

at $(1,2)$ $\partial z/\partial y = 1/6$

at $(0,0)$ $\partial z/\partial y = 0$.

4.

a) $f(x,y) = \dfrac{2xy}{(x^2+y^2)^2} = 2xy \cdot (x^2+y^2)^{-2}$

$$\frac{\partial f}{\partial x} = 2y\cdot(x^2+y^2)^{-2} + 2xy \cdot -2(x^2+y^2)^{-3}\cdot 2x$$

$$= \frac{2y}{(x^2+y^2)^2} + \frac{-8x^2y}{(x^2+y^2)^3}$$

b) $f(x,y) = \frac{x}{y} + \frac{y}{x} = xy^{-1} + yx^{-1}$

$$\frac{\partial f}{\partial x} = \frac{1}{y} + \frac{-y}{x^2} = \frac{x - y^2}{x^2 y}$$

continuous for $x \neq 0$ and $y \neq 0$

$$\frac{\partial f}{\partial y} = \frac{-x}{y^2} + \frac{1}{x} = \frac{y^2 - x^2}{xy^2}$$

continuous for $x \neq 0$.

5. Find equation of plane tangent to the surface

$$z = x^2 + y^3 \quad \text{at } (3,1,10)$$

$\partial z / \partial x = 2x = 3(3) = 9 \quad x_0 = 3,\ y_0 = 1,\ z_0 = f(x_0, y_0) = 10$

$\partial z / \partial y = 3y^2 = 3(1^2) = 3$

$$z = 9(x-3) + 3(y-1) + 10.$$

7. Compute the matrix of partial derivs.

c. $f: \mathbb{R}^3 \to \mathbb{R}^2 \quad f(x,y,z) = (x + e^z + y,\ yx^2)$

$f_1(x,y,z) = x + e^z + y \quad f_2(x,y,z) = yx^2$

$$Df(x,y,z) = \begin{array}{c} \begin{array}{ccc} \partial f_1/\partial x_1 & \partial f_1/\partial y_1 & \partial f_1/\partial z \end{array} \\ \begin{bmatrix} e^z + y & x + e^z & e^z \\ 2xy & x^2 & 0 \end{bmatrix} \end{array}$$